Estimation of the Generalized Exponential Distribution Parameters Under Constant-Stress Partially Accelerated Life Testing Using Type I Censoring

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Abstract

Testing the lifetime of items under normal use condition often requires a long period of time, particularly for a products having high reliability. To minimize the costs involved in testing without reducing the quality of the data obtained, the items run at higher than usual level of stresses to induce early failure. This article concerns with constant-stress partially accelerated life tests (CS-PALT) based on type I censoring in which each test item is observed until it fails before a predetermined time, keeping all the stress factors at constant level. The lifetime of items is assumed to follow generalized exponential distribution. Maximum likelihood method is used to estimate the model parameters and acceleration factor of lifetime distribution from the test data. Netwon Raphson method is applied to solve numerically the non-linear likelihood equations using Mathcad (2001). Confidence intervals of the estimators are constructing. In addition, an asymptotic variance and covariance matrix of the estimators is obtained. Simulation studies are carried out to investigate the performance of the estimators.

Key words: *Reliability; Constant-stress, Partially accelerated life test; Accelerated factor, Generalized exponential distribution.*

1. Introduction

In several technological areas, it is more difficult to test or estimate the life time of today's products at normal condition. To overcome such difficulties, the test units may be subjected to more level stresses to assure rapid failure. Partial accelerated life test (PALT) is a widely used approach for reliability demonstration and prediction of components at normal operating conditions using data obtained at accelerated condition. It consists of a variety of test methods for shorting the lifetime of products or hastening the degradation of their performance to quickly get information over a range of condition, which are encountered in practice.

Nelson (1990) mentioned that the stress can be applied in various ways, the step-stress partially accelerated life test (SS-PALT) and constant-stress partially accelerated life test (CS-PALT) are commonly used methods. In SS-PALT a test item is first run at use condition and, if it does not fail for a specified time, then the unfailed units are subjected to accelerated conditions until the failure occurs or the observation is censored. While in CS-PALT each test item is run at a constant- stress level until the test is terminated.

SS-PALT was studied extensively and applied for estimating the parameters of many distributions by many authors [for example, Bhattacharyya and Soejoeti, (1989); Xiong (1998)*;* Xiong and Milliken(1999)*;* Abdel-Ghaly et al (2002); Abdel-Ghani (2004); Ismail (2004)]. For an overview of the CS-PALT, Bai and Chung (1992) used the maximum likelihood method to estimate the scale parameter and acceleration factor for the exponential distribution under two types of PALT which are step and constant stresses in case of type I censoring. In addition, they provided the optimal design for CS-PALT and SS-PALT that terminates at a predetermine time. Abdel-Ghani (1989) considered the estimation problem of the parameters and acceleration factor for Weibull distribution under CS-PALT. Ismail (2004) used maximum likelihood approach for estimating the acceleration factor and parameters of compound Pareto distribution. This work is conducted under CS-PALT based on type I and type II censoring. In addition, the problem of optimal design was considered for this type of PALT. Ismail (2006) has provided the optimum design of CS-PALT under type II censoring assuming the lifetime at design stress has a Weibull distribution.

This article is devoted for estimating failure time data for items under generalized exponential distribution in CS-PALT. The CS-PALT allows the test to be run either at use or accelerated condition only at specified time. Point and interval estimates of the parameters and acceleration factor will be obtained using maximum likelihood method. In order to study the precision and variation of maximum likelihood estimators (RABias); which is the absolute difference between the mean estimates and its true value divided by the true value of the parameter, and the mean square error (MSE) of estimators. Furthermore**,** the asymptotic variance covariance matrix will be obtained.

The organization of this article is as follows. In Section 2 generalized exponential distribution is introduced as the lifetime model and the assumption of the CS-PALT are described. The detailed procedure for maximum likelihood parameters estimates will be obtained in Section 3. In Section 4 the approximate asymptotic variances and covariances matrix are developed. Simulation studies are carried out in Section 5. Finally, conclusions are included in Section 6. Tables are displayed at the end of this article.

