Game theory

Lecture 8

Introduction

- Life is full of conflict and competition. Numerous examples involving adversaries in conflict include political campaigns, advertising and marketing campaigns by competing business firms, and so forth.
- A basic feature in many of these situations is that the final outcome depends primarily upon the combination of strategies selected by the adversaries.

Introduction

- Game theory is a mathematical theory that deals with the general features of competitive situations like these in a formal, abstract way.
- It places particular emphasis on the decision-making processes of the adversaries.
- Game theory has applications in a variety of areas, including in business and economics.

Introduction

- The 1994 Nobel Prize for Economic Sciences was won by John F. Nash, Jr. (whose story is told in the movie *A Beautiful Mind),* John C. Harsanyi, and Reinhard Selton for their analysis of equilibria in the theory of noncooperative games.

Application

- the focus in this Lecture is on the simplest case, called **two-person, zero-sum games. As the name implies,** these games involve only two adversaries or *players (who may be teams, firms, and so* on).
- They are called *zero-sum games because one player wins whatever the other one* loses, so that the sum of their net winnings is zero.

Two-person, zero-sum games

- To illustrate the basic characteristics of two-person, zero-sum games, consider the game called *odds and evens.*
- This game consists simply of each player simultaneously showing either one finger or two fingers. If the number of fingers matches, so that the total number for both players is even, then the player taking evens (say, player 1) wins the bet (say, $1) from the player taking odds (player 2).

Two-person, zero-sum games

- A primary objective of game theory is the development of *rational criteria for selecting* a strategy. Two key assumptions are made:
  - *Both players are rational.*
  - *Both players choose their strategies solely to promote their own welfare (no compassion* for the opponent).

Example

- Two politicians are running against each other for the U.S. Senate. Campaign plans must now be made for the final two days, which are expected to be crucial because of the closeness of the race.
- Therefore, both politicians want to spend these days campaigning in two key cities, Bigtown and Megalopolis. To avoid wasting campaign time, they plan to travel at night and spend either one full day in each city or two full days in just one of the cities.

Example

- Therefore, each politician has asked his campaign manager in each of these cities to assess what the impact would be (in terms of votes won or lost) from the various possible combinations of days spent there by himself and by his opponent.
- He then wishes to use this information to choose his best strategy on how to use these two days.

Example

- To formulate this problem as a two-person, zero-sum game, we must identify the two *players (obviously the two politicians), the strategies for each player, and the payoff table.*
- As the problem has been stated, each player has the following three strategies:
  - Strategy 1 spend one day in each city.
  - Strategy 2 spend both days in Bigtown.
  - Strategy 3 spend both days in Megalopolis.

Example

- Each entry in the payoff table for player 1 represents the *utility to player 1 (or the* negative utility to player 2) of the outcome resulting from the corresponding strategies used by the two players.
- From the politician's viewpoint, the objective is to *win votes,* and each additional vote (before he learns the outcome of the election) is of equal value to him.

Example (alternative 1)

- three alternative sets of data for the payoff table to illustrate how to solve three different kinds of games.

Strategy		Player 2		
		1	2	3
Player 1	1	1	2	4
	2	1	0	5
	3	0	1	−1

The dominant strategy