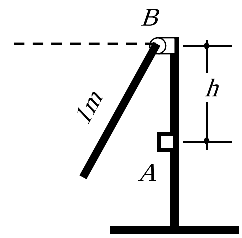
EXAMPLE (6.8)

In material strength testing machine, a testing rod (m= 20 kg, L= 1m) is released from rest from the horizontal position to strikes the sample at A, and rebounds after impact with a speed of 3 rad/sec. Determine the value of the distance h that minimizes the reaction at hinge B during the impact and find the average value of the impact force if the impact time is 0.02 sec.	

SOLUTION:

In this problem, when the testing rod is released from the horizontal position, its velocity will increase during descending till it strikes the sample at ***A***, then it will rebound. To calculate the testing force, the impact of the testing rod with the sample at ***A*** is to be studied.

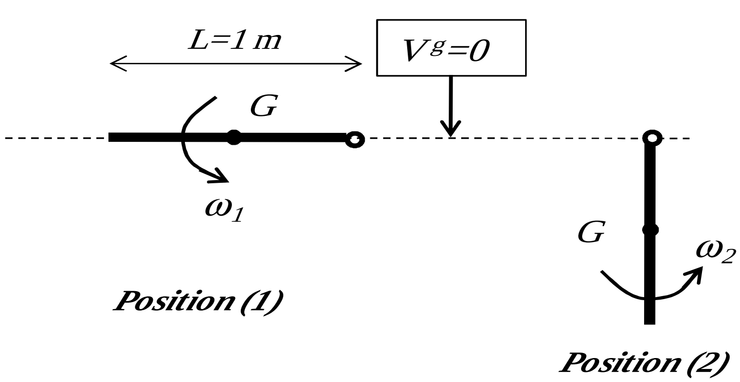
During the impact, the impulsive forces acting on the rod are:

- Horizontal reaction at sample (***A***);
- Two components of reactions at hinge ***B.***

Since these forces are represent 3-unknowns, then all other velocity components of the rod just before impact and just after impact must be known, i.e. either given or calculated.

The velocities of the rod just before impact:

To obtain the velocities of the rod just before striking the sample, the energy principles may be used. Applying the principle of mechanical energy (***E=T+V)*** and work of nonconservative forces $W_{1-2}^{N.C.F}$ between pos.(1) and pos. (2):



The principle of mechanical energy and work of nonconservative forces (reaction at B) between pos.(1) and pos. (2):

$$T_1 + V_1 + W_{1-2}^{N.C.F} = T_2 + V_2$$

where:

$$T_1 = 0 \; , \; V_1 = 0 \text{ and } W_{1-2}^{N.C.F} = 0$$

$$T_2 = \frac{1}{2} I_B \omega_2^2 \; , V_2 = mg\left(-\frac{L}{2}\right)$$

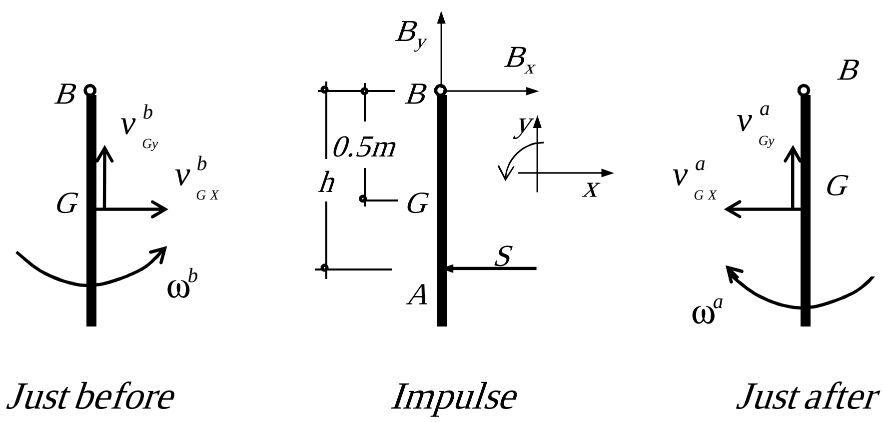
$$I_B = \frac{mL^2}{3} = \frac{(20)(1)^2}{3} = 6.667 \;\; kg.m^2$$

$$\therefore \; \omega_2 = \left(\frac{3g}{L}\right)^{0.5} = \left(\frac{30}{1}\right)^{0.5} = 5.477 \, rad / sec \; (c.w.)$$

