# Estimation of the Lifetime Performance Index with Burr Type III Distribution Under Type II Censoring

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## Abstract

Assessing the lifetime performance index of products is one of the most important topics in the manufacturing industries. Uniformly minimum variance unbiased estimator (UMVUE) of the lifetime performance index based on the right Type II censored sample for Burr Type III distribution is obtained. Then, the UMVUE of lifetime performance index is utilized to develop the new hypothesis testing procedure in the condition of known specification limit. One practical example and Monte Carlo simulation are given to illustrate the use of testing procedure under given significance level.

**Keywords**: Process capability index, Lifetime performance index, Burr Type III distribution, Type-II censoring, Uniformly minimum variance unbiased estimator.

## 1. Introduction

The globalization of business activities has intensified competitions among business organizations and resulted in quality consciousness, manufacture defect-free products and offer greater reliability of their products. These results are then compared with those of industry leaders for competitive bench marking. One metric popularly used is the Process capability index (PCI). Process capability analysis has the following benefits: (*i*) continuously monitoring the process quality through process capability indices in order to assure the products manufactured are conforming to the specifications; (*ii*) supplying information on product design and process quality improvement for engineers and designer; and (*iii*) providing the basis for reducing the cost of product failures (see Pan and Wu 1997).

Controlling and improving quality of products is important for many companies. In particular, lifetime of products is a relevant quality characteristic that producers have to monitor. As a measure of quality for life times, Montgomery (1985) and Kane (1986) proposed the process capability index, $C_L$ for evaluating the lifetime performance of electronic components, where $L$ is the lower specification limit. Statistical inferences for $C_L$ on the basis of complete sample for different lifetime distributions have been considered in the literature. For example, Tong et al. (2002) obtained the UMVUE for $C_L$ and considered hypothesis testing procedure for the one parameter exponential distribution. Chen et al. (2002) also used the UMVUE of $C_L$ to develop the confidence interval under an exponential distribution.

Statistical studies for $C_L$ based on Type II censored samples have been considered by several researchers. For instance, Hong et al. (2007) constructed a maximum likelihood estimator of $C_L$ under the Pareto distribution based on the right Type II censored sample. Lee et al. (2010) constructed the UMVUE of $C_L$ and developed a testing procedure for the performance index of products for the exponential distribution based on the Type II right censored sample. Lee et al. (2015) derived maximum likelihood estimator and constructed a hypothesis testing procedure for lifetime performance index from the normal distribution but sample data modeled by fuzzy numbers.

In life testing experiments, there are time limits and/or other restrictions including money, material resources, mechanical or experimental difficulties on data collection. In these situations, the exact lifetime of products put on the test is not observable and the censored data is used. There are several types of censoring schemes in survival analysis and the Type-II censoring is one of the most common for consideration. So, in this paper, we construct a UMVUE of $C_L$ for Burr Type III distribution with Type II right censored sample. The UMVUE of $C_L$ is then utilized to develop a new hypothesis testing procedure under the condition of a known lower specification limit $L$. The new testing procedure can be employed to assess whether the lifetime of products (or items) adheres to the required level in the condition of known $L$.

The rest of this paper is organized as follows. Section 2 introduces some properties of the lifetime performance index for lifetime of product (or item) and discusses the relationship between the lifetime performance index and conforming rate. Section 3 presents the UMVUE of the lifetime performance index and its statistical properties. Section 4 develops a new hypothesis testing procedure for the lifetime performance index. In the same section, the power of the statistical test and the one sided confidence interval of $C_L$ are obtained. A Monte Carlo simulation algorithm of power function and real data example are provided in Sections 5 and 6 respectively. The article ends with concluding remarks.

## 2. Lifetime Performance Index and Conforming Rate

In 1942, Burr suggested twelve types of cumulative distribution function, one of them, was Burr Type III distribution. It is a very important lifetime model in the analysis of equipment failure data, vehicle and informational studies, as well as in models of stress and durability.

Suppose that the lifetime $X$ of product has a two-parameter Burr Type III distribution with the cumulative distribution function (cdf) and probability distribution function (pdf) defined respectively as

$$F(x;\lambda,\beta) = \left[1 + x^{-\lambda}\right]^{-\beta}, x > 0 \quad, \lambda, \beta > 0, \tag{1}$$

where $\lambda$ and $\beta$ are shape parameters. The corresponding pdf of Burr Type III distribution is,

$$f(x;\lambda,\beta)=\lambda\beta x^{-\lambda-1}\left[1+x^{-\lambda}\right]^{-\beta-1};x>0. \tag{2}$$

To obtain the performance index $C_L$ for the Burr Type III distribution, a random variable $X$ will be converted to exponential distribution through the transformation $Y=\log_e(1+X^{-\lambda})$. Hence, the pdf and cdf of $Y$ are obtained respectively as follows,

$$f(y;\beta)=\beta e^{-\beta y},y>0\quad,\beta>0, \tag{3}$$

and

$$F(y;\beta)=1-e^{-\beta y},\beta>0.$$

Also, it is known that the mean and variance of $Y$ are ${}^{1}\!/\!{}_{\beta}$ and ${}^{1}\!/\!{}_{\beta^2}$ respectively.

