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Composite right/left-handed circular meta-waveguide

Tamer M. Abuelfadl

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Abstract A new realization of a cylindrical meta-waveguide is proposed. The guide is a metallic cylinder loaded azimuthally with conducting rods short circuited to the outer cylinder. It can be simplified to a planar periodically cascaded coupled lines. However, the actual circuit is described in cylindrical coordinates as screw or helically periodic structure. The dispersion relation was obtained from the circuit model and from the actual structure using EM simulation, with a good agreement between both. An 8-cell structure was simulated to show the overall guide transmission characteristic. The major advantage of this new guide is that it does not contain any dielectrics, which makes it favorable in applications involving electron beams such as Backward Wave Oscillator BWO, Gyrotron BWO, and Cherenkov backward wave detector.

1 Introduction

Left-Handed (LH) transmission lines and waveguides are the 1D version of metamaterial [1], where the guided wave has opposite phase and group velocities. So, they are also named backward wave transmission lines. They are usually realized on conventional Right-Handed (RH) lines using series capacitors and shunt inductors [2]. Hence, they exhibit RH characteristic on a portion of their operating frequency and a LH characteristic on another portion, and so they are named Composite Right/Left-Handed CRLH transmission lines [1]. They can be realized using lumped or semi-distributed elements [3]. Coupled lines have been also employed to achieve the CRLH transmission lines [4–6]. The LH waveguide version has been realized using dielectric filled corrugations in rectangular waveguide and named metaguide or meta-waveguide [7].

In this paper, a CRLH circular meta-waveguide that has the first (dominant) mode propagating as a backward wave or LH is realized by periodically loading a circular waveguide with coupled lines. The loading is done by azimuthal placement of conducting rods close to the circular guide surface. These rods act as coupled lines and the outer conducting cylinder as ground.

This paper is organized as follows: Sect. 2 presents the equivalent circuit of the short-circuited ideal coupled lines, and their dispersion relation. In Sect. 3 the actual metaguide which is a circular waveguide loaded with coupled conducting rods is described, and the screw or helical periodicity of the structure is demonstrated. The dispersions relations and the transmission characteristics are discussed in Sect. 4. The main conclusions are presented in Sect. 5.

2 Composite right/left-handed transmission lines using coupled lines

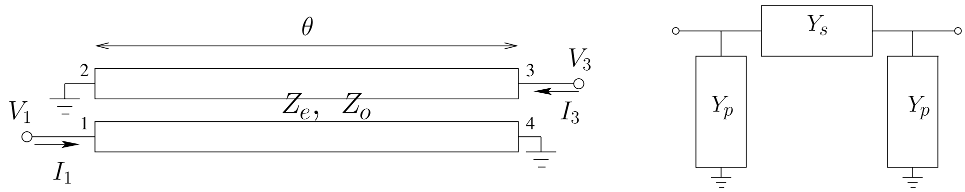
The proposed CRLH line is a periodic structure. The unit cell for such a structure shown in Fig. 1(a) can be obtained from the simple coupled line by terminating two ports (port 2 and 4 in Fig. 1(a)) with short circuit. The resulting 2-port admittance matrix written in terms of the 4-port network Y-parameters [8],

$$\begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{13} \\ Y_{31} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \end{bmatrix}, \quad (1)$$

where $Y_{11} = Y_{33} = -j(Y_o + Y_e)\cot\theta/2$, $Y_{13} = Y_{31} = -j(Y_o - Y_e)\csc\theta/2$, and $\theta = kL$ is the electrical length

T.M. Abuelfadl (✉)
Electronics and Electrical Communications Dept.,
Faculty of Engineering, Cairo University, Giza, Egypt
e-mail: telfadl@ieee.org