Note that: the nonconservative forces are the reactions components at ***B*** which do no work as the point ***B*** has no displacement and the ground acceleration *g=10 m/sec.*

The impact equations:

To derive the impact equations, the impact figures is to be drawn as follow:



The impact figures

In the x-direction:

$$\rightarrow \quad \sum S_X = m\left[v_{GX}^a - v_{GX}^b\right]$$

$$\therefore \quad -S + B_x = m\left(v_{GX}^a - v_{GX}^b\right) = m\left(-\frac{L}{2}\omega^a - \frac{L}{2}\omega^b\right) = -84.77 \qquad (1)$$

In the y-direction:

$$\uparrow \quad \sum S_y = m\left[v_{Gy}^{a} - v_{Gy}^{b}\right]$$

$$\therefore \quad B_y = m\left(v_{Gy}^{a} - v_{Gy}^{b}\right) = m\left(0-0\right) = 0 \qquad (2)$$

Moment equation about G:

$$c.w. \quad \sum M_S^B = I_B\left[\omega^a - \omega^b\right]$$

$$(c.w.) \quad S\left(h\right) = \left(\frac{mL^2}{3}\right)\left(\omega^a + \omega^b\right) = 56.513$$

$$\therefore S = \frac{56.513}{h} \qquad (3)$$

Equations Solving:

From Eq. (1) and (3):

$$B_X = S - 169.54h = \frac{56.513}{h} - 84.77$$

$$\therefore B_X = 0 \ \ when \ \ \frac{56.513}{h} = 84.77 \rightarrow \text{for min. reaction } B_x \ \rightarrow \ \therefore h = 0.667\,m$$

From Eq. (3):

$$\therefore S = \frac{56.513}{h} = 84.77$$

The average value of impulsive force at A:

$$\therefore S = \left(F_{imp}\big|_{av}\right)\cdot \Delta t \ \rightarrow \ F_{imp}\big|_{av} = \frac{S}{\Delta t} = \frac{84.77}{0.02} = 4238.5\,N$$

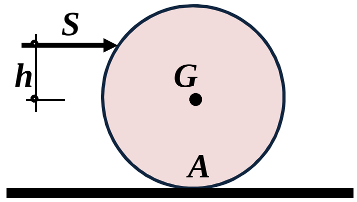
The loss in rod kinetic energy

The loss in the rod kinetic energy due to impact is given by:

$$\Delta T = T_b - T_a = \frac{1}{2}I_B(\omega_b^2 - \omega_a^2) = \frac{1}{2}\left(\frac{20(1)^2}{3}\right)\left((5.477)^2 - (3)^2\right) = 70 \text{ Joule}$$

EXAMPLE (6.9):

A sphere (m=25 kg, r=2 m, k_G = (2/5) mr^2 $kg.m^2$) resting on a rough horizontal plane (μ=0.6), is struck by a horizontal blow S=500 N.sec along a line in the vertical plane passes through the centre G and at a distance h above the centre. Find the distance h that makes the sphere starts motion without sliding (i.e. starts its motion by rolling). Also calculate the kinetic energy of the sphere after impact.



SOLUTION:

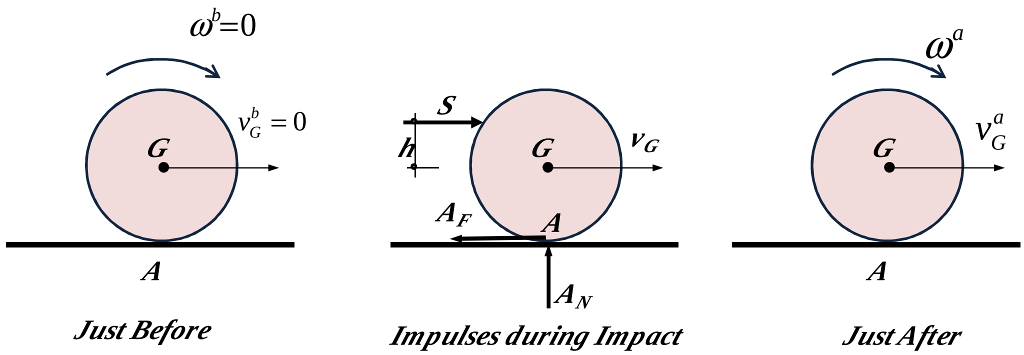
To solve the problem using impact equations, one should identify the impulsive forces and draw the impact diagrams first, and then derive the impact equations and solve it to obtain the required unknowns.