The failure rate function $r(y;\beta)$ is defined by

$$r(y;\beta)=\frac{f(y;\beta)}{1-F(y;\beta)}=\frac{\beta e^{-\beta y}}{e^{-\beta y}}=\beta,\qquad \beta>0. \tag{4}$$

So, the life time performance index $C_L$ is obtained by substituting mean and variance as follows

$$C_L=\frac{\mu-L}{\sigma}=\frac{{}^{1}\!/\!{}_{\beta}-L}{{}^{1}\!/\!{}_{\beta}}=1-\beta L,\quad -\infty<C_L<1, \tag{5}$$

where the process mean μ= ${}^{1}\!/\!{}_{\beta}$ , the process standard deviation σ = $\sqrt{Var(Y)}$= ${}^{1}\!/\!{}_{\beta}$ , and $L$ is the lower specification limit.

When the mean lifetime of products $(1/\beta)>L$, then the life time performance index $C_L>0$. From Equations (4) and (5), it can be seen that the larger the mean $1/\beta$, the smaller the failure rate and the larger the lifetime performance index $C_L$. Therefore, the life time performance index $C_L$ reasonably and accurately represents the lifetime performance of products.

If the lifetime of a product $Y$; which $Y\geq L$ and $\beta>0$ exceeds the lower specification limit $L$, then the product is defined as a conforming product. The ratio of conforming products is known as the conforming rate $P_r$ and can be defined as

$$P_r=P(Y\geq L)=\int_L^\infty f(y)\,dy=\int_L^\infty \beta e^{-\beta y}dy=e^{-\beta L}.$$

From Equation (5), $\beta L=1-C_L$, then $P_r$ is as follows

$$P_r=e^{(C_L-1)},\qquad -\infty<C_L<1. \tag{6}$$

Obviously, a strictly increasing relationship exists between the conforming rate $P_r$ and the lifetime performance index $C_L$. Thus, the larger the index value $C_L$, the larger conforming rate $P_r$. Table 1 lists various $C_L$ values and the corresponding $P_r$. For the $C_L$ values which are not listed in Table1, the conforming rate $P_r$ can be obtained through interpolation. The conforming rate can be calculated by dividing the number of conforming products by the total number of products sampled.

To accurately estimate the conforming rate, Montgomery (1985) suggested that the sample size must be large. However, a large sample size is usually not practical from the perspective of cost, since collecting the lifetime data of new products (or items) need many monies. In addition, a complete sample is also not practical due to time limitation and /or other restrictions (such as lack of funds, lack of material resources, mechanical or experimental difficulties, etc.) on data collection.

Since a one-to one mathematical relationship exists between the conforming rate $P_r$ and the lifetime performance index $C_L$. Therefore, utilizing the one-to-one relationship between $P_r$ and $C_L$, lifetime performance index can be a flexible and effective tool, not only evaluating product quality, but also for estimating the conforming rate $P_r$.

**Table1.** The lifetime performance index versus the conforming rate

| $C_L$ | $P_r$ | $C_L$ | $P_r$ |
|---|---|---|---|
| $-\infty$ | 0.00000 | 0.15 | 0.42741 |
| −9.00 | 0.00004 | 0.20 | 0.44933 |
| −8.00 | 0.00012 | 0.25 | 0.47237 |
| −7.00 | 0.00033 | 0.30 | 0.49659 |
| −6.00 | 0.00091 | 0.35 | 0.52205 |
| −5.00 | 0.00248 | 0.40 | 0.54881 |
| −4.50 | 0.00409 | 0.45 | 0.57695 |
| −4.00 | 0.00673 | 0.50 | 0.60653 |
| −3.50 | 0.01111 | 0.55 | 0.63763 |
| −3.00 | 0.01832 | 0.60 | 0.67032 |
| −2.50 | 0.03019 | 0.65 | 0.70469 |
| −2.00 | 0.04979 | 0.70 | 0.74082 |
| −1.50 | 0.08208 | 0.75 | 0.77880 |
| −1.00 | 0.13534 | 0.80 | 0.81873 |
| −0.50 | 0.22313 | 0.85 | 0.86071 |
| 0.00 | 0.36788 | 0.90 | 0.90484 |
| 0.05 | 0.38674 | 0.95 | 0.95123 |
| 0.10 | 0.40657 | 1.00 | 1.00000 |