2. Model and Test Model

Generalized exponential distribution or exponentiated exponential distribution has introduced and studied quite extensively by (Gupta and Kundu ;1999, 2001a, 2001b), see also Ragab and Ahsanullah (2001). The generalized exponential (GE) distribution function has the form:

$$F(t;\alpha,\lambda) = (1 - e^{-\alpha t})^{\lambda}; \qquad \alpha,\lambda, t > 0; \qquad (2.1)$$

The corresponding density function is:

$$f(t;\alpha,\lambda) = \alpha\lambda(1 - e^{-\alpha t})^{\lambda-1} e^{-\alpha t}. \qquad \alpha,\lambda, t > 0; \qquad (2.2)$$

where, α is the scale and λ is the shape parameters. For different values of the shape parameter, the probability density function has different shapes and that are quite similar to the Weibull distribution. The hazard function of a GE distribution can be increasing, decreasing or constant depending on the shape parameter λ. It has been observed that the GE can be used quite effectively to analyze skewed data set and it is a good alternative to the Weibull distribution. In many situations the GE distribution might provide a better data fit than the Weibull distribution

The concept of the CS-PALT was given by Bai and Chung (1992). In this type of test, the test units of the total sample size n are divided into two parts with certain proportion π. The $n\pi$ items randomly chosen among the n test items are subjected to stress condition, while the remaining $n(1-\pi) = n\bar{\pi}$ are allocated at the normal condition, where $\pi + \bar{\pi} = 1$. Each test item run until censoring time η is reached and the test condition is not changed.

Thus the total lifetime of test items, passes through two stages, which are the normal and accelerated conditions. Let the lifetime of test item is assumed to follow GE with scale parameter α and shape parameter λ. Therefore, the probability density function of lifetime, T_i, $i = 1,2,...,n(1-\pi)$ at use condition, is given by the equation (2.2) and its survival function at censoring time η is given by

$$S(\eta;\alpha,\lambda) = 1-(1\text{-}\mathrm{e}^{-\alpha\,\eta})^{\lambda}; \qquad \alpha,\lambda\ ,\eta > 0 \tag{2.3}$$

While the probability density function of lifetime, X_j, $j = 1,2,...,n\pi$ at accelerated condition, is given by

$$f(x;\alpha,\lambda,\beta) = \alpha\,\lambda\beta(1\text{-}\mathrm{e}^{-\alpha\beta\,\mathrm{x}})^{\lambda-1} e^{-\alpha\,\beta\,x}. \qquad \alpha,\lambda\ ,x > 0 \tag{2.4}$$

where, $X = T\,\beta^{-1}$, and β is the acceleration factor (usually $\beta > 1$) which is the ratio of the mean life at use condition to that at accelerated condition. Its survival function at censoring time η is given by

$$S(\eta;\alpha,\lambda,\beta) = 1-(1\text{-}\mathrm{e}^{-\alpha\beta\eta})^{\lambda}; \qquad \alpha,\lambda\ ,\eta > 0 \tag{2.5}$$

The lifetimes T_i and X_j are assumed to be mutually statistically independent.

3. Parameters Estimates in Constant Stress PALT

The method of maximum likelihood is the most widely used for most theoretical models and kinds of censored data. Maximum likelihood estimates have the desirable properties of being consistent and asymptotically normal for large samples under regularity conditions. In this section the point and interval estimates of the parameters and acceleration factor for the generalized exponential lifetime distribution in CS-PALT based on type I censoring will be obtained.

In CS-PALT, a sample of size $n(1-\pi)$ units randomly chosen among n units are tested under normal condition and the test terminates when the censoring time η is reached. The observed ordered failures times at normal conditions are $t_{(1)} \leq ... < t_{(n_u)} \leq \eta$, and n_u is the number of items failed at normal conditions. The remaining $n\pi$ units are subjected to accelerated condition until censoring time η is reached. The observed ordered failures times at accelerated conditions are $x_{(1)} \le ... < x_{(n_a)} \le \eta$, and n_a is the number of items failed at accelerated conditions.