Due to the impulse ***S***, two impulses due to impulsive forces will be exerted at the point of contact ***A***, one is the impulse of the normal reaction $\boldsymbol{A_N}$ and the other is the impulse of the friction force $\boldsymbol{A_F}$, where the rolling condition is:

$$A_F \leq \mu_S A_N$$

During the impact, the impulsive forces acting on the sphere are:

- The impulse ***S***;
- The impulses $\boldsymbol{A_N}$ and $\boldsymbol{A_F}$ between the sphere and the horizontal surface.



Impact diagrams

The Impact equations:

In the x-direction:

$$\rightarrow \quad \sum S_x = m\left[v_{Gx}^{a} - v_{Gx}^{b}\right]$$

$$\therefore \quad 500 - A_F = 25\left[v_{Gx}^{a} - 0\right] \qquad (1)$$

In the y-direction:

$$\uparrow \quad \sum S_y = m\left[v_{Gy}^{a} - v_{Gy}^{a}\right]$$

$$\therefore \quad A_N = m\left[0 - 0\right] \qquad (2)$$

Moment equation about G:

$$c.w. \quad \sum M_S^G = I_G\left[\omega^a - \omega^b\right]$$

$$\therefore \quad S\left(h\right) + A_F\left(r\right) = I_G\left[\omega^a - 0\right] \qquad (3)$$

Equations Solution:

Note that the sphere moment of inertia : $I_G = \frac{2}{5} m r^2 = \frac{2}{5}(25)(2)^2 = 40 \ kg.m^2$

From Eq. (2):

$$A_N = 0 \quad \rightarrow \therefore A_F = 0$$

From Eq. (1):

$$\therefore \quad 500 - 0 = 25\left[v_{Gx}^{a} - 0\right] \quad \rightarrow \quad v_{Gx}^{a} = v_G = \frac{500}{25} = 20 \ m/sec$$

From Eq. (3):

$$\therefore \quad Sh - 0 = 40\left[\omega^a - 0\right] = 40\omega^a \ \rightarrow \ \therefore h = \frac{40\omega^a}{500} \qquad (4)$$

Rolling Condition:

$$v_A = 0 \quad \rightarrow \quad \therefore \ v_G^a = \omega^a \ r \ \rightarrow \therefore \quad \omega^a = \frac{v_G^a}{r} = \frac{20}{2} = 10 \ rad/sec \qquad (5)$$

Sub. Eq. (5) into Eq. (4):

$$\therefore \quad h = \frac{40\omega^a}{500} = \frac{40(10)}{500} = 0.8 \ m$$

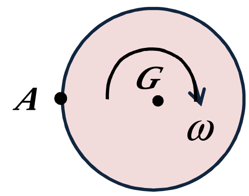
Sphere kinetic energy:

The ball will make general plane motion after impact:

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \ \omega^2 = 0.5(25)(20)^2 + 0.5(40)(10)^2 = 7000 \text{ Joule}$$

EXAMPLE(6.10):

A circular disc (m=16 kg, r=0.4 m) is rotating in its plane about its center ***G*** with angular velocity ω=10 rad/sec (c.w.). Suddenly, a point ***A*** on its perimeter is set to be fixed. Calculate the angular velocity of the disk just after ***A*** fixation and the loss in the kinetic energy of the disk due to the sudden fixation of ***A***.

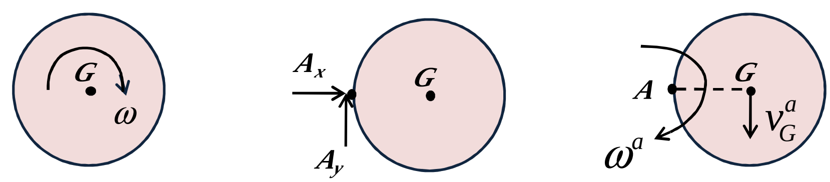


SOLUTION

When a point in a moving rigid body is set to be fixed suddenly, an impulsive force with unknown direction exerts at that point. We can solve the equations to obtain the impulse of this force and its direction, but solving equations contains unknown angles is difficult and it is recommended to replace this force by two perpendicular components in ***x*** and ***y*** directions. Then the unknowns are the two impulses of the components of fixation force and the angular velocity of the disc after fixation.

