Goodness-Of-Fit For The Generalized Exponential Distribution

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Abstract

Recently a new distribution called generalized exponential or exponentiated exponential distribution was introduced and studied quite extensively by the authors (see Gupta and Kundu, 1999, 2001a, 2001b, 2002, 2003). A class of goodness-of-fit tests for the generalized exponential distribution with estimated parameter is proposed. The tests are based on the empirical distribution function. These test statistics are available when the hypothesized distribution is completely specified. When the parameters of the generalized exponential distribution are not known and must be estimated from the sample data, the standard Tables for these test statistics are not valid. This article uses Monte Carlo and Pearson system techniques to create Tables of critical values for such situations. Moreover, the power of the proposed test statistics is investigated for a number of alternative distributions. The results of the power studies showed that the test statistic proposed by Liao and Shimokawa (1999) is the most powerful goodness-of-fit test among the competitors.

Key words: Anderson-Darling test statistic, Cramer–von Mises test statistic, Kolmogorov-Smirnov statistic, Watson statistic, Critical values, Generalized exponential distribution, Power test.

1. Introduction

Recently a new distribution, named generalized exponential distribution or exponentiated exponential distribution was introduced and studied quite extensively by Gupta and Kundu (1999, 2001a, 2001b, 2002, 2003). The generalized exponential has the distribution function

$$F(x,\alpha,\lambda) = (1 - e^{-\lambda x})^{\alpha}; \qquad \alpha, \lambda, x > 0 \tag{1.1}$$

with the density function

$$f(x,\alpha,\lambda) = \alpha\lambda(1 - e^{-\lambda x})^{\alpha-1} e^{-\lambda x}. \qquad \alpha, \lambda, x > 0 \tag{2.1}$$

Where α is the shape parameter and λ is the scale parameter. When the shape parameter α equals one it reduces to a one-parameter exponential distribution, that is, generalized exponential is a generalization of a one-parameter exponential distribution. Generalized exponential distribution with shape parameter α and scale parameter λ will be denoted by

GE(α, λ). It is observed in Gupta and Kundu (1999) that the two-parameter GE(α, λ) can be used quite effectively in analyzing many lifetime skewed data, and the properties of the two-parameter GE(α, λ) distribution are quite close to the corresponding properties of the two-parameter gamma distribution. Gupta and Kundu (2001a) estimate the unknown parameters α and λ using different methods of estimation. They compare maximum likelihood estimators with moment estimators, least square estimators, weighted least square estimators, estimators based on percentiles, and estimators based on the linear combination of order statistics in terms of their bias and mean square error. They concluded from their simulation that the percentile estimators have smaller bias in almost all cases for estimating α and λ followed by the least square estimators and weighted least square estimators. For the mean square error, the maximum likelihood estimators have smaller mean square error compared to other estimators.

Goodness-of-fit tests are designed to measure the compatibility of a random sample with a theoretical probability distribution function. Several goodness-of-fit tests are available in the literature such as those of Kolmogorov-Smirnov (K-S) statistic, Cramer-von-Mises (C-M) statistic, Anderson-Darling (A-D) statistic , Watson test statistic, and L_n test statistic which introduced by Liao and Shimokawa (1999). These test statistics are generally measure, in different ways the distance between a continuous distribution function F(x) and the empirical distribution function $F_n(x)$. They are also called empirical distribution function test statistics. However, these tests require continuous underlying distributions with known parameters. Moreover, goodness-of-fit tests are not distribution free when the parameters must be estimated from the sample data. In the last two decades, many authors (for example, Lawless, (1982); Liao and Shimokawa, (1999b); Littell *et al* (1979); Park *et al* (1994); Stephens, (1974)) have reported that the A-D and C-M test statistics are more powerful than the K-S test. Liao and Shimokawa (1999) concluded that the L_n test statistic is the most powerful goodness-of-fit test among the corresponding K-S, C-M and A-D test statistics for testing the type-I extreme-value and 2- parameter Weibull distributions with estimated parameters. Hassan (1999) concluded that the A-D test statistic is more powerful than the K-S and C-M test statistics for testing the generalized gamma distribution. Another class of goodness of fit tests based on the empirical Laplace transform was discussed by many authors (for example Baringhaus and Henze (1991), Henze and Meintanis (2002a), Henze and Meintanis (2002b) Meintanis and Iliopoulosl (2003)).