Let δ_{ui} and δ_{aj} be indicator functions such that

$$\delta_{ui} = \begin{cases} 1 & T_i \le \eta \\ 0 & otherwise \end{cases} \qquad i = 1,2,...,n(1-\pi)$$

and,

$$\delta_{aj} = \begin{cases} 1 & X_j \le \eta \\ 0 & otherwise \end{cases} \qquad j = 1,2,...,n\pi$$

For simplifying $t_{(i)}$ can be expressed by t_i and $x_{(j)}$ can also be expressed by x_j. In view of equations (2.1) and (2.3), the likelihood function for (t_i, δ_{ui}) is given by:

$$L_{ui}(\alpha, \lambda | t_i, \delta_{ui}) = [\alpha\lambda e^{-\alpha t_i}(1-e^{-\alpha t_i})^{\lambda-1}]^{\delta_{ui}}[1-(1-e^{-\alpha\eta})^{\lambda}]^{\bar{\delta}_{ui}}$$

Also, in view of equations (2.4) and (2.5), the likelihood function for (x_j, δ_{aj}) is given by:

$$L_{aj}(\alpha, \lambda, \beta | x_j, \delta_{aj}) = [\alpha\lambda\beta e^{-\alpha x_j}(1-e^{-\alpha\beta x_j})^{\lambda-1}]^{\delta_{aj}}[1-(1-e^{-\alpha\beta\eta})^{\lambda}]^{\bar{\delta}_{aj}}$$

Since the lifetimes of $t_1,...,t_{nu}$ and $x_1,...,x_{na}$ are independent and identically distributed random variables, then the total likelihood function for $(t_1;\delta_{u1},...,t_{n(1-n\pi)};\delta_{un(1-n\pi)}, x_1, \delta_{a1},..., x_{n\pi};\delta_{an\pi})$ is given by

$$L(\alpha, \lambda, \beta) = \prod_{i=1}^{n\bar{\pi}} \alpha\lambda e^{-\alpha t_i}(1-e^{-\alpha t_i})^{\lambda-1}]^{\delta_{ui}}[1-(1-e^{-\alpha t_i})^{\lambda}]^{\bar{\delta}_{ui}}$$
$$\prod_{j=1}^{n\pi}[\alpha\lambda\beta e^{-\alpha\beta x_j}(1-e^{-\alpha\beta x_j})^{\lambda-1}]^{\delta_{aj}}[1-(1-e^{-\alpha\beta\eta})^{\lambda}]^{\bar{\delta}_{aj}} \tag{3.1}$$

where $\bar{\delta}_{ui} = 1-\delta_{ui}$, $\bar{\delta}_{aj} = 1-\delta_{aj}$ and $\bar{\pi} = 1-\pi$.

The maximum likelihood estimators, $\hat{\alpha}$, $\hat{\lambda}$ and $\hat{\beta}$, of the parameters and acceleration factor α, λ and β are the values which maximize the likelihood function. The logarithm of the likelihood function is given by:

$$\ln L = n_0 \ln\alpha + n_0 \ln\lambda + n_a \ln\beta - \alpha\left\{\sum_{i=1}^{n\bar{\pi}}\delta_{ui}t_i + \beta\sum_{j=1}^{n\pi}\delta_{aj}x_j\right\} + (\lambda-1)\{\sum_{i=1}^{n\bar{\pi}}\delta_{ui}\ln(1-e^{-\alpha t_i}) +$$
$$\sum_{j=1}^{n\pi}\delta_{aj}\ln(1-e^{-\alpha\beta x_j})\} + (n\bar{\pi}-n_u)\ln[1-(1-e^{-\alpha\eta})^{\lambda}] + (n\pi-n_a)\ln[1-(1-e^{-\alpha\eta\beta})^{\lambda}]$$

(3.2)

where, $n_u = \sum_{i=1}^{n\bar{\pi}} \delta_{ui}$, $n_a = \sum_{j=1}^{n\pi} \delta_{aj}$ and $n_0 = n_u + n_a$

The first derivatives of the logarithm of the likelihood function (3.2) with respect to α, β and λ are given by:

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n_0}{\alpha} - \sum_{i=1}^{n\bar{\pi}} \delta_{ui}\, t_i - \beta \sum_{j=1}^{n\pi} \delta_{aj} x_j + (\lambda - 1)\left(\Psi_1 + \beta\,\Psi_2\right) - \lambda\eta\varphi_1 - \lambda\beta\eta\varphi_2, \quad (3.3)$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{n_a}{\beta} - \alpha \sum_{j=1}^{n\pi} \delta_{aj} x_j + \alpha(\lambda - 1)\Psi_2 - \lambda\alpha\eta\varphi_2, \quad (3.4)$$

and,

$$\frac{\partial \ln L}{\partial \lambda} = \frac{n_0}{\lambda} + \sum_{i=1}^{n\bar{\pi}} \delta_{ui} \ln\left(1 - e^{-\alpha t_i}\right) + \sum_{j=1}^{n\pi} \delta_{aj} \ln\left(1 - e^{-\alpha\beta x_j}\right) - M_1 - M_2. \quad (3.5)$$

where, $\Psi_1 = \sum_{i=1}^{n\bar{\pi}} \frac{\delta_{ui}\, t_i e^{-\alpha t_i}}{\left(1 - e^{-\alpha t_i}\right)}$, $\Psi_2 = \sum_{j=1}^{n\pi} \frac{\delta_{aj}\, x_j e^{-\alpha\beta x_j}}{\left(1 - e^{-\alpha\beta x_j}\right)}$, $\varphi_1 = \frac{(n\bar{\pi} - n_u) a_1^{\lambda-1} e^{-\alpha\eta}}{[1 - a_1^{\lambda}]}$

$\varphi_2 = \frac{(n\pi - n_a) a_2^{\lambda-1} e^{-\alpha\beta\eta}}{[1 - a_2^{\lambda}]}$, $a_1 = \left(1 - e^{-\alpha\eta}\right)$ $a_2 = \left(1 - e^{-\alpha\beta\eta}\right)$

$M_1 = \frac{(n\bar{\pi} - n_u) a_1^{\lambda} \ln a_1}{[1 - a_1^{\lambda}]}$ and $M_2 = \frac{(n\pi - n_a) a_2^{\lambda} \ln a_2}{[1 - a_2^{\lambda}]}$

Therefore, the maximum likelihood estimates of α, β and λ are obtained by setting Equations (3.3)-(3.5) to be equal zero, hence

$$\frac{n_0}{\hat{\alpha}} - \sum_{i=1}^{n\bar{\pi}} \delta_{ui}\, t_i - \hat{\beta} \sum_{j=1}^{n\pi} \delta_{aj} x_j + \left(\hat{\lambda} - 1\right)\left(\Psi_1 + \hat{\beta}\,\Psi_2\right) - \hat{\lambda}\eta\varphi_1 - \hat{\lambda}\hat{\beta}\eta\varphi_2 = 0 \quad (3.6)$$

$$\frac{n_a}{\hat{\beta}} - \hat{\alpha} \sum_{j=1}^{n\pi} \delta_{aj} x_j + \hat{\alpha}\left(\hat{\lambda} - 1\right)\Psi_2 - \hat{\lambda}\hat{\alpha}\eta\varphi_2 = 0 \quad (3.7)$$

and,

$$\frac{n_0}{\hat{\lambda}} + \sum_{i=1}^{n\bar{\pi}} \delta_{ui} \ln\left(1 - e^{-\hat{\alpha} t_i}\right) + \sum_{j=1}^{n\pi} \delta_{aj} \ln\left(1 - e^{\hat{\alpha}\hat{\beta} x_j}\right) - M_1 - M_2 = 0 \quad (3.8)$$

Obviously, it is very difficult to obtain a closed form solution for the three non-linear equations. So, iterative procedures will be used to solve these equations numerically using Mathcad (2001). Newton Raphson method is used to get the maximum likelihood estimators of α, β and λ.

For large sample size, the maximum likelihood estimates are consistent and asymptotically normally distributed. Therefore, the two sided approximate 100γ % confidence limits for the maximum likelihood estimates α^*, β^* and λ^* of a population parameter α, β and λ can be obtained, such that

$$p[\alpha - z\sigma(\alpha^*) \leq \alpha^* \leq \alpha + z\sigma(\alpha^*)] \cong \gamma,$$

$$p[\beta - z\sigma(\beta^*) \leq \beta^* \leq \beta + z\sigma(\beta^*)] \cong \gamma \tag{3.9}$$

and, $$p[\lambda - z\sigma(\lambda^*) \leq \lambda^* \leq \lambda + z\sigma(\lambda^*)] \cong \gamma$$

where, z is the $[\frac{100(1-\gamma)}{2}]$ standard normal percentile and $\sigma(.)$ is the standard deviation for the maximum likelihood estimates. Therefore, the two sided approximate confidence limits for α, β and λ will be constructed with confidence levels 95 % and 99 %.