During the impact, the impulsive forces acting on the disc are:

- The impulses components $\boldsymbol{S_X}$ and $\boldsymbol{S_y}$;



Just Before ***Impulses during Impact*** ***Just After***

Impact diagrams

The Impact equations:

In the x-direction:

$$\rightarrow \quad \sum S_x = m\left[v_{Gx}^{a} - v_{Gx}^{b}\right]$$

$$\therefore \quad A_X = 16[0-0] \qquad (1)$$

In the y-direction:

$$\uparrow \quad \sum S_y = m\left[v_{Gy}^{a} - v_{Gy}^{a}\right]$$

$$\therefore \quad A_y = m\left[-v_G^{a} - 0\right] \qquad (2)$$

Moment equation about G:

$$c.w. \quad \sum M_S^G = I_G\left[\omega^a - \omega^b\right]$$

$$\therefore \quad A_y(0.4) = \frac{1}{2}(16)(0.4)^2\left[\omega^a - 10\right] \qquad (3)$$

Equations Solution:

From Eq. (1):

$A_x = 0$

From kinematics:

$$v_G^a = r\,\omega^a \qquad (4)$$

Sub. in Eq. (2)

$$\therefore \quad A_y = -16(0.4)\omega^a = -6.4\,\omega^a \qquad (5)$$

Sub. in Eq. (3)

$$\therefore \quad \left(-6.4\,\omega^a\right)(0.4) = \frac{1}{2}(16)(0.4)^2\left[\omega^a - 10\right] = 1.28\,\omega^a - 12.8$$

$$\therefore \quad -2.56\,\omega^a = 1.28\,\omega^a - 12.8 \;\rightarrow\; \therefore\; \omega^a = 3.333 \text{ rad/sec}$$

Loss in kinetic energy:

The disc rotates with ω^b about ***G*** before impact and will make rotational motion about ***A*** with ω^a after impact, then the loss in its kinetic energy is given by:

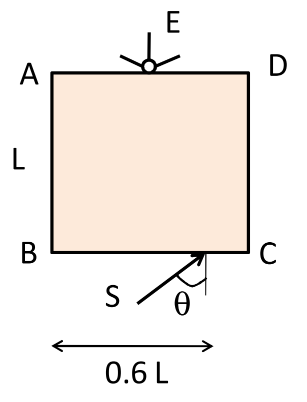
$$\Delta T = T_1 - T_2 = \frac{1}{2}I_G\,\omega^b - \frac{1}{2}I_A\,\omega^a = 0.5\left(\frac{1}{2}(16)(0.4)^2\right)(10)^2 - 0.5\left(\frac{3}{2}(16)(0.4)^2\right)(3.33)^2$$

$$= 64 - 21.333 = 42.667 \text{ Joule}$$

EXAMPLE(6.11):

A square plate ***ABCD*** (m=300 kg, L=2 m) free to rotate in a vertical plane about a hinge at point ***E***; the midpoint of its side ***AD***. An impulse ***S***=800 N.sec strikes the plate as shown (θ=45), calculate:

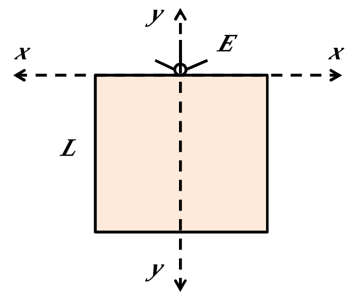
- The polar moment of inertia I_E .
- The impulse of reactions at ***E***.
- ω and ***T*** of the plate just after impulse.
- The rotation angle of the plate due to the impact θ_1.
- The impulse ***S*** to rotate the plate such that the side DC just reaches the horizontal position.



SOLUTION

1) The polar moment of inertia I_E

The polar moment of inertia I_E is the sum of the moment of inertia about two perpendicular axes passing through the point E, say the selected axes x-x and y-y as shown in the opposite figure.



$$I_E = I_{xx} + I_{yy} = \frac{1}{3} m L^2 + \frac{1}{12} m L^2 = \frac{5}{12} m L^2$$

