Lecture 1:

Introduction to LP theory and applications

What is Operations Research?

- **World War II : British military leaders asked scientists and engineers to analyze several military problems**
  - **Deployment of radar**
  - **Management of convoy, bombing, antisubmarine, and mining operations.**

What is Operations Research?

Operations Research (O.R.) is the discipline of applying advanced analytical methods to help make better decisions.

The value of Operation Research?

- Making sense of data
  - big data
  - social network info, transactional data, polls
- Dealing with complexity and uncertainty
  - understanding systems
  - making good choices in an uncertain world
- Using mathematical models to augment our own thinking.
  - develop insights
  - develop plans

Applications

- **Personal choices**
  - best career choices
  - best use of our time
  - best strategies
- **Company choices**
  - maximize value to shareholders
  - determine optimal mix of products or services
  - minimize production costs
  - minimize cost of getting product to customers

Phases for implementing OR

- **Definition of the problem**
  - This function should be carried out by the entire OR team.
- **Construction of the model**
  - translate the problem definition into mathematical relationships.
- **Solution of the model**
  - the use of well-defined optimization algorithm
- **Validation of model**
  - checks whether or not the proposed model does what it purports to do-that is, does it predict adequately the behaviour of the system under study?

The Optimization model

- A mathematical program is a *linear program (LP)* if the objective is a linear function and the constraints are linear equalities or inequalities.

$$\text{e.g.,} \quad 3x_1 + 4x_2 - 3x_4 \geq 7$$
$$x_1 - 2x_5 = 7$$

A ***non-linear program*** is permitted to have a non-linear objective and constraints.

- maximize $f(x,y) = xy$
- subject to $x - y^2/2 \geq 10$
  $3x - 4y \leq 2$
  $x \geq 0, y \geq 0$

The Optimization model

- **Decision variables:** the elements that are under the control of the decision maker.

- **A single objective function (of the decision variables)**
  - minimize cost
  - maximize return
  - make the last semester as enjoyable as possible

The Optimization model

- **Constraints:** restrictions on the decision variables

– "**Business rules**"
  - no worker can work more than 5 consecutive days
  - There is at most 2% investment in any stock in the portfolio

– "**Physical laws**"
  - No worker can work a negative amount of time
  - The amount of a goods in inventory at the end of period j is the amount of goods arriving during period j plus the amount of goods in inventory in period j-1 minus the amount of goods that are sold in the period.

Example

- A farm produces corn and soybeans from two raw materials (seeds and fertilizers)

- The daily usage of seeds is 6 kilo of per ton of corn and 4 kilo per ton soybean with daily availability of 24. The daily usage of fertilizers is 1 kilo of fertilizer per ton of corn and 2 tons per ton soybean with daily availability of 6

- The price of Corn is 5 thousand LE and the price of Soybean is 4 thousand LE

Properties of the LP Model

- **Proportionality**
  - This property requires the contribution of each decision variable in both the objective function and the constraints to be directly proportional to the value of the variable.
- **Additivity**
  - requires the total contribution of all the variables in the objective function and in the constraints to be the direct sum of the individual contributions of each variable.
- **Certainty**
  - All the objective and constraint coefficients of the LP model are deterministic (not stochastic LP and sensitivity analysis).