In this article, extensive Tables of goodness-of-fit critical values for the generalized exponential distribution are developed through simulation for the K-S, C-M, A-D, Watson statistic, and L_n test statistic. We concentrate on the most practical case in which the parameters are not known. This problem is studied through three different cases, when one of the two parameters is unknown and when both parameters are unknown. Using a Mathcad (2001), critical values for these test statistics will be obtained using two different techniques. The first method is based on the Monte Carlo simulation, while the second method used Pearson system to obtain the sampling distributions of the proposed test statistics, from the resulting sampling distributions critical values for the test statistics are obtained. In addition, power comparisons of test statistics are investigated.

The paper is organized as follows. **Section** 2 deals with the estimation of unknown parameters under three cases. **Section** 3 discusses the problem of obtaining the critical values for the test statistics by using two different methods. **Section** 4 gives power comparisons among the K-S, C-M, A-D, Watson, and L_n test statistics. Finally conclusions are shown in **Section** 5.

2. Estimation Of The Unknown Parameters

This **Section** is concerned with the maximum likelihood estimation of the unknown parameters α and λ for the GE (α, λ). This problem is studied through three cases.

Case 1, the maximum likelihood estimators in which both parameters α and λ are unknown. Let X_1, X_2,…,X_n be a random sample from a generalized exponential distribution with unknown parameters α and λ. The maximum likelihood estimator of λ, say $\hat{\lambda}_1$ can be obtained as a solution of the equation

$$\frac{n}{\hat{\lambda}_1}-[\frac{n}{\sum_{i=1}^{n}\ln(1-e^{-\hat{\lambda}_1 x_i})}+1][\sum_{i=1}^{n}\frac{x_i e^{-\hat{\lambda}_1 x_i}}{(1-e^{-\hat{\lambda}_1 x_i})}]-\sum_{i=1}^{n}x_i=0. \qquad (2.1)$$

The exact solution for equation (2.1) requires iterative technique. Once the maximum likelihood estimator $\hat{\lambda}_1$ is obtained the maximum likelihood estimator of α, say $\hat{\alpha}_1$, can be obtained as

$$\hat{\alpha}_1=-\frac{n}{\sum_{i=1}^{n}\ln(1-e^{-\hat{\lambda}_1 x_i})}. \qquad (2.2)$$

Case 2, the maximum likelihood estimator of λ when the shape parameter α is known. For known α Gupta and Kundu (2001) obtained the maximum likelihood estimator of λ as a fixed point solution of equation $v(\lambda)=\lambda$, where

$$v(\lambda)=\left(\frac{1}{n}\sum_{i=1}^{n}\frac{x_i(1-\alpha e^{-\lambda x_i})}{(1-e^{-\lambda x_i})}\right)^{-1}. \qquad (2.3)$$

Case 3, the maximum likelihood estimator of α, when the scale parameter λ is known. Without loss of generality Gupta and Kundu (2001) take λ =1. If λ is known they obtained the maximum likelihood estimator of α, as

$$\hat{\alpha}=-\frac{n}{\sum_{i=1}^{n}\ln(1-e^{-x_i})}. \qquad (2.4)$$

3. Critical Values Calculations

A goodness-of-fit test is used to test the null hypothesis H_0: the random sample X_1, X_2, …, X_n comes from distribution (1.1). In this **Section**, the Kolmogorov-Smirnov statistic D_n, Cramer-von-Mises statistic W_n^2, Anderson-Darling statistic A_n^2, Watson test statistic U_n^2, and L_n test statistic which introduced by Liao and Shimokawa *(*1999) will be described. The A-D statistic is a modification of C-M statistic giving more weight to observations in the tail of the distribution, which is useful in detecting outliers (see Anderson and Darling (1954), Stephens (1977)). The Watson statistic is a modification of the C-M test statistic; it is also measure the discrepancy between the empirical distribution function and the hypothesized distribution function. L_n test statistic measures the average of all weighted distances over the entire range of x, which combines the characteristic of the K-S, C-M and A-D statistics (see, Liao and Shimokawa (1999)).

The aim in this **Section** is to obtain Tables of goodness-of-fit critical values for all test statistics using two different methods. The first method by using Monte Carlo simulation. The second method by obtaining the sampling distributions for the proposed test statistics using Pearson system technique. From the resulting sampling distributions the critical values for the test statistics will be obtained. The two methods are carried out via Mathcad (2001) package.

3.1 Method A

Monte Carlo Simