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Optimal Progressive Group-Censoring Plans for Weibull Distribution in Presence of Cost Constraint

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Abstract

This article discusses a life test under progressive type-I group-censoring. We use maximum likelihood method to obtain the point estimator of the unknown parameter of lifetime distribution. We proposed that scale parameter α is known and fixed. In order to obtain a precise estimate of shape parameter of the Weibull distribution, one needs to design an optimal life test. Thus, this article proposes an approach to determine the number of test units, number of inspections, and length of inspection interval of a life test under a pre-determined budget of experiment such that the asymptotic variance of estimator of shape parameter is minimum. The method will be applied to a numerical example and the sensitivity analysis will be investigated.

Keywords: Grouped data; Progressive censoring; Sensitivity analysis; Type-I censoring; Variance optimality

1. Introduction

The Weibull distribution has been recognized as an appropriate model in reliability studies and life testing because of its versatility in fitting time to failure distributions of a rather extensive variety of complex mechanisms. Since it has a variety of shapes, this make the Weibull distribution flexible for fitting data and has been used as a model for the distribution of time to failure of products. Also, in reliability engineering, the Weibull shape parameter is often tied to the failure mechanism of the product.

In industrial life testing and medical survival analysis, it is very often that object is lost or withdrawn before failure or the object lifetime is only known within an interval. Hence, the obtained sample is called a censored sample. Aggarwala (2001) introduced type-I interval and progressive censoring and developed the statistical inference for the exponential distribution based on progressively type-I interval censored data. Under progressive type-I interval censoring, observations are only known within two consecutively pre-scheduled times and items would be allowed to withdrawn at pre-scheduled time points.

2. Progressive censoring

Progressively type-I interval censored sample is a union of type-I interval and progressive censoring. Such a sample is collected as follows: n units are put on life test at time 0. Units are observed at pre-set times $\tau_1, \tau_2, \ldots, \tau_k$ (k is also fixed). At these times, $r_1, r_2, ..., r_k$ live units are removed from experimentation, respectively. The values $r_1, r_2, ..., r_k$ may be pre-specified as percentages of the remaining live units since the numbers of units remaining at times $\tau_1, \tau_2, \ldots, \tau_k$ are random variables or positive integers, with the provision that at times T_i, there are r_i units available for removal. In this case, the number of live units removed at time τ_i is r_i^{obs} = min (r_i, number of units remaining), $i = 1, 2, ..., k-1$. Again, r_k^{obs} = all remaining units at time τ_k, when experimentation is scheduled to terminate. Let $n_1, n_2, ..., n_k$ denote the observed data, which will be the number of units known to have failed in the intervals (0, τ_1], (τ_1, τ_2], …, (τ_{k-1}, τ_k] respectively. The likelihood function will be:

$$L(x;\theta) = \prod_{i=1}^{k} [F(T_i) - F(T_{i-1})]^{n_i} [1 - F(T_i)]^{r_i^{obs}} \tag{2.1}$$

where $\tau_0 = 0$, and $\sum_{i=1}^{k}(n_i + r_i) = n$.

Extensive publications can be found in the literature which discuss the statistical inference for progressively censored data under various lifetime distributions. Some of them are Balasooriya and Saw (1998), Balasooriya et al. (2000), Balakrishnan and Aggarwala (2000), Ali Mousa and Jaheen (2002), Wu (2003), Gouno et al. (2004), and Lin et al. (2006). Discussion of grouped data can be found in Cheng and Chen (1988), Chen and Mi (1996), Aggarwala (2001), Xiong and Ming (2004), Xiang and Tse (2005), Yang and Tse (2005), and Wu et al. (2008).

To conduct a progressive type-I group-censored life test more efficiently, one has to address the problem of determining the number of test units, the number of inspections, and the length of the inspection intervals. In practice, the budget of a life experiment is limited. In this study, we will obtain the optimal settings of a progressive type-I group-censored life test by minimizing the asymptotic variance of shape parameter of the Weibull distribution under the constraint that the total experimental cost does not exceed a pre-determined budget.

Numerous problems in applied statistics can be formulated as optimization problems. In practice, optimization techniques are widely used for solving those problems. In various practical fields, the decision problem of obtaining appropriate number of test units, number of inspections, and length of inspection interval under restricted budget of experiment is important for experimenters especially in reliability analysis and quality control.

The rest of the paper is organized as follows: Section 3 describes the model and some necessary assumptions. We use the maximum likelihood method to obtain the point estimator of the shape parameter. Section 4 proposes a procedure to determine the number of units to test, the number of inspections and the length of inspection intervals. Section 5 applies the proposed procedure to a numerical example, and Section 6 studies the sensitivity analysis of the proposed procedure and some conclusions are presented.

3. Parameter Estimation

Suppose a progressively type-I group-censored sample is collected, beginning with a random sample of *n* units with a Weibull lifetime distribution. Let n_i be the number of units known to have failed in the interval $(\tau_{i-1}, \tau_i]$ and let r_i be the number of surviving units being withdrawn from the test at time τ_i, for $i = 1, 2, ..., k$, where $\tau_0 = 0$. The values of $r_1, r_2, ..., r_k$ are determined by the pre-specified percentages of the remaining live units $p_1, p_2, ..., p_{k-1}$ and $p_k = 1$. That is, $r_i = (m_i - n_i)p_i$, for $i = 1, 2, ..., k$, where $m_i = n - \sum_{j=1}^{i-1} n_j - \sum_{j=1}^{i-1} r_j$ is the number of non-removed surviving units at the beginning of the ith stage. Then, we have the fact that

$$n_i | n_{i-1}, ..., n_1, r_{i-1}, ..., r_1 \sim \text{binomial}(m_i, q_i) . \quad (3.1)$$

where $q_i = \frac{F(T_i) - F(T_{i-1})}{1 - F(T_{i-1})}$ is the probability that a unit fails in the time interval $(\tau_{i-1}, \tau_i]$, for $i = 1,\ 2,\ ...,\ k$. The form of the Weibull distribution considered here has probability density function as follows

$$f(x;\alpha,\beta)=\frac{\beta}{\alpha}(\frac{x}{\alpha})^{\beta-1}\exp(-(\frac{x}{\alpha})^{\beta}) \ , \qquad x>0,\alpha>0,\beta>0,$$

where α is the scale parameter and β is the shape parameter and the cumulative distribution function is :

$$F(x)=1-\exp\,(-(\frac{x}{\alpha})^{\beta}) \ , \qquad x>0. \tag{3.2}$$

In the case of α is known and β is unknown, the likelihood function under a progressively type-I group-censored sample with binomial removal becomes

$$L(\alpha,\beta)=\prod_{i=1}^{k}\left[1-e^{-\frac{1}{\alpha^{\beta}}\left(\tau_i^{\beta}-\tau_{i-1}^{\beta}\right)}\right]^{n_i}\left[e^{-\frac{1}{\alpha^{\beta}}\left(\tau_i^{\beta}-\tau_{i-1}^{\beta}\right)}\right]^{m_i-n_i}. \tag{3.3}$$

Let $h_i=\left(\tau_i^{\beta}-\tau_{i-1}^{\beta}\right), i=1,\ 2,\ ...,\ k.$ Then

$$L(\alpha,\beta)=\prod_{i=1}^{k}\left[1-e^{\frac{-h_i}{\alpha^{\beta}}}\right]^{n_i}\left[e^{\frac{-h_i}{\alpha^{\beta}}}\right]^{m_i-n_i}. \tag{3.4}$$

Since

$$q_i=\frac{F(\tau_i)-F(\tau_{i-1})}{1-F(\tau_{i-1})}, \qquad i=1,\ 2,\ ...,\ k. \tag{3.5}$$

$$q_i=1-\exp\left[-\frac{1}{\alpha^{\beta}}\left(\tau_i^{\beta}-\tau_{i-1}^{\beta}\right)\right], \qquad i=1,2,...,k.$$

So that, The logarithm of the likelihood function (3.4) will be

$$\log L(\alpha,\beta)=\sum_{i=1}^{k}\left[n_i\log\left(1-e^{-\frac{h_i}{\alpha^{\beta}}}\right)-(m_i-n_i)\left(\frac{h_i}{\alpha^{\beta}}\right)\right]. \tag{3.6}$$

The first partial derivative for β is obtained as follows

$$\frac{d}{d\beta}\log L(\alpha,\beta)=\sum_{i=1}^{k}\left\{n_i\left(\frac{1}{\alpha}\right)^{\beta}\frac{e^{-\frac{h_i}{\alpha^{\beta}}}\left[\frac{dh_i}{d\beta}+h_i\log\left(\frac{1}{\alpha}\right)\right]}{\left(1-e^{-\frac{h_i}{\alpha^{\beta}}}\right)}-(m_i-n_i)\left(\frac{1}{\alpha}\right)^{\beta}\left[\frac{dh_i}{d\beta}+h_i\log\left(\frac{1}{\alpha}\right)\right]\right\}. \tag{3.7}$$

where

$$\frac{dh_i}{d\beta}=\tau_i^{\beta}\log\tau_i-\tau_{i-1}^{\beta}\log\tau_{i-1}\ , \qquad i=1,2,...,k.$$

The second partial derivative for β is obtained as follows

$$\frac{d^2}{d\beta^2}\log L(\alpha,\beta)=\sum_{i=1}^{k}\left\{\begin{array}{l} n_i\left(\frac{1}{\alpha}\right)^{\beta}\left[\begin{array}{l}\dfrac{e^{-\frac{h_i}{\alpha^\beta}}\xi(h_i)\log\left(\frac{1}{\alpha}\right)-\left(\frac{1}{\alpha}\right)^{\beta}e^{-\frac{h_i}{\alpha^\beta}}\left(\xi(h_i)\right)^2+e^{-\frac{h_i}{\alpha^\beta}}\xi'_\beta(h_i)}{q_i} \\ -\dfrac{\left(e^{-\frac{h_i}{\alpha^\beta}}\right)^2\left[\frac{dh_i}{d\beta}+h_i\left(\frac{1}{\alpha}\right)^{\beta}\log\left(\frac{1}{\alpha}\right)\right]\left[\xi(h_i)\right]}{q_i^2}\end{array}\right] \\ -(m_i-n_i)\left(\frac{1}{\alpha}\right)^{\beta}\left[\log\left(\frac{1}{\alpha}\right)\left[\xi(h_i)\right]+\xi'_\beta(h_i)\right]\end{array}\right\} \quad (3.8)$$

where

$$\xi(h_i)=\left[\frac{dh_i}{d\beta}+h_i\log\left(\frac{1}{\alpha}\right)\right],\ \xi'_\beta(h_i)=\left[\frac{d^2h_i}{d\beta^2}+\frac{dh_i}{d\beta}\log\left(\frac{1}{\alpha}\right)\right] \text{ and}$$

$$q_i=1-\exp\left(-\frac{h_i}{\alpha^\beta}\right),\ i=1,2,\dots,k.$$

In the special case where the inspection intervals are of equal length, say, $\tau_i = i\tau$, then we can find that

$$h_i=[i^\beta-(i-1)^\beta]\tau_i^\beta,$$

$$\frac{dh_i}{d\beta}=\left\{[i^\beta-(i-1)^\beta]\log\tau+[i^\beta\log i-(i-1)^\beta\log(i-1)]\right\}\tau^\beta,\ i=1,2,\dots,k,$$

and

$$\frac{d^2h_i}{d\beta^2}=\left\{[i^\beta-(i-1)^\beta]\log\tau+[i^\beta\log i-(i-1)^\beta\log(i-1)]\right\}\tau^\beta\log\tau+$$
$$\left\{[i^\beta\log i-(i-1)^\beta\log(i-1)]\log\tau+[i^\beta(\log i)^2-(i-1)^\beta(\log(i-1))^2]\right\}\tau^\beta,\ i=1,2,\dots,k.$$

The Fisher's information is as follows

$$I(\beta)=n\sum_{i=1}^{k}\frac{\left(\frac{dq_i}{d\beta}\right)^2}{q_i(1-q_i)}\prod_{j=1}^{i-1}(1-q_j)(1-p_j)\ . \quad (3.9)$$

where

$$\frac{dq_i}{d\beta}=\frac{d}{d\beta}\left[1-\exp\left[-\left(\frac{h_i}{\alpha^\beta}\right)\right]\right],\qquad i=1,2,\dots,k.$$

In the special case where the inspection intervals are of equal length, say, $\tau_i = i\tau$, and the percentages of removals are the same (i.e. $p_i = p$). The asymptotic variance of $\hat{\beta}$ can be found as follows

$q_i = 1 - e^{-[i^\beta - (i-1)^\beta]\left(\frac{\tau}{\alpha}\right)^\beta}$, and

$$\frac{dq_i}{d\beta} = \left\{[i^\beta - (i-1)^\beta]\log\left(\frac{\tau}{\alpha}\right) + [i^\beta \log i - (i-1)^\beta \log(i-1)]\right\}\left(\frac{\tau}{\alpha}\right)^\beta e^{-[i^\beta - (i-1)^\beta]\left(\frac{\tau}{\alpha}\right)^\beta}. \quad (3.10)$$

By substituting in (3.9), we have

$$Var(\hat{\beta}) = \left[I(\beta)\right]^{-1}. \quad (3.11)$$

$$Var(\hat{\beta}) = \left[n \left[\frac{\left(B'_\beta e^{-B}\right)^2}{e^{-B}(1-e^{-B})} - \sum_{i=2}^{k} \left[\frac{\left[\left(\psi'_\beta(i)B + \psi(i)B'_\beta\right)e^{-\psi(i)B}\right]^2}{e^{-\psi(i)B}(1-e^{-\psi(i)B})} (1-p)^{i-1} e^{-T^\beta \sum_{j=1}^{i-1}\psi(j)} \right]\right]\right]^{-1}$$

where

$B = \left(\frac{\tau}{\alpha}\right)^\beta$, $B'_\beta = \left(\frac{\tau}{\alpha}\right)^\beta \log\left(\frac{\tau}{\alpha}\right)$, $\psi(i) = [i^\beta - (i-1)^\beta]$

and $\psi'_\beta(i) = [i^\beta \log i - (i-1)^\beta \log(i-1)]$.

Then, we can be obtained the asymptotic variance numerically.

4. Planning of Life Test for Weibull Distribution

This section discussed a life test under progressive type-I group censoring plans for Weibull distribution. In order to obtain a precise estimate of shape parameter β, one needs to design an optimal life test. We used an approach which proposed by Wu and Huang (2010) to determine the number of test units, number of inspections, and length of inspection interval of a life test under a pre-determined budget of experiment such that the asymptotic variance of estimator of shape parameter β is minimum. We assumed that the lengths of inspection intervals are all equal for simplicity of discussion. The equi-length assumption is also convenient for practitioners.

Let n denote the number of units on test, k be the number of inspections and τ be the length of inspection interval. Obviously, the decision variables (n, k, τ) affect both the cost of experiment and the precision of estimating mean lifetime. Let $TC(n, k, \tau)$ denote the total cost of conducting a progressive group censoring experiment. It involves four parts:

1. The cost of installing all test units in the beginning of a life experiment, say C_a.
2. The cost of test units is nC_s, where C_s denotes the cost of one test unit.

3. The cost of inspection is kC_I, where C_I denotes the cost of one inspection.
4. The cost of operating an experiment is $k\tau C_o$, where C_o is the operation cost in the time interval between two inspections.

Therefore, the total cost of experiment is $TC(n, k, \tau) = C_a + n\, C_s + k\, C_I + k\, \tau\, C_o$.

In order to obtain a precise estimator of the shape parameter β of lifetime distribution, a typical decision problem can be formulated as follows:

$$\begin{aligned} &\text{minimize } Var(\hat{\beta}) \\ &\text{subject to } C_a + nC_S + kC_I + k\tau C_0 \le C_r \\ &\qquad k, n \in \mathrm{N}, k \ge 1, \text{and } \tau > 0, \end{aligned} \tag{4.1}$$

where C_r denotes the budget for the experiment and N is the set of positive integers. Since the decision variables n and k are integer, the decision variable τ is real, and the objective function and constraint are both nonlinear functions of n, k, and τ, the nonlinear mixed integer programming can be used to find the optimal solution. An excellent review of nonlinear mixed integer programming can be found in Grossmann (2002).

5. Illustrative Example

To illustrate the use of our method to obtain variance optimality, the following example is discussed. We apply the proposed methods as in Wu and Huang (2010) to a sample which we generated from the Weibull distribution, with scale parameter α = 1 and shape parameter $\beta = 2$ by using Mathcad 13 package. The sample size n = 15, number of inspections k = 4 and length of inspection interval τ = 2. The pre-determined percentages of removals are ($p1$, $p2$, $p3$, $p4$) =(0.25, 0.25, 0.5, 1). The data are presented in Table 1.