Fig. 1 (**a**) Schematic of ideal coupled lines, and (**b**) equivalent circuit when port 3 and 4 are short circuited



a Ideal Coupled Lines. **b** Equivalent Circuit

of the coupled lines of physical length L. The considered coupled line has no dielectric filling, hence its propagation factor $k = \omega/c$, where c is the speed of light in vacuum. The parameters Y_o and Y_e, are the odd and even admittance, respectively. The proposed unit cell can be modeled by the Π network shown in Fig. 1(b). In this equivalent circuit the series Y_s and parallel Y_p are given in terms of the elements of the admittance matrix (1) by [9]

$$\begin{aligned} Y_p &= Y_{11} + Y_{13} = -j(Y_e + Y_o)\frac{\cos\theta + \cos\theta_c}{2\sin\theta}, \\ Y_s &= -Y_{13} = j(Y_e + Y_o)\frac{\cos\theta_c}{2\sin\theta}, \end{aligned} \tag{2}$$

where $\cos\theta_c = (Y_o - Y_e)/(Y_o + Y_e)$. The electrical length θ is directly proportional to the operating frequency, so it can be considered as the normalized frequency. In the first range of $\theta < \pi - \theta_c$, the series admittance Y_s in (2) is pure capacitive, while the parallel admittance Y_p is pure inductive, making the cell acts as a left-handed one.

For a unit cell with period d, the dispersion relations and the Bloch impedance can be written as [1, 10],

$$\begin{aligned} \cos\beta d &= 1 + \frac{Y_p}{Y_s} = -\frac{\cos\theta}{\cos\theta_c}, \\ Z_B &= \frac{1}{\sqrt{Y_p(Y_p + 2Y_s)}} = \frac{2|\sin\theta|}{(Y_o - Y_e)\sqrt{1 - \cos^2\theta/\cos^2\theta_c}}. \end{aligned} \tag{3}$$

Both the propagation factor β and the Bloch impedance Z_B, in (3), are real in the first frequency range $\theta_c < \theta < \pi - \theta_c$, and as previously mentioned the periodic line will be left-handed in this range of frequency. The solid curve in Fig. 3(b) shows this dispersion relation between the normalized frequency θ and the propagation factor represented in βd, which is the phase shift per cell in the periodically infinite line.

One problem with this realization in its planar configuration is that the cell periodicity is directly related to the electrical length of the coupled lines ($d = L = \theta/k$, where θ cannot exceed π). So, in order to operate at small frequencies very long lines have to be employed. A solution to this problem is to circularly bend the structure, such that the periodicity of the structure becomes the spacing between the line, where the periodicity now will be accompanied with a rotation by a certain angle, and the structure is called screw or helically periodic.

3 Circular waveguide loaded with rod coupled lines

In principle, the coupled lines geometry can be taken of arbitrary cross section, depending on the required admittances Y_e and Y_o, and the application. As an example, circular rod coupled lines, Fig. 2(a), are considered in this paper, however other types can be treated in a similar fashion. The planar circular rod coupled lines, Fig. 2(a), are bent to form a cylinder as shown in Figs. 1(b), and 1(c), where the ground plan becomes the outer cylinder of the bent structure and the coupled rods are short circuited with small rods connecting between the bent lines and the outer cylinder, which at the same time act as mechanical supports for them.

After bending the structure it became no longer planar but a three dimensional screw or helically periodic structure. As an example, Fig. 2 shows the structure is periodic when we make a simultaneous rotation of 90° and a translation of distance d, respectively, around and along the z-axis. In general, for a screw or helically periodic structure the periodicity is achieved with simultaneous rotation of angle ϕ_0 around the z-axis along the ϕ direction and translation with distance d along the z-axis. Hence, in cylindrical coordinates (ρ, ϕ, z) Floquet theory applied with these simultaneous shift and translation is expressed as [11]

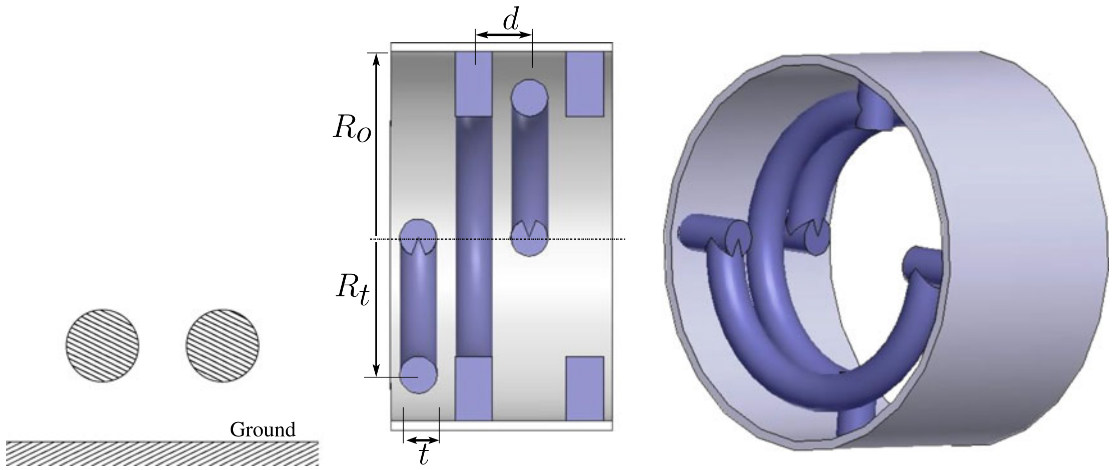
$$\Psi(\rho, \phi + \phi_0, z + d) = e^{-j\beta d}\Psi(\rho, \phi, z), \tag{4}$$

where the Ψ function stands for any of the field components. It can be shown, using (4) and the cylindrical fields expansion in [12], that the fields can be written as

$$\begin{aligned} &\Psi(\rho, \phi, z) \\ &\quad = e^{-j\beta z}\sum_{n,m} a_{n,m} J_n(k_{c_{n,m}}\rho) e^{jn(\phi - z\phi_0/d)} e^{-j2m\pi z/d}, \end{aligned} \tag{5}$$

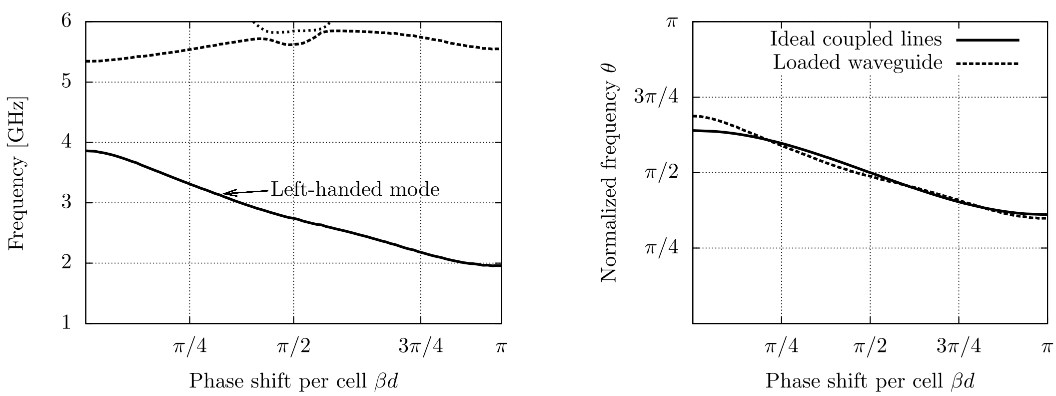
where $k_{c_{n,m}} = \sqrt{k^2 - (\beta + n\phi_0/d + 2m\pi/d)^2}$. In principle, the angle ϕ_0 can have any real value and the structure is still screw or helically periodic. However, in order to have periodicity along z, the angle ϕ_0 must be representable as $\phi_0 = 2\pi p/q$, where $p < q$ and both p and q are integers satisfying, $\gcd(p, q) = 1$. For this case the structure will be periodic along z, with "macro-period" qd.

The coefficients $a_{n,m}$ in (5), at least in principle, can be obtained through applying the boundary conditions on all



a Planar Rode Coupled Lines. **b** Cross Section. **c** 3D View.

Fig. 2 (**a**) Cross section of planar coupled circular rods, (**b**) cross section of cylindrical waveguide loaded with rode coupled lines, and (**c**) 3D view of subfigure (**b**)



a Dispersion Curves. **b** Normalized Dispersion for First Mode.