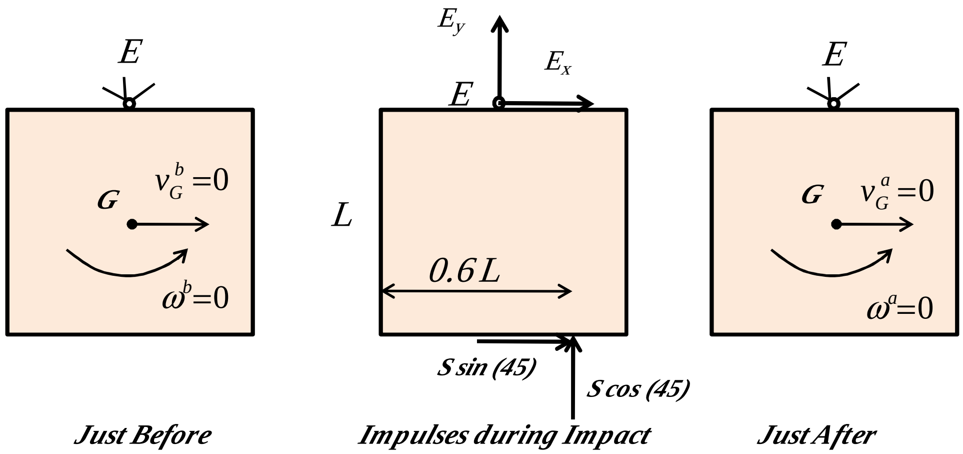
$$= \frac{5}{12}(300)(2)^2 = 500 \quad kg.m^2$$

2) The impulse of reactions at E.

When the blow ***S*** struck the plate, an impulsive reactions will be exerted at point ***E***, this impulsive reaction can replaced by two perpendicular components, say in ***x*** and ***y*** direction. Then the unknowns are the impulses of the components of impulsive reactions at ***E***, say E_X and E_y and the angular velocity of the plate just after impact ω^a.

During the impact, the impulsive forces acting on the sphere are:

- The given impulse ***S***, replaced by two components $S \sin\theta$ and $S \cos\theta$;
- The impulses of the ***E***-reaction components E_X and E_y.



Impact diagrams

The Impact equations:

In the x-direction:

$$\rightarrow \quad \sum S_X = m\left[v_{Gx}^a - v_{Gx}^b\right]$$

$$\therefore \quad \frac{S}{\sqrt{2}} + E_X = 300\left[v_G^a - 0\right] \qquad (1)$$

In the y-direction:

$$\uparrow \quad \sum S_y = m\left[v_{Gy}^a - v_{Gy}^a\right]$$

$$\therefore \quad \frac{S}{\sqrt{2}} + E_y = m\left[0 - 0\right] \qquad (2)$$

Moment equation about G:

$$c.c.w. \quad \sum M_S^E = I_E\left[\omega^a - \omega^b\right]$$

$$\therefore \quad \frac{S}{\sqrt{2}}(L) + \frac{S}{\sqrt{2}}(0.1L) = I_E\left[\omega^a - 0\right] \qquad (3)$$

Equations Solution:

From Eq. (2):

$$E_y = -\frac{S}{\sqrt{2}} = -565.685\ N.sec$$

From Eq. (3)

$$\therefore \quad \omega^a = \frac{1.1L\,S}{I_E\sqrt{2}} = 2.489\ \ rad/sec \qquad (4)$$

From kinematics:

$$v_G^a = \left(\frac{L}{2}\right)\omega^a \qquad (5)$$

Sub. in Eq. (1)

$$\therefore \quad E_X = \frac{S}{\sqrt{2}} - 300\left[v_G^a\right] = \frac{800}{\sqrt{2}} - 300\left[(2)(2.489)\right] = -927.715\ N.sec$$

The Impulse at E

$$\therefore \quad E = \sqrt{E_x^2 + E_x^2} = \sqrt{(-927.715)^2 + (-565.685)^2} = 1086.6\ N.sec$$

3) The kinetic energy:

The plate makes rotational motion about ***E***, then:

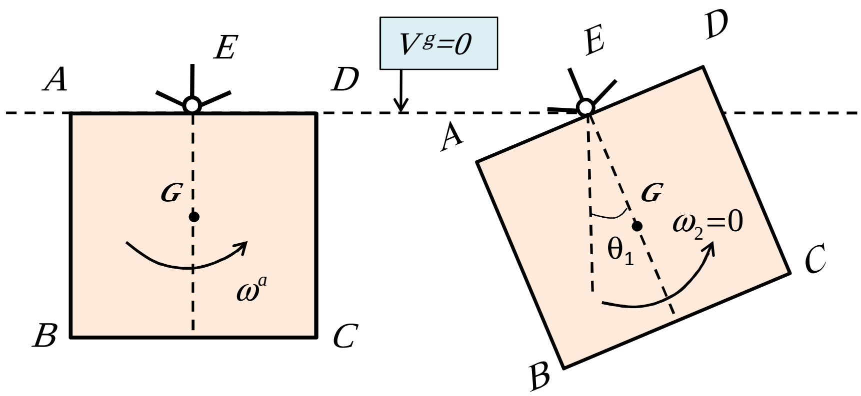
$$T^a = \frac{1}{2} I_E \left(\omega^a\right)^2 = 0.5(500)(2.489)^2 = 1548.78\ \text{Joule}$$