## 3. UMVUE of Lifetime Performance Index

In this section, Type II censored is considered, let $n$ denote the sample size of the test, $m$ denote the number of failures in the test. Consider that $y_{(1)}$ , $y_{(2)}$ ,…, $y_{(m)}$ is the corresponding Type II censored sample. The likelihood function for observed samples $y_{(1)}$ , $y_{(2)}$ ,…, $y_{(m)}$, is obtained as follows

$$\begin{aligned} L &= \frac{n!}{(n-m)!}\prod_{i=1}^{m} f\left(y_{(i)},\beta\right)\left[1-F\left(y_{(\mathrm{m})}\right)\right]^{n-m} . \\ &= \frac{n!}{(n-m)!}\prod_{i=1}^{m}\left(\beta e^{-\beta y_{(i)}}\right)\left[1-\left(1-e^{-\beta y_{(m)}}\right)\right]^{n-m} \\ &= \frac{n!}{(n-m)!}\beta^{m} e^{-\beta\left(\sum_{i=1}^{m} y_{(i)}+(n-m)y_{(m)}\right)} . \end{aligned} \tag{7}$$

To show that $S(y)=\sum_{i=1}^{m} y_{(i)}+(n-m)y_{(m)}$ is a complete and sufficient statistic from Equation (7), it must be rewritten as the exponential family as follows

$$L = \exp\left[\left(ln\frac{n!}{(n-m)!}\right)+m\,ln\,\beta-\left(\beta\left(\sum_{i=1}^{m} y_{(i)}+(n-m)y_{(m)}\right)\right)\right], \tag{8}$$

where, $c(\beta)=-\beta$ , $S\equiv S(y)=\sum_{i=1}^{m} y_{(i)}+(n-m)y_{(m)}$, $d(\beta)=m\ ln\beta$, $A=ln\left[\frac{n!}{(n-m)!}\right]$,

$I_A(y)=\prod_{i=1}^{m}(1,\infty)$ , $y=y_{(1)}$ , $y_{(2)}$ ,…, $y_{(m)}$.

Hence, based on Lehmann (1983), and Hogg et al. (2005), $S=\sum_{i=1}^{m} y_{(i)}+(n-m)y_{(m)}$ is a complete and sufficient statistic for $\beta$. In addition, by using Theorem 4.1.1 and Corollary 4.1.1 of Lawless (2003) also can obtain that $2\beta S\sim\chi^2_{(2m)}$.

By using the invariance property of UMVUE, hence the UMVUE estimator of $C_L$ say $\hat{C}_L$ , can be written as

$$\begin{aligned} \hat{C}_L &= 1-\hat{\beta}L , \\ &= 1-\frac{(m-1)L}{\sum_{i=1}^{m} y_{(i)}+(n-m)y_{(m)}} . \end{aligned} \tag{9}$$

Meanwhile, the $k^{th}$ moment of $\hat{C}_L$ is obtained as follows

$$E\left(\hat{C}_L\right)^k = E\left[1-\frac{(m-1)L}{\sum_{i=1}^{m} y_{(i)}+(n-m)y_{(m)}}\right]^k$$

$$= E\left[1-\frac{2\beta(m-1)L}{2\beta S}\right]^k,$$

where $2\beta S$ has a chi-square distribution with 2*m* degrees of freedom.

$$E\left(\hat{C}_L\right)^k = \sum_{i=0}^{k}\binom{k}{i}(-1)^i\left[2\beta(m-1)L\right]^i E\left(2\beta S\right)^{-i},$$

where,

$$E\left(2\beta S\right)^{-i} = \frac{\Gamma(m-i)}{\Gamma(m)2^i}.$$

Hence, the $k^{th}$ moment of $\hat{C}_L$ takes the form

$$E\left(\hat{C}_L\right)^k = \sum_{i=0}^{k}\binom{k}{i}(-1)^i\frac{\left[(m-1)L\right]^i\beta^i\Gamma(m-i)}{\Gamma(m)}. \tag{10}$$

In particular, the expected value and the variance of $\hat{C}_L$ can be obtained respectively as

$$E\left(\hat{C}_L\right) = 1-\beta L, \tag{11}$$

and

$$Var(\hat{C}_L) = \frac{\beta^2 L^2}{m-2}, m > 2. \tag{12}$$

## 4. Testing Procedure for the Lifetime Performance Index

In this section, the power of the statistical test and at given specified significance level $\gamma$, the level $(1-\gamma)$ one-sided confidence interval for $C_L$ are discussed.

To assess whether the lifetime performance index adheres to the required level, a statistical testing procedure is constructed. Assuming that the required index value of lifetime performance is larger than $c^*$, where $c^*$ denotes the target value, the null hypothesis

$$H_0: C_L \le c^* \text{ (the product is unreliable),}$$

the alternative hypothesis

$$H_1: C_L > c^* \text{ (the product is reliable),}$$

are constructed. By using $\hat{C}_L$ , the UMVUE of $C_L$ as the test statistic, the rejection region can be expressed as $\left\{\hat{C}_L \,\middle|\, C_L > C_0^*\right\}$. Given the specified significance level $\gamma$ , the critical value $C_0^*$ can be calculated as follows

$$P(\hat{C}_L > C_0^* \,|\, C_L \le c^*) = \gamma \,,$$

$$P(1 - \frac{(m-1)L}{S} > C_0^* \,|\, C_L \le c^*) = \gamma \,,$$

$$P(1 - \frac{2\beta(m-1)L}{2\beta S} > C_0^* \,|\, C_L \le c^*) = \gamma \,,$$

$$\left( Since\; P\left( 2\beta S > \frac{2(m-1)\left(1-c^*\right)}{1-C_0^*} \right) as\, C_L \le c^* \right),$$

$$P\left( 2\beta S \le \frac{2(m-1)\left(1-c^*\right)}{1-C_0^*} \right) = 1-\gamma \,, \tag{13}$$

where $2\beta S \sim \chi^2_{(2m)}$. From Equation (13), utilizing inverse-chi-square (INVCHI) (1-$\gamma$ , 2$m$) function which represents the lower $1-\gamma$ percentile of $\chi^2_{(2m)}$, then

$$\frac{2(m-1)\left(1-c^*\right)}{1-C_0^*} = \text{INVCHI}\,(1-\gamma,\; 2m)\,,$$

is obtained. Thus, the critical value can be derived as;

$$C_0^* = 1 - \frac{2(m-1)\left(1-c^*\right)}{\text{INVCHI}\left(1-\gamma,\, 2m\right)}, \tag{14}$$

where $c^*$ , $\gamma$ and $m$ are denoted to the target value, the specified significance level and the number of observed failures before termination, respectively.

## 4.1 Power Function of the Test

The power of a statistical test is the probability of correctly rejecting a false null hypothesis. The procedure used in practice is to limit the probability of Type I error to some pre-assigned level $\gamma$ (usually 0.01or 0.05) that is small and to maximize the power of a statistical test. The null hypothesis $H_0$: $C_L \le c^*$, and the alternative hypothesis $H_1$: $C_L > c^*$, are constructed. The power of a statistical test is derived as follows:

Under Type II censoring scheme, we get a size $\gamma$ test with the rejection region $\left\{\hat{C}_L \,\middle|\, \hat{C}_L > 1 - \frac{2(m-1)(1-c^*)}{\text{INVCHI }(1-\gamma, 2m)}\right\}$, for the number of observed failures before termination $m$ and sample size $n$ ($m \le n$). The power $P(C_L)$ of the test at this point $C_L >$ $c^*$ defined as;

$$\begin{aligned} P\ (C_L) &= p\left(\hat{C}_L > 1 - \frac{2(m-1)(1-c^*)}{\text{INVCHI}(1-\gamma, 2m)}\right) \\ &= p\left(1 - \frac{(m-1)L}{S} > 1 - \frac{2(m-1)(1-c^*)}{\text{INVCHI}(1-\gamma, 2m)} \middle| \beta = \frac{1-C_L}{L}\right) \\ &= p\left(2\beta S > \frac{\beta L\ \ \text{INVCHI}(1-\gamma, 2m)}{(1-c^*)}\right) \\ &= p\left(2\beta S > \frac{(1-C_L)\,\text{INVCHI}(1-\gamma, 2m)}{(1-c^*)}\right), \end{aligned} \tag{15}$$

where $2\beta S \sim \chi^2_{(2m)}$.