4. Asymptotic Variances and Covariances of Estimates

The asymptotic variances covariances of maximum likelihood estimates are given by the elements of the inverse of the Fisher information matrix

$$I_{ij}(\underline{\theta}) \cong E\{-\partial^2 \ln L / \partial\theta_i \partial\theta_j\}, i, j = 1,2,3 \text{ and } (\underline{\theta}) = (\alpha, \beta, \lambda) \tag{4.1}$$

Unfortunately, the exact mathematical expressions for the above expectation are very difficult to obtain. Therefore, according to Cohen (1965) the observed Fisher information matrix is obtained by dropping the expectation on operation E. The approximate (observed) asymptotic variance covariance matrix H for the maximum likelihood estimates can be written as follows

$$H = I_{ij}(\underline{\theta}) \cong \{-\partial^2 \ln L / \partial\theta_i \partial\theta_j\}, \quad i, j = 1,2,3 \text{ and } (\underline{\theta}) = (\alpha, \beta, \lambda) \tag{4.2}$$

The elements of the matrix H are obtained by taking the second derivatives of the logarithm of likelihood function and substituting $\hat{\beta}$ for β, $\hat{c}$ for c and $\hat{k}$ for k:

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = -\frac{n_0}{\alpha^2} - (\lambda - 1)[\sum_{i=1}^{n\bar{\pi}} \frac{\delta_{ui} e^{-\alpha t_i} t_i^2}{\left(1 - e^{-\alpha t_i}\right)^2} + \beta^2 \sum_{i=1}^{n\pi} \frac{\delta_{aj} e^{-\alpha \beta x_j} x_j^2}{\left(1 - e^{-\alpha \beta x_j}\right)^2}] - \lambda \eta A_1 \varphi_1 - \lambda \beta^2 \eta A_2 \phi_2 \tag{4.3}$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \lambda} = \Psi_1 + \beta \Psi_2 - \eta \varphi_1 - \lambda \eta M_1 B_1 - \eta \beta \phi_2 - \lambda \beta \eta M_2 B_2 \tag{4.4}$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \beta} = -\sum_{j=1}^{n\pi} \delta_{aj} x_j + (\lambda - 1)[\Psi_2 - \alpha \beta \sum_{j=1}^{n\pi} \frac{\delta_{aj} x_j^2 e^{-\alpha \beta x_j}}{\left(1 - e^{-\alpha \beta x_j}\right)^2}] - \lambda \eta \varphi_2 - \lambda \eta \beta \alpha \phi_2 A_2 \tag{4.5}$$

$$\frac{\partial^2 \ln L}{\partial \beta \partial \lambda} = \alpha \Psi_2 - \alpha \eta M_2 B_2 - \alpha \eta \varphi_2 \tag{4.6}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -\frac{n_a}{\beta^2} - \alpha^2 (\lambda - 1) \left[\sum_{i=1}^{n\pi} \frac{\delta_{aj} x_{aj}^2 \ e^{-\alpha \beta x_j}}{\left(1 - e^{-\alpha \beta x_j}\right)^2} \right] - \alpha^2 \eta \varphi_2 A_2 \lambda \tag{4.7}$$

and,

$$\frac{\partial^2 \ln L}{\partial \lambda^2} = -\frac{n_0}{\lambda^2} - \frac{M_1 \ln a_1}{[1 - a_1^{\lambda}]} - \frac{M_2 \ln a_2}{[1 - a_2^{\lambda}]} \tag{4.8}$$

where,

$$A_1 = \eta[\frac{e^{-\alpha \eta}\left(a_1^{\lambda} + \lambda - 1\right) - a_1\left(1 - a_1^{\lambda}\right)}{a_1\left(1 - a_1^{\lambda}\right)}] , \ A_2 = \eta[\frac{e^{-\alpha \eta \beta}\left(a_2^{\lambda} + \lambda - 1\right) - a_2\left(1 - a_2^{\lambda}\right)}{a_2\left(1 - a_2^{\lambda}\right)}]$$

$$B_1 = \frac{e^{-\alpha \eta}}{a_1\left(1 - a_1^{\lambda}\right)} \ \text{and,} \ B_2 = \frac{e^{-\alpha \beta \eta}}{a_2\left(1 - a_2^{\lambda}\right)}$$

Consequently, the maximum likelihood estimators of α, β and λ have an asymptotic variance covariance matrix defined by inverting the Fisher information matrix H as indicated above.