Table 1 Progressively type-I group-censored sample from the Weibull distribution

i	1	2	3	4
n_i	5	3	1	1
r_i	2	1	1	1

From (3.6), we obtain the maximum likelihood estimate of β to be $\hat{\beta} = 0.348$. We use this value in the design of our new experiment. Assume that the percentages of removals are $p_1 = p_2 = p_3 = 0.25$ and $p_4 = 1$. Suppose further that the values of cost parameters are as follows: Ca = \$800, Cs = \$85 per unit, CI = \$40, Co = \$8 per 10 h, and Cr = \$6000. Consider the objective function to be the asymptotic variance of the shape parameter β. Thus, the optimal design problem is:

Minimize $Var(\hat{\beta})$
subject to $800+85n+40k+8k\tau \leq 6000$.

By using the algorithm proposed in Wu and Huang (2010). We can obtain the optimal design as follows:

$$n^* = 52,\quad k^* = 4 \text{ and } \tau^* = 19.375.$$

That is, the optimal number of test units is 52, the optimal number of inspections is 4, and the optimal length of inspection interval is 193.75 h. Thus, the termination time of the experiment is 775 h. The value of $Var(\hat{\beta})$ is 0.0033. From this obtained optimal design, we can find the following result. When the percentages of removals are all set to be 0.25 and the pre-determined budget of experiment is \$6000, one needs more number of test units, longer length of inspection interval than those in the original design in order to get the minimum asymptotic variance of the shape parameter β.

6. Sensitivity analysis & Conclusion

The sensitivity study of the optimal solution to change in the values of the different parameters is an important issue to the planning of life test. These parameters consist of two parts: (1) the percentages of removals, i.e., p_i ; and (2) the parameters of experimental cost, i.e., *Ca*, *Cs* , *CI* , *Co*, and *Cr* . We will investigate the effects of the percentages of removals, and the cost parameters on the optimal solution in Sections. 6.1 and 6.2, respectively. The sensitivity study is based on the last Example in Section 5.

6.1. The effect of percentages of removals

In this subsection, we assume that the pre-specified percentages of removals from the
remaining live units at each stage are all equal. That is, $p_1 = p_2 = \dots = p_{k-1} = p$ and $p_k = 1$. Under fixed cost parameters (*Ca, Cs ,CI ,Co, Cr*) = (800, 85, 40, 8, 6000) used in above example, the optimal solutions of *n*, *k*, and τ for different choices of percentage of removals *p* are presented in Table 2.

Table 2 Optimal values of *n*, *k* and τ under various combinations of β and *p*

β	p	n	K	τ	$k\tau$	$Var(\hat{\beta})$
0.348	0.05	53	4	16.719	66.875	0.0030
	0.10	52	4	19.375	77.5	0.0031
	0.25	52	4	19.375	77.5	0.0033

From Table 2, we can find that:

1. The value of n decreases as p increases.
2. The length of inspection interval τ is an increasing function of p.
3. The value of $Var(\hat{\beta})$ increases as p increases.

6.2. The effect of cost parameters

Changes in cost parameters can also affect the optimal solutions. Let us consider the value of distribution parameter $\beta = 0.348$ and the cost parameters (Ca ,Cs ,CI ,Co, Cr) = (800, 85, 40, 8, 6000). Using the same value of distribution parameter, the influence of cost parameters on n, k and τ is investigated.

Table 3 Optimal values for different costs values under $\boldsymbol{\beta = 0.348}$ and $\boldsymbol{p = 0.05}$

C_a	C_s	C_I	C_0	C_r	n	k	τ	$k\tau$	$Var(\hat{\beta})$
800	85	40	8	4000	30	4	15.313	61.25	0.0054
				5000	41	4	17.344	69.375	0.0039
				6000	53	4	16.719	66.875	0.0030
				7000	64	4	18.75	75	0.0025
				8000	76	4	18.125	72.5	0.0021
800	85	40	6	6000	54	4	18.75	75	0.0030
			7		53	4	19.107	76.429	0.0030
			8		53	4	16.719	66.875	0.0030
			9		52	4	17.222	68.889	0.0031
			10		52	4	15.5	62	0.0031
800	85	30	8	6000	53	4	17.969	71.875	0.0030
		35			53	4	17.344	69.375	0.0030
		40			53	4	16.719	66.875	0.0030
		45			52	4	18.75	75	0.0031
		50			52	4	18.125	72.5	0.0031
800	65	40	8	6000	69	4	17.344	69.375	0.0023
	75				60	4	16.875	67.5	0.0027
	85				53	4	16.719	66.875	0.0030
	95				47	4	17.969	71.875	0.0034
	105				42	4	19.688	78.75	0.0038
600	85	40	8	6000	55	4	17.656	70.625	0.0029
700					54	4	17.188	68.75	0.0030
800					53	4	16.719	66.875	0.0030
900					51	4	18.906	75.625	0.0031
1000					50	4	18.438	73.75	0.0032