## 4.2 Confidence Interval for $C_L$

In this section, given the specified significance level $\gamma$, the level $(1-\gamma)$ one-sided confidence interval for $C_L$ can be derived as follows:

Since the pivotal quantity $2\beta S$, where $2\beta S \sim \chi^2_{2m}$ and INVCHI $(1-\gamma, 2m)$ which represents the lower $1-\gamma$ percentile of $\chi^2_{2m}$. So

$$P\left(2\beta S \le \text{INVCHI}(1-\gamma, 2m)\right) = 1-\gamma, \tag{16}$$

where $C_L = 1 - \beta L$ and $\hat{C}_L = 1 - \frac{(m-1)L}{S}$. Multiply inequality (16) by $\frac{\hat{\beta} L}{2(m-1)}$, then

$$P\left(\frac{2\beta S\, \hat{\beta} L}{2(m-1)} \le \frac{\hat{\beta} L \ \mathrm{INVCHI}(1-\gamma, 2m)}{2(m-1)}\right) = 1-\gamma. \tag{17}$$

But, $\hat{\beta} = \frac{(m-1)}{S}$,

$$P\left(\frac{\beta\hat{\beta} L}{\hat{\beta}} \le \frac{\hat{\beta} L \ \mathrm{INVCHI}(1-\gamma, 2m)}{2(m-1)}\right) = 1-\gamma,$$

$$P\left(1-\beta L \ge 1 - \frac{\hat{\beta} L \ \mathrm{INVCHI}(1-\gamma, 2m)}{2(m-1)}\right) = 1-\gamma, \tag{18}$$

From Equation (18), then

$$C_L \ge 1 - \frac{\left(1-\hat{C}_L\right)\mathrm{INVCHI}(1-\gamma, 2m)}{2(m-1)} \tag{19}$$

is the level $(1-\gamma)$ one-sided confidence interval for $C_L$. Thus, the level $(1-\gamma)$ lower confidence bound (*LB*) for $C_L$ can be written as

$$LB = 1 - \frac{\left(1-\hat{C}_L\right)\mathrm{INVCHI}(1-\gamma, 2m)}{2(m-1)}, \tag{20}$$

where $\hat{C}_L$, $\gamma$ and *m* denote the UMVUE of $C_L$, the specified significance level and the number of observed failures before termination, respectively.

In addition, the proposed testing procedure can be constructed with the one-sided confidence interval too. The decision rule of statistical test is ‘‘If performance index value $c^* \notin (LB, \infty)$, it is concluded that the lifetime performance index of products meets the required level’’.

## 5. Numerical Study

The numerical study is designed to obtain the critical values of the power test. The simulated procedures are described as follows

### Step 1:

**(a)** Generate a random sample $u_1, u_2, \ldots, u_m$ of size $n$ from a uniform (0,1), then the uniform random numbers can be transformed to Burr III random numbers by using the following transformation

$$x_i = \left( u_i^{\frac{-1}{\beta}} - 1 \right)^{\frac{-1}{\lambda}}, i = 1, 2, \ldots, m .$$

The generated data for $y_i = \log_e(1 + x_i^{-\lambda})$ , $i$ = 1,...,$m$, ( $x_1$ , $x_2$ ,…, $x_m$ ) is a random sample from the Burr Type III distribution

**(b)** Set ( $y_{(1)}$ , $y_{(2)}$ ,…, $y_{(m)}$ ) as a Type II right censored sample from a one-parameter exponential distribution with pdf (3).

**(c)** Given $c^* = 0.1, 0$ $C_L$ =0.1 (0.1) 0.9, $\lambda = 1.7,$ $L$=0.5, and the indicated significance levels of $\gamma$ = 0.01 and 0.05. The selected values of ($n$, $m$) are ($n$, $m$)=(10,2), (20,4), (30,6), (50,10), where $c^* < C_L$ < 1 and $m$<$n$.

**(d)** The value of $\hat{C}_L$ is calculated by $\hat{C}_L = 1 - \dfrac{(m-1)L}{\sum_{i=1}^{m} y_{(i)} + (n-m)\,y_{(m)}}$.

**(e)** If $\hat{C}_L > C_0^*$, the critical value is obtained by using $C_0^* = 1 - \dfrac{2(m-1)(1-c^*)}{\mathrm{INVCHI}(1-\gamma, 2m)}$.

**Step 2: (a)** Step1 is repeated 1000 times.

**(b)**The estimation of the power $p(C_L)$, is $\hat{P}(C_L) = \dfrac{TotalCount}{1000}$.

**Step 3: (a)** Step 2, based on 100 estimations of the power $P(C_L)$ can be obtained as follows: $\hat{P}_1(C_L), \hat{P}_2(C_L), \ldots\ldots, \hat{P}_{100}(C_L)$.

**(b)**The mean $\overline{\hat{P}(C_L)}$ of $\hat{P}_1(C_L), \hat{P}_2(C_L), \ldots\ldots, \hat{P}_{100}(C_L)$, that is $\overline{\hat{P}(C_L)} = \dfrac{\sum_{i=1}^{100} \hat{P}_i(C_L)}{100}$, is calculated.

