Linear programming (Simplex method)

Lecture 2

Introduction

- Briefly, the most common type of application involves the **general problem of allocating limited resources among competing activities in a best possible** (i.e., optimal) way.

- A remarkably efficient solution procedure, called the simplex method, is available for solving linear programming problems of even enormous size.

Introduction

$$\text{Maximize} \quad Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n,$$

subject to the restrictions

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\le b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\le b_2 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\le b_m,
\end{aligned}$$

and

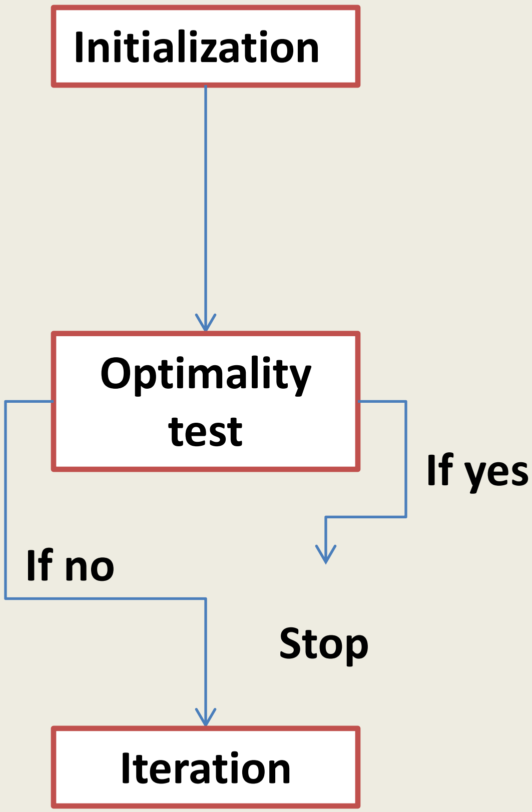
$$x_1 \ge 0, \quad x_2 \ge 0, \quad \ldots, \quad x_n \ge 0.$$

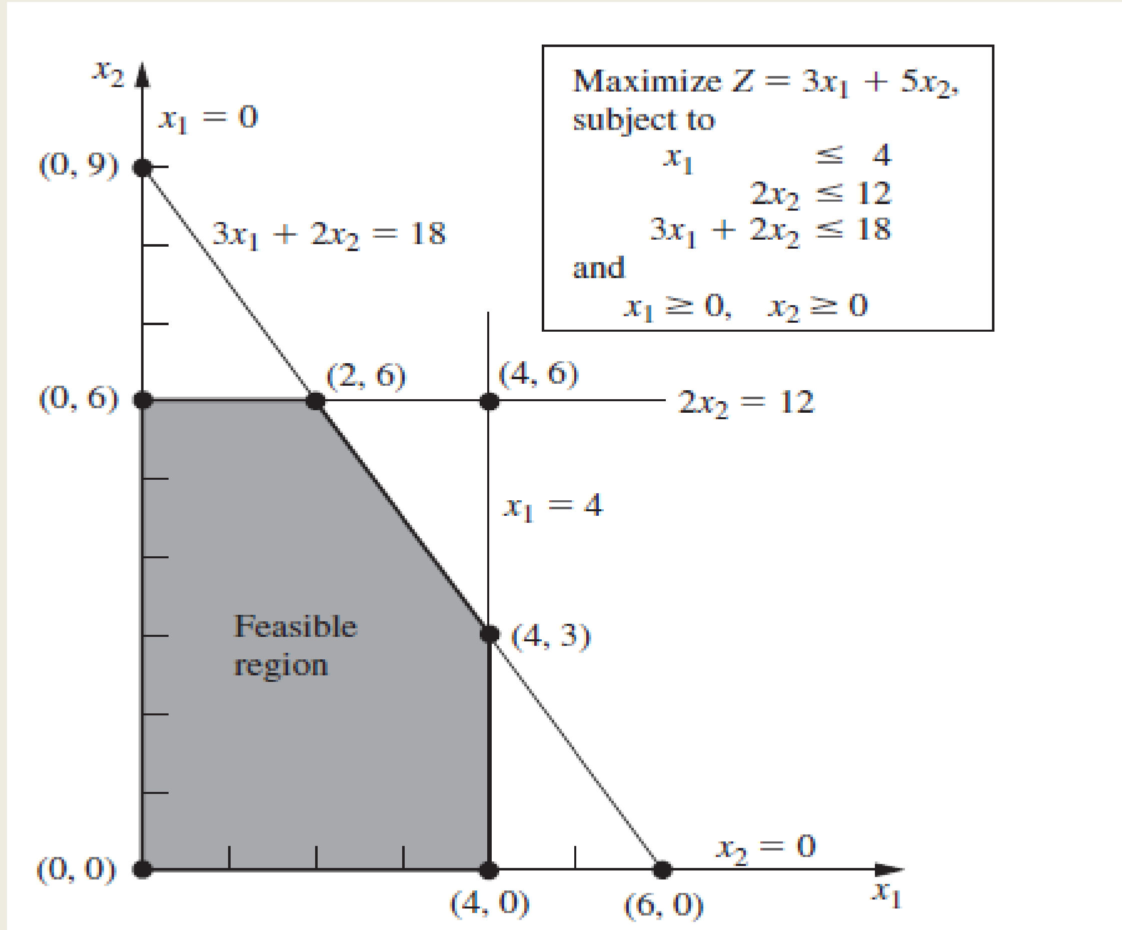
The Simplex Method

- A general procedure for solving linear programming problems. Developed by George Dantzig in 1947, it has proved to be a remarkably efficient method that is used routinely to solve huge problems on today's computers.

- The simplex method is an algebraic procedure with underlying geometric concepts.

Geometry of Simplex method





Algebra of Simplex method

1. Convert inequality to equality (add slack variable)
2. Set up the model in the following **augmented form**

Maximize Z,

subject to

$$
\begin{aligned}
&(0) \quad Z - 3x_1 - 5x_2 &&= 0\\
&(1) \quad x_1 + x_3 &&= 4\\
&(2) \quad 2x_2 + x_4 &&= 12\\
&(3) \quad 3x_1 + 2x_2 + x_5 &&= 18
\end{aligned}
$$

and

$x_j \geq 0, \quad$ for $j = 1, 2, \ldots, 5.$

The Simplex method Tabular form

- Lets solve the following problem using the tabular form

Original Form of the Model	*Augmented Form of the Model*[4]
Maximize $Z = 3x_1 + 5x_2$,	Maximize $Z = 3x_1 + 5x_2$,
subject to	subject to
$x_1 \leq 4$	(1) $x_1 + x_3 = 4$
$2x_2 \leq 12$	(2) $2x_2 + x_4 = 12$
$3x_1 + 2x_2 \leq 18$	(3) $3x_1 + 2x_2 + x_5 = 18$
and	and
$x_1 \geq 0, \quad x_2 \geq 0.$	$x_j \geq 0, \quad$ for $j = 1, 2, 3, 4, 5.$