# Applications of LP (Assignment problem)

Lecture 6

# Introduction

- The **assignment problem** is a special type of linear programming problem where assignees are being assigned to perform tasks.
- For example, the assignees might be employees who need to be given work assignments.
- Assigning people to jobs is a common application of the assignment problem. However, the assignees need not be people. They also could be machines, or vehicles, or plants, or even time slots to be assigned tasks.

# Assignment problem assumptions

- The number of assignees and the number of tasks are the same. (This number is denoted by *n.)*
- Each assignee is to be assigned to exactly *one task.*
- Each task is to be performed by exactly *one assignee.*
- There is a cost *cij associated with assignee i (i = 1, 2, . . . , n) performing task j ( j = 1, 2, . . . , n).*
- The objective is to determine how all *n assignments should be made to minimize the* total cost.

# Assignment problem model

By letting $Z$ denote the total cost, the assignment problem model is

$$\text{Minimize} \qquad Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij},$$

subject to

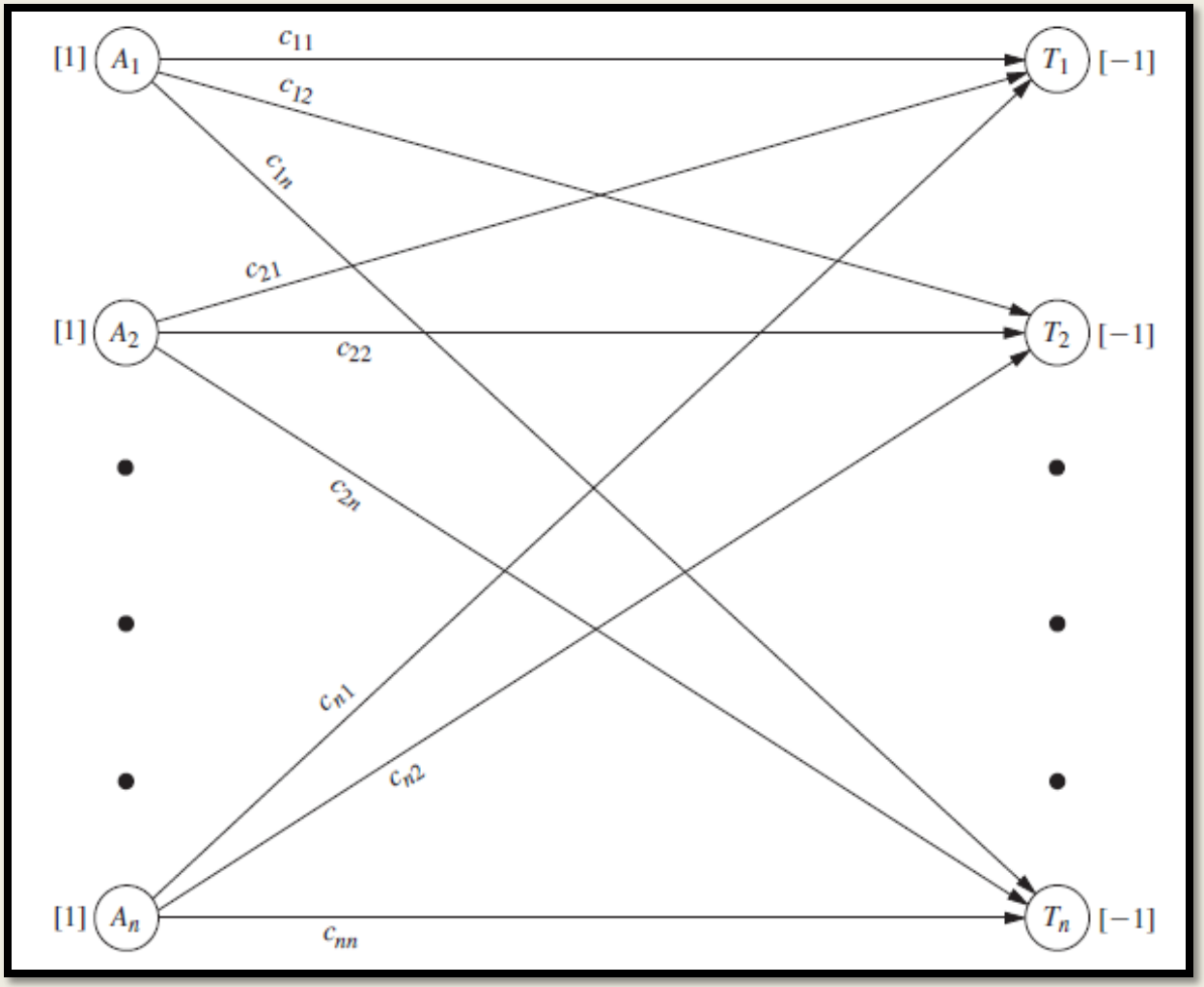
$$\sum_{j=1}^{n} x_{ij} = 1 \qquad \text{for } i = 1, 2, \ldots, n,$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad \text{for } j = 1, 2, \ldots, n,$$

and

$$x_{ij} \geq 0, \qquad \text{for all } i \text{ and } j$$
$$(x_{ij} \text{ binary}, \qquad \text{for all } i \text{ and } j).$$

# Network representation of the assignment problem



# Example

| | | Unit Cost ($) for Product | | | | Capacity Available |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | |
| Plant | 1 | 41 | 27 | 28 | 24 | 75 |
| | 2 | 40 | 29 | — | 23 | 75 |
| | 3 | 37 | 30 | 27 | 21 | 45 |
| Production rate | | 20 | 30 | 30 | 40 | |

Can be solved by assignment model and a transportation mode, how?

# Example

| | | Cost per Unit Distributed | | | | | |
|---|---|---|---|---|---|---|---|
| | | Destination (Product) | | | | | |
| | | 1 | 2 | 3 | 4 | 5(*D*) | Supply |
| *Source (Plant)* | 1 | 41 | 27 | 28 | 24 | 0 | 75 |
| | 2 | 40 | 29 | *M* | 23 | 0 | 75 |
| | 3 | 37 | 30 | 27 | 21 | 0 | 45 |
| Demand | | 20 | 30 | 30 | 40 | 75 | |

| | | Task (Product) | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5(*D*) |
| *Assignee (Plant)* | 1*a* | 820 | 810 | 840 | 960 | 0 |
| | 1*b* | 820 | 810 | 840 | 960 | 0 |
| | 2*a* | 800 | 870 | *M* | 920 | 0 |
| | 2*b* | 800 | 870 | *M* | 920 | 0 |
| | 3 | 740 | 900 | 810 | 840 | *M* |