Fig. 3 (**a**) Dispersion curves of the metaguide obtained using FEM, where the *dotted lines* are for higher modes, and (**b**) normalized dispersion of LH mode of subfigure (**a**) compared to that obtained from circuit

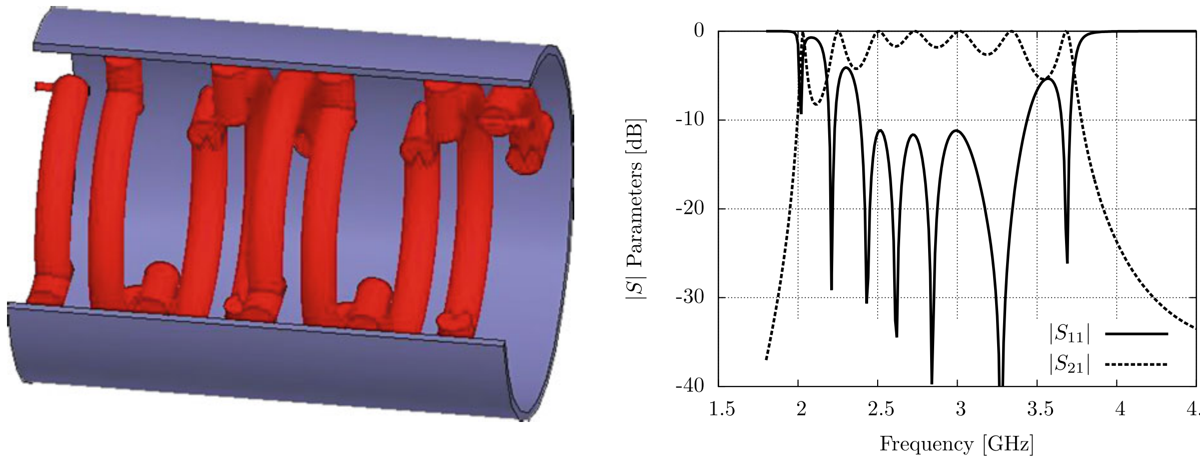
the cell boundaries, however it is not easy for a complicated geometry like ours. Although the expansion (5) is not that helpful by itself in solving the screw periodic structure fields and in obtaining the mode dispersion (ω–β relation), it demonstrates the screw symmetry of the structure fields and it can be used together with other methods of solution such as FEM to extract the different spatial harmonics amplitudes $a_{m,n}$ which are important in studying possible application devices such as Backward Wave Oscillators (BWO) and Gyrotron BWO [11].

4 Results and discussion

In this section we present the dispersion characteristics for the proposed coupled line loaded cylindrical waveguide, both from the circuit analysis discussed in Sect. 2 and from the screw periodic guide described in Sect. 3, where in the latter case the electromagnetic problem is solved using the FEM HFSS code. The dispersion is obtained from the analysis of one unit cell of the guide, however the multi-cell guide composed from 8-cells is also solved numerically to see how the overall structure will behave. The unit cell used, see Fig. 2, is composed of a hosting conducting cylinder with radius $R_o = 2$ cm, loaded with rod coupled lines that form quarter of a torus. The torus radius is $R_t = 1.5$ cm, the rod diameter is $t = 4$ mm and the cell period is $d = 6$ mm.

4.1 Dispersion characteristic

In order to get the dispersion characteristic from the planar circuit, the coupled line parameters Y_o and Y_e need to be calculated. The cross section of the planar coupled lines considered in this calculations is shown in Fig. 2(a). The rod



a 8-Cells Backward Wave Waveguide (inside view). **b** $|S|$ Parameters.