**(c)**The MSE of $\hat{P}_1(C_L), \hat{P}_2(C_L), \ldots\ldots, \hat{P}_{100}(C_L)$ is obtained as follows

$$\mathrm{MSE} = \frac{\sum_{i=1}^{100} (\hat{P}_i(C_L) - P(C_L))^2}{100}.$$

**Step 4:** The results of simulated data are listed in Tables 2 and 3.

**Table 2** The values of $P(C_L)$, $\overline{\hat{P}(C_L)}$ and MSE for the test at and $\gamma = 0.01$

| (*n*,*m*) | $C_L$ | $P(C_L)$ | $\overline{\hat{P}(C_L)}$ | MSE |
|---|---|---|---|---|
| (10,2) | 0.1 | 0.102 | 0.11464 | 0.00008 |
| | 0.2 | 0.158 | 0.16223 | 0.00015 |
| | 0.3 | 0.225 | 0.22444 | 0.00011 |
| | 0.4 | 0.311 | 0.30162 | 0.00018 |
| | 0.5 | 0.419 | 0.39794 | 0.00018 |
| | 0.6 | 0.513 | 0.50769 | 0.00031 |
| | 0.7 | 0.64 | 0.62132 | 0.00019 |
| | 0.8 | 0.733 | 0.73375 | 0.00019 |
| | 0.9 | 0.815 | 0.83224 | 0.00011 |
| (20,4) | 0.1 | 0.036 | 0.03791 | 0.00003 |
| | 0.2 | 0.066 | 0.06969 | 0.00007 |
| | 0.3 | 0.11 | 0.12378 | 0.00007 |
| | 0.4 | 0.206 | 0.20793 | 0.00017 |
| | 0.5 | 0.32 | 0.33202 | 0.00023 |
| | 0.6 | 0.505 | 0.50244 | 0.00022 |
| | 0.7 | 0.686 | 0.68711 | 0.00018 |
| | 0.8 | 0.859 | 0.84906 | 0.00011 |
| | 0.9 | 0.955 | 0.95012 | 0.00004 |
| (30,6) | 0.1 | 0.009 | 0.0083 | 0.00000 |
| | 0.2 | 0.03 | 0.01897 | 0.00001 |
| | 0.3 | 0.058 | 0.04483 | 0.00003 |
| | 0.4 | 0.111 | 0.10084 | 0.00008 |
| | 0.5 | 0.226 | 0.20733 | 0.00015 |
| | 0.6 | 0.409 | 0.39547 | 0.00022 |
| | 0.7 | 0.654 | 0.64797 | 0.00022 |
| | 0.8 | 0.843 | 0.86947 | 0.00011 |
| | 0.9 | 0.983 | 0.97845 | 0.00002 |
| (50,10) | 0.1 | 0.002 | 0.00028 | 0.00000 |
| | 0.2 | 0.002 | 0.00102 | 0.00000 |
| | 0.3 | 0.003 | 0.00368 | 0.00000 |
| | 0.4 | 0.01 | 0.01401 | 0.00001 |
| | 0.5 | 0.05 | 0.05054 | 0.00004 |
| | 0.6 | 0.157 | 0.17075 | 0.00014 |
| | 0.7 | 0.452 | 0.45792 | 0.00022 |
| | 0.8 | 0.853 | 0.83797 | 0.00013 |
| | 0.9 | 0.989 | 0.99154 | 0.00000 |