Table 4 Optimal values for different costs values under $\boldsymbol{\beta = 0.348}$ and $\boldsymbol{p = 0.10}$

C_a	C_s	C_I	C_0	C_r	N	k	$\boldsymbol{\tau}$	$k\tau$	$Var(\hat{\beta})$
800	85	40	8	4000	30	4	15.313	61.25	0.0056
				5000	41	4	17.344	69.375	0.0040
				6000	52	4	19.375	77.5	0.0031
				7000	64	4	18.75	75	0.0026
				8000	75	4	20.781	83.125	0.0022
800	85	40	6	6000	54	4	18.75	75	0.0030
			7		53	4	19.107	76.429	0.0031
			8		52	4	19.375	77.5	0.0031
			9		52	4	17.222	68.889	0.0032
			10		51	4	17.625	70.5	0.0032
800	85	30	8	6000	53	4	17.969	71.875	0.0031
		35			53	4	17.344	69.375	0.0031
		40			52	4	19.375	77.5	0.0031
		45			52	4	18.75	75	0.0031
		50			52	4	18.125	72.5	0.0032
800	65	40	8	6000	69	4	17.344	69.375	0.0024
	75				59	4	19.219	76.875	0.0028
	85				52	4	19.375	77.5	0.0031
	95				47	4	17.969	71.875	0.0035
	105				42	4	19.688	78.75	0.0039
600	85	40	8	6000	55	4	17.656	70.625	0.0030
700					53	4	19.844	79.375	0.0031
800					52	4	19.375	77.5	0.0031
900					51	4	18.906	75.625	0.0032
1000					50	4	18.438	73.75	0.0033

Table 5 Optimal values for different costs values under $\boldsymbol{\beta = 0.348}$ and $\boldsymbol{p = 0.25}$

C_a	C_s	C_I	C_0	C_r	n	k	$\boldsymbol{\tau}$	$k\tau$	$Var(\hat{\beta})$
800	85	40	8	4000	29	4	17.968	71.875	0.0061
				5000	40	4	20	80	0.0043
				6000	52	4	19.375	77.5	0.0033
				7000	63	4	21.406	85.625	0.0027
				8000	74	4	23.437	93.75	0.0023
800	85	40	6	6000	53	4	22.292	89.167	0.0032
			7		52	4	22.143	88.571	0.0033
			8		52	4	19.375	77.5	0.0033
			9		51	4	19.583	78.333	0.0034
			10		50	4	19.75	79	0.0035
800	85	30	8	6000	52	4	20.625	82.5	0.0033

		35			52	4	20	80	0.0033
		40			52	4	19.375	77.5	0.0033
		45			51	4	21.406	85.625	0.0034
		50			51	4	20.781	83.124	0.0034
800	65	40	8	6000	68	4	19.375	77.5	0.0026
	75				59	4	19.219	76.875	0.0029
	85				52	4	19.375	77.5	0.0033
	95				46	4	20.938	83.75	0.0037
	105				42	4	19.687	78.748	0.0041
600	85	40	8	6000	54	4	20.313	81.252	0.0032
700					53	4	19.844	79.376	0.0033
800					52	4	19.375	77.5	0.0033
900					50	4	21.562	86.248	0.0034
1000					49	4	21.094	84.376	0.0035

From Tables 3, 4 and 5, we find the following results:

1. A higher value of *Cr* leads to a higher value of n.

2. The number of test units n is a decreasing function of *Ca*.

3. The number of test units n is insensitive to changes in *CI* and *Co*.

4. A higher value of *Cs* causes a lower value of n.

5. The length of inspection interval τ and the termination time $k\tau$ are sensitive to the changes in these values of cost parameters when p is small.

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