5. Simulation Studies

Performances of CS-PALT and model assumptions are usually evaluated by the properties of the maximum likelihood of model parameters. Simulation studies have been performed to investigate the performance of the estimators for items having GE lifetime distribution based on type I censoring. The investigated properties are relative absolute bias (RABias) and mean square error (MSE). Furthermore, the asymptotic variance and covariance matrix and two-sided confidence intervals of the acceleration factor and two parameters are obtained. All the computation is performed via Mathcad statistical package. The algorithm for the CS-PALT parameter estimation can be summarized in the following steps:

Step (1): Following Ismail(2006) a random sample, $x_1, x_2, ..., x_n$, of sizes n = 100 (100) 500 were selected. These random samples were generated from the GE distribution by using the transformation $z_i = (-\frac{1}{\alpha})\ln[1-(u_i)^{\frac{1}{\lambda}}]$, i = 1,2,…,n where $u_1,...,u_n$ are random sample from uniform(0,1).

Step (2): Three sets of parameters will be selected which are $(\alpha = 0.5, \lambda = 0.5, \beta = 3)$, $(\alpha = 1, \lambda = 1.5, \beta = 2)and(\alpha = 2, \lambda = 1, \beta = 1.5)$. The censoring time η will be taken to be 10 throughout the tests.

Step (3): The n test items are divided with certain sample proportion π, let it taken to be $\pi = 0.5$, such that $n\pi$ items are allocated at accelerated condition and the remaining $n\bar{\pi}$ are allocated at normal condition ; where $\bar{\pi} + \pi = 1$

Step (4): Maximum likelihood method will be used to estimate the parameters and acceleration factor for the two stress level in each sample size with the same censoring time η.

Step (5): The nonlinear equations (3.6)-(3.8) of the maximum likelihood estimates will be solved iteratively using Netwon Raphson method.

Step (6): The resulting estimates of the parameters and acceleration factor will be used to construct confidence limits with confidence level at $\gamma = 0.95$ and $\gamma = 0.99$, also asymptotic variance and covariance matrix of the estimators was obtained.

Step (7): The performance of the estimators can be evaluated through some measures of accuracy which are RABias and MSE.

Step (8): Steps from 1 to7 will be repeated 1000 times for each sample size and for the selected sets of parameters.

Simulation results are summarized in Tables 1-5. From these Tables, the following conclusions can be observed on the properties of estimated parameters from the GE lifetime distribution in CS-PALT:

1. The RABiases and MSEs for the set of parameters $(\alpha = 0.5, \beta = 3, \lambda = 0.5)$ are the smallest among the three sets of data.

2. For the three sets of data, the maximum likelihood estimates approximate the true value as the sample increase.

3. As the sample size increases the RABiases and MSEs of the estimated parameters decrease. This indicates that the maximum likelihood estimates provide asymptotically normally distributed and consistent estimators for the parameters and acceleration factor. The asymptotic variances of the estimators are decreasing when the sample size increasing (see Table4).

4. The interval of the estimators decreases when the sample size is increasing. Also, the interval of the estimator at $\gamma = 0.95$ is smaller than the interval of estimator at $\gamma = 0.99$ (see Table5).

6. Concluding Remarks

ALT or PALT are used to estimate the lifetime of highly realizable products within a reasonable testing time. The test units are run at higher than usual levels of stress to induce early failures. The test data obtained at the accelerated conditions are analyzed in terms of a model, and then extrapolated to design stress to estimate the life distribution. The main assumption in the ALT is that the relationship between the mean lifetime and the stress is known, while In PALT this relationship is not known.

Stress can be applied in various ways, commonly used stress are SS-PALT and CS-PALT. For some materials and products, accelerated test models for constant stress are well developed and experimentally established. Examples of a constant stress are temperature, voltage, and current. It is easier to maintain a constant-stress level in most tests. Also, Yang (1994) mentioned that constant-stress are widely used to save time and cost.

This study deals with estimating the failure time data under type I censoring in CS-PALT, assuming the lifetime at design condition has GE distribution. In a CS-PALT, each test item is subjected to constant stress level until the censoring time is reached. The maximum likelihood estimates of the acceleration factor and parameters were obtained numerically. Performance of constant-stress testing plans and model assumptions are usually evaluated by the properties of the maximum likelihood estimates of model parameters. The asymptotic variance and covariance of estimators are obtained. In addition, the two-sided confidence limits of the model parameters are constructed.

Table1: The Estimates, Variances, RABias and MSE for different sized sample of the Parameters ($\alpha = 1$, $\beta = 2$ and $\lambda = 1.5$). Using Type I Censoring in CS-PALT Given $\pi = 0.5$

n	Parameters	True	Estimates	Variances	RABias	MSE
100	α	1.000	0.943	0.0240	0.0570	0.0270
	β	2.000	2.121	0.0858	0.0600	0.1000
	λ	1.500	1.469	0.0501	0.0210	0.0510
200	α	1.000	0.981	0.0176	0.0190	0.0180
	β	2.000	2.054	0.0256	0.0270	0.0290
	λ	1.500	1.479	0.0380	0.0140	0.0380
300	α	1.000	0.978	0.0151	0.0220	0.0150
	β	2.000	2.040	0.0161	0.0200	0.0180
	λ	1.500	1.481	0.0327	0.0130	0.0330
400	α	1.000	0.982	0.0151	0.0180	0.0160
	β	2.000	2.022	0.0065	0.0110	0.0070
	λ	1.500	1.486	0.0313	0.0095	0.0320
500	α	1.000	0.987	0.0136	0.0220	0.0140
	β	2.000	2.016	0.0046	0.0078	0.0048
	λ	1.500	1.494	0.0262	0.0042	0.0270

Table2: The Estimates, Variances, RABias and MSE for different sized sample of the Parameters ($\alpha = 2$, $\beta = 1.5$ and $\lambda = 1$). Using Type I Censoring in CS-PALT Given $\pi = 0.5$

n	Parameters	True	Estimates	Variances	RABias	MSE
100	α	2.000	2.125	0.1436	0.0620	0.1590
	β	1.500	1.544	0.0240	0.0290	0.0260
	λ	1.000	1.068	0.0376	0.0680	0.0420
200	α	2.000	2.100	0.0967	0.0550	0.1090
	β	1.500	1.514	0.0039	0.0075	0.0041
	λ	1.000	1.051	0.0240	0.0510	0.0270
300	α	2.000	2.068	0.0660	0.0340	0.0710
	β	1.500	1.506	0.0018	0.0037	0.019
	λ	1.000	1.031	0.0174	0.0310	0.0180
400	α	2.000	2.042	0.0488	0.0210	0.0510
	β	1.500	1.502	0.0002	0.0011	0.0000
	λ	1.000	1.016	0.0112	0.0160	0.0120

Continued Table 2

500	α	2.000	2.027	0.0390	0.0140	0.0380
	β	1.500	1.501	0.0000	0.0006	0.0000
	λ	1.000	1.013	0.0085	0.0130	0.0082

Table3: The Estimates, Variances, RABias and MSE for different sized sample of the Parameters ($\alpha = 0.5$, $\beta = 3$ and $\lambda = 0.5$). Using Type I Censoring in CS-PALT Given $\pi = 0.5$

n	Parameters	True	Estimates	Variances	RABias	MSE
100	α	0.500	0.504	0.0026	0.0089	0.0026
	β	3.000	3.005	0.0000	0.0015	0.0000
	λ	0.500	0.519	0.0061	0.0380	0.0064
200	α	0.500	0.503	0.0012	0.0055	0.0012
	β	3.000	3.005	0.0000	0.0015	0.0000
	λ	0.500	0.507	0.0030	0.0150	0.0031
300	α	0.500	0.496	0.0007	0.0036	0.0007
	β	3.000	3.005	0.0000	0.0016	0.0000
	λ	0.500	0.502	0.0018	0.0046	0.0018
400	α	0.500	0.497	0.0006	0.0084	0.0006
	β	3.000	3.005	0.0000	0.0016	0.0000
	λ	0.500	0.503	0.0013	0.0025	0.0013
500	α	0.500	0.498	0.0005	0.0068	0.0005
	β	3.000	3.005	0.0000	0.0015	0.0000
	λ	0.500	0.499	0.0012	0.0058	0.0011

Table 4: Asymptotic Variances and Covariances of Estimates under Type I Censoring for the Sets of Data

	$(\alpha = 0.5, \beta = 3, \lambda = 0.5)$			$(\alpha = 1, \beta = 2, \lambda = 1.5)$			$(\alpha = 2, \beta = 1.5, \lambda = 1)$		
n	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$
100	0.0110	0.0026	-0.054	0.0200	0.0170	-0.031	0.0128	0.0260	-0.073
	0.0026	0.0037	0.0002	0.0170	0.0430	-0.002	0.0260	0.0200	-0.002
	-0.054	0.0002	0.6470	-0.031	-0.002	0.133	-0.073	-0.002	0.0990
200	0.0054	0.0013	-0.027	0.0100	0.0087	-0.015	0.0610	0.013	-0.033
	0.0013	0.0018	0.0002	0.0087	0.0220	-0.000	0.0130	0.0095	-0.000
	-0.027	0.0002	0.3270	-0.015	-0.000	0.0600	-0.033	-0.000	0.0460
300	0.0037	0.0008	-0.018	0.0067	0.0057	-0.009	0.0390	0.0076	-0.021
	0.0008	0.001	-0.000	0.0057	0.0150	-0.000	0.0076	0.0061	-0.000
	-0.018	0.000	0.222	-0.009	-0.000	0.0390	-0.021	-0.000	0.0300
400	0.0027	0.0006	-0.014	0.0051	0.0043	-0.007	0.0290	0.0057	-0.016
	0.0006	0.0008	0.000	0.0043	0.0110	-0.000	0.0057	0.0044	-0.000
	-0.014	0.0000	0.1690	-0.007	-0.000	0.0290	-0.016	-0.000	0.0300

Continued Table 4

500	0.0022	0.0005	-0.011	0.0040	0.0034	-0.005	0.0220	0.0045	-0.012
	0.0005	0.0007	0.0000	0.0034	0.0087	0.0000	0.0045	0.0035	-0.000
	-0.011	0.0000	0.1330	-0.005	0.0000	0.0230	-0.012	-0.000	0.0180

Table 5: Confidence Bounds of the Estimates at Confidence level 0.95 and 0.99

n	Parameters	$(\alpha=0.5,\beta=3,\lambda=0.5)$		$(\alpha=1,\beta=2,\lambda=1.5)$		$(\alpha=2,\beta=1.5,\lambda=1)$	
		Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound
100	α	0.405 0.374	0.604 0.635	0.639 0.542	1.247 1.344	1.383 1.148	2.867 3.102
	β	2.958 2.948	3.026 3.036	1.547 1.365	2.695 2.876	1.240 1.144	1.848 1.944
	λ	0.366 0.317	0.672 0.721	1.030 0.891	1.907 2.046	0.688 0.568	1.449 1.569
200	α	0.434 0.412	0.571 0.593	0.720 0.637	1.242 1.325	1.500 1.307	2.720 2.913
	β	2.998 2.996	3.011 3.013	1.740 1.641	2.368 2.467	1.388 1.349	1.634 1.673
	λ	0.339 0.365	0.615 0.650	1.112 0.992	1.875 1.996	0.747 0.651	1.356 1.452
300	α	0.468 0.448	0.594 0.614	0.738 0.662	1.219 1.295	1.564 1.405	2.572 2.732
	β	3.003 3.002	3.007 3.007	1.790 1.711	2.289 2.368	1.421 1.394	1.590 1.617
	λ	0.429 0.402	0.604 0.631	1.130 1.018	1.841 1.954	0.773 0.691	1.290 1.372
400	α	0.447 0.431	0.545 0.560	0.740 0.664	1.224 1.300	1.609 1.472	2.475 2.612
	β	3.003 3.002	3.006 3.007	1.863 1.813	2.181 2.232	1.474 1.465	1.529 1.538
	λ	0.429 0.407	0.569 0.591	1.134 1.025	1.828 1.937	0.808 0.742	1.224 1.290
500	α	0.452 0.438	0.541 0.555	0.748 0.676	1.208 1.281	1.645 1.524	2.410 2.531
	β	3.004 3.004	3.006 3.006	1.883 1.841	2.148 2.190	1.482 1.476	1.520 1.526
	λ	0.437 0.416	0.569 0.590	1.161 1.061	1.797 1.898	0.834 0.777	1.192 1.249

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