**Table 3** The values of $P(C_L)$, $\overline{\hat{P}(C_L)}$ and MSE for the test at $\gamma = 0.05$

| *(n, m)* | $C_L$ | $P(C_L)$ | $\overline{\hat{P}(C_L)}$ | MSE |
|---|---|---|---|---|
| (10,2) | 0.1 | 0.287 | 0.29048 | 0.00015 |
| | 0.2 | 0.348 | 0.35118 | 0.00028 |
| | 0.3 | 0.417 | 0.41815 | 0.00022 |
| | 0.4 | 0.453 | 0.49504 | 0.00025 |
| | 0.5 | 0.583 | 0.57597 | 0.00021 |
| | 0.6 | 0.636 | 0.65109 | 0.00019 |
| | 0.7 | 0.749 | 0.7288 | 0.00018 |
| | 0.8 | 0.801 | 0.79704 | 0.00018 |
| | 0.9 | 0.858 | 0.85668 | 0.00009 |
| (20,4) | 0.1 | 0.132 | 0.13298 | 0.00012 |
| | 0.2 | 0.176 | 0.20031 | 0.00013 |
| | 0.3 | 0.273 | 0.29113 | 0.00018 |
| | 0.4 | 0.395 | 0.40436 | 0.00024 |
| | 0.5 | 0.504 | 0.53828 | 0.00028 |
| | 0.6 | 0.676 | 0.67845 | 0.00018 |
| | 0.7 | 0.801 | 0.80758 | 0.00018 |
| | 0.8 | 0.887 | 0.90554 | 0.00009 |
| | 0.9 | 0.964 | 0.96457 | 0.00003 |
| (30,6) | 0.1 | 0.031 | 0.03931 | 0.00003 |
| | 0.2 | 0.08 | 0.07566 | 0.00006 |
| | 0.3 | 0.132 | 0.13826 | 0.00012 |
| | 0.4 | 0.25 | 0.24643 | 0.00018 |
| | 0.5 | 0.425 | 0.40484 | 0.00022 |
| | 0.6 | 0.605 | 0.60371 | 0.00022 |
| | 0.7 | 0.774 | 0.79464 | 0.00018 |
| | 0.8 | 0.927 | 0.92893 | 0.00005 |
| | 0.9 | 0.986 | 0.98654 | 0.00001 |
| (50,10) | 0.1 | 0.002 | 0.00199 | 0.00002 |
| | 0.2 | 0.004 | 0.00591 | 0.00000 |
| | 0.3 | 0.016 | 0.0181 | 0.00001 |
| | 0.4 | 0.064 | 0.05213 | 0.00004 |
| | 0.5 | 0.131 | 0.14097 | 0.00010 |
| | 0.6 | 0.342 | 0.34203 | 0.00022 |
| | 0.7 | 0.635 | 0.65975 | 0.00027 |
| | 0.8 | 0.906 | 0.92025 | 0.00006 |
| | 0.9 | 0.998 | 0.9961 | 0.00000 |

From Tables 2 and 3, we observe the following:

- For fixed value of *m*, the simulation power $\overline{\hat{P}(C_L)}$ and the power $P(C_L)$ increase as $C_L$ increases (see Figure 1).

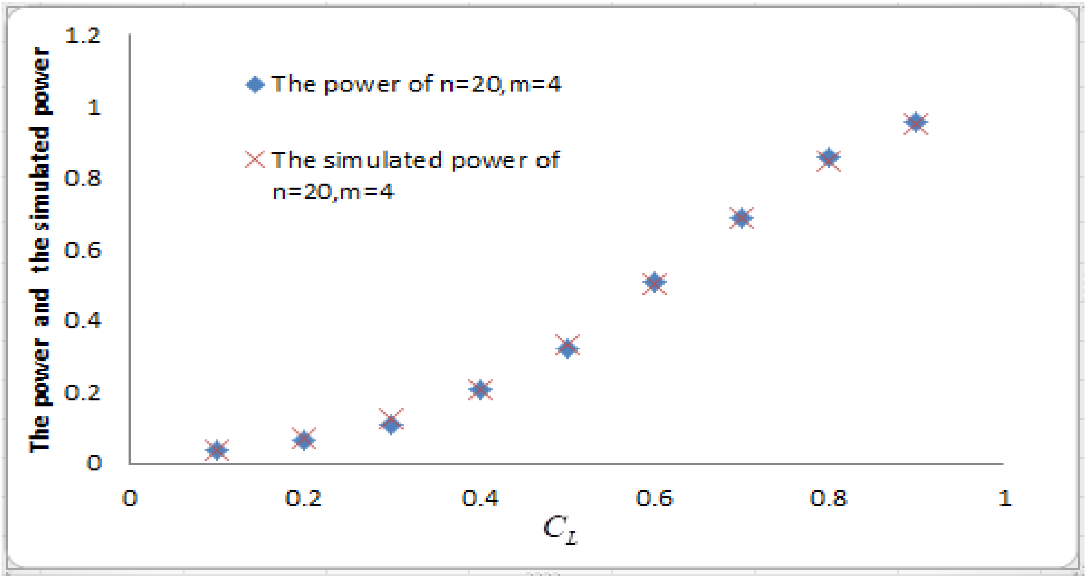

**Figure 1** The power function and simulated power function at $\gamma = 0.01$ for *n*=20, *m*=4 and

- All of the simulated power function $\overline{\hat{P}(C_L)}$ close to the power $P(C_L)$, for any value of $C_L$.
- The power values, $P(C_L)$ increases when $C_L$ increases for different values of *n* and *m* (see Figure 2).