4) The angle of rotation

As the plate rotates after impact, its angular velocity will decrease gradually till it stop and then returns back to its original position. The maximum angle reached by the plate is called the angle of rotation due to impact. Using principle of work and energy, considering pos.(1) is the position of the plate just after impact pos. (2) is the position where the plate stops.

The non-conservative forces acting on the plate during moving from pos. (1) to pos. (2) are the reactions at E and its work equals zero as the point ***E*** has no displacement.

The angle of rotation θ_1 may be obtained as follow:



Just after Impact Pos. (1) ***When the plate stopped Pos. (2)***

The work - Energy principle is:

$$T_1 + V_1 + W_{1-2}^{N.C.F} = T_2 + V_2$$

$$\therefore \frac{1}{2} I_E \omega_a^2 + mg\left(-\frac{L}{2}\right) + 0 = 0 + mg\left(-\frac{L}{2} cos\left(\theta_1\right)\right)$$

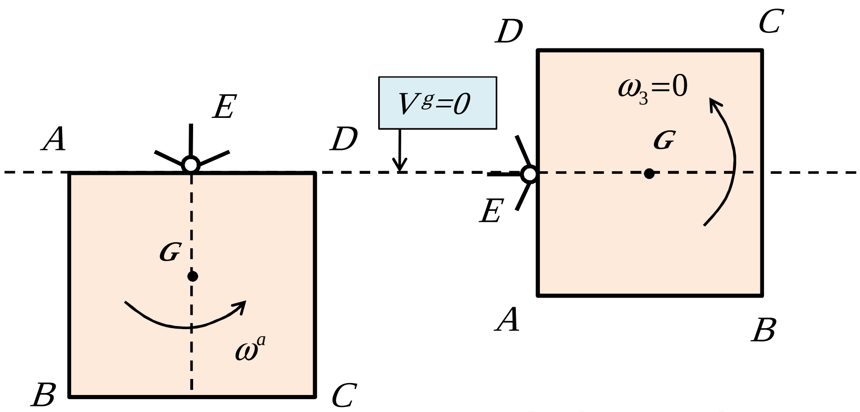
$$\therefore \frac{1}{2} I_E \omega_a^2 + mg\left(-\frac{L}{2}\right) + 0 = 0 + mg\left(-\frac{L}{2} cos\left(\theta_1\right)\right)$$

$$\therefore\ cos\left(\theta_1\right) = \frac{I_E \omega_a^2 - mgL}{-mgL} = \frac{(500)(2.489)^2 - (300)(10)(2)}{-(300)(10)(2)} = 0.4837$$

$$\therefore\ \theta_1 = cos^{-1}(0.4837) = 61.07^o$$

*5) The value of **S** so that DC reaches the horizontal position*

When the blow ***S*** increases the angle of rotation of the plate due to impact increases. Using the principle of work and energy, considering pos.(1) is the position of the plate just after impact and pos. (3) is position when the plate stops so that the side ***DC*** reaches the horizontal position, then the value of the angular velocity of the plate can be calculated and then using the impact equations the value of the impulse ***S*** can be calculated as follow:



Just after Impact
Pos. (1)

The plate stops when DC reaches the horizontal position
Pos. (3)

The work - Energy principle is:

$$T_1 + V_1 + W_{1-2}^{N.C.F} = T_3 + V_3$$

$$\therefore \frac{1}{2} I_E \omega_a^2 + mg\left(-\frac{L}{2}\right) + 0 = 0 + 0 = 0 \rightarrow \therefore \omega_a^2 = \frac{mgL}{I_E} = \frac{6000}{500} = 12$$

$$\therefore \omega^a = \sqrt{12} = 3.464 \ rad/sec$$

Sub. in Eq. (4):

$$\therefore \omega^a = \frac{1.1L\,S}{I_A \sqrt{2}} \rightarrow \therefore S = \frac{\omega^a I_A \sqrt{2}}{1.1L} = 1113.4 \ N.sec$$