Fig. 4 (**a**) 3D inside view of 8-cells metaguide, and (**b**) $|S|$ parameters obtained by FEM

diameter taken is $t = 4$ mm, the spacing is $d = 6$ mm, and the rods hight from the ground plane is $R_o - R_t = 5$ mm. In order to calculate Y_e and Y_o we solved the 2D electrostatic problem in the cross section using FEM (Maxwell 2D SV). The resulting admittances are $Y_e = 7.8$ m$\mho$ and $Y_o = 19.27$ m$\mho$. The dispersion relation for planar periodic structure formed from those coupled lines is the solid curve shown in Fig. 3(b), where $\theta = \omega L/c$ is the normalized frequency, and L is the coupled line length. The dispersion of cylindrical waveguide loaded with the coupled lines can be calculated from the electromagnetic fields. The structure unit cell resonance frequencies are calculated subject to the Floquet boundary conditions, discussed in Sect. 3, when applied between the two ends of the cell. The βd phase shift relates the fields at one end of the unit cell with the ϕ_0 rotated points at the other end. In the example considered in this paper the rotation angle $\phi_0 = 90°$ and the phase βd is varied between 0 and π to scan the first Brillouin zone. The FEM code HFSS was used to solve this eigenvalue problem. The resulting dispersion curves shown in Fig. 3(b), reveals that the first propagation mode is a backward or left-handed mode. This left-handed mode frequency range is below the cut-off frequency of the first mode (TE_{11}) of the empty guide which is 4.395 GHz. Operation below the cut-off frequency was also adopted before in designing left-handed rectangular guide [7].

Comparison between the dispersion relation obtained from circuit and from the cylinder loaded coupled lines, required normalizing the calculated frequency from the EM dispersion to $\theta = \omega L_{\text{eff}}/c$. In Fig. 2 the coupled line lengths fill a quarter of the circumference, with a length $L_{\text{eff}} = \pi R_e/2$, where the effective radius R_e satisfies $R_t < R_e < R_o$ (for our structure $R_e = 1.7$ cm is taken). The normalized dispersion curve for the cylinder loaded coupled line structure formed is shown in Fig. 3(b) to have good agreement with that obtained from the planar circuit.

4.2 Deployment of the meta-waveguide

The performance of the proposed cylindrical waveguide loaded with coupled lines is investigated by studying the transmission through an 8-cell composed guide. The guide termination ports could be mounted on the side cylindrical surface, or fixed on the end flat faces of the tube. In the proposed structure in Fig. 4(a), the end flat faces act as transmission line ports with inner conductor diameter chosen such that port impedance matches the structure Bloch impedance at phase $\pi/2$. Such a choice of the way of coupling the ports is not unique, it depends on the application involved and the range of frequency of interest to work at. Figure 4(b) shows a transmission along the guide in the frequency range 2–3.7 GHz, which is the same left-handed frequency range shown in Fig. 3(a).

Among the possible applications of this metaguide is its use in Backward Wave Oscillator (BWO), Gyrotron BWO and accelerator applications [13]. In these applications the use of all metallic structure is favorable as it does not block any portion of the electron beam, like in waveguides filled with metamaterial [13, 14]. Even with a clear beam path the absence of dielectric is favorable in these applications, where dielectrics beside being not easy to integrate with metals, they deteriorate the vacuum level inside these tubes. Of course for such applications the coupled lines have to be tailored to provide enough axial field for interaction with the electron beam. Another candidate application is in charged particle detection through reverse Cherenkov radiation [15].

5 Conclusion

We presented a new type of left-handed waveguide (metaguide), which is a cylindrical waveguide loaded with short-circuited coupled lines. The proposed guide has the characteristic of supporting a backward wave at frequencies below the empty guide cut-off. The guide is considered as a bent version of the planar periodically cascaded short-circuited coupled lines. Hence, it can be analyzed using the symmetric coupled line circuit theory, which enables the design of the operating Floquet frequency band and the Bloch impedance for the matching purpose of the periodic structure.

Although the circuit modeling was a simple 1D Floquet analysis, our actual guide is a screw (helically) periodic structure. Using the screw periodic Floquet condition, the dispersion relation using electromagnetic simulation was calculated and compared to that obtained with a simple circuit coupled lines model, where good agreement between both models was observed.

The introduced metaguide has the advantage of not containing any dielectrics, which makes it very suitable for vacuum tube applications. Backward Wave Oscillator (BWO), Gyrotron BWO and Cherenkov backward radiation detectors are among the possible applications for this guide.

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