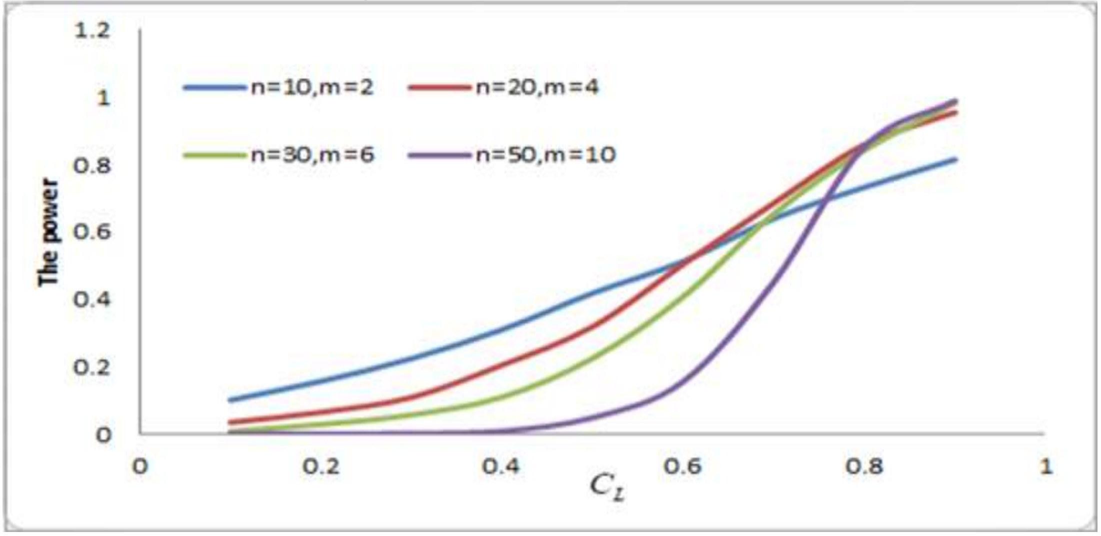

**Figure 2.** The power function of test at $\gamma = 0.01$ for different values of *n* and *m*

- Values of MSE are enough small and the scope of MSE is between 0.00000 and 0.00029.
- These results illustrate that the performance of our proposed method is acceptable.

## 6. Real Data Example

A real data set given by Mann and Fertig (1973) by (Real life data) is considered here. This data set is also studied by Altindag et al. (2017) who proposed the estimation and prediction problems for the Burr Type III distribution under Type II censored data.

The data set contains 10 failure times of airplane components of total 13 items. The censoring scheme is Type II censoring with $m = 10$ and $n = 13$. The observed failure times and the Type II censoring scheme are given in Table 4.
Then by using the transformed Type II censored sample with transformation $y = \log_e(1 + x^{-\lambda})$ for $\hat{\lambda} = 1.4990$ , the transformed values are calculated.

**Table 4.** The observed failure times and censoring scheme

| $i$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $x_i$ | 0.22 | 0.50 | 0.88 | 1.00 | 1.32 |
| $i$ | 6 | 7 | 8 | 9 | 10 |
| $x_i$ | 1.33 | 1.54 | 1.76 | 2.50 | 3.00 |

**Step 1:** The lower lifetime limit $L$=0.1. To deal with concerns about the lifetime performance, the conforming rate $P_r$ of products is required to exceed 80%. Referring to Table 1, the $C_L$ value is required to exceed 0.80.Thus, the performance index value is set at $c^*$ = 0.80. The testing hypothesis $H_0$: $C_L \le c^*$ versus $H_1$: $C_L > c^*$ is constructed.
**Step 2:** Specify a significance level $\gamma = 0.05$.
**Step 3:** Calculate the value of test statistic

$$\hat{C}_L = 1 - \frac{(10-1)\times 0.1}{7.39 + (13-10)2.37} = 0.94.$$

**Step 4:** Obtain the critical value $C_0^*$ = 0.885 by using Equation (14), according to $c^*$ = 0.80, $m$=10 and the significance level $\gamma = 0.05$.

Step5. Because of $\hat{C}_L = 0.94 > C_0^* = 0.885$, so we reject to the null hypothesis $H_0$: $C_L \leq c^*$

The lower confidence bound (0.90, ∞) for Type II censored. So, the performance index value to $c^* = 0.80$ don't belong to lower confidence bound, it is also concluded that the lifetime performance index of airplane components meets the required level.

## 7. Concluding Remarks

This study constructs a UMVUE estimator of $C_L$ under the assumption of Burr Type III distribution in case of Type II censored sample. The UMVUE estimator of $C_L$ is then utilized to develop the new hypothesis testing procedure in the condition of known L. The proposed testing procedure is easily applied and can effectively evaluate whether the lifetime of products meets requirements. In addition, this study provides a table of the lifetime performance index with its corresponding conforming rate. Hence, for any specified conforming rate, a corresponding $C_L$ can be obtained, and the hypotheses of the proposed testing procedure can also be expressed in terms of the conforming rate under $L$ is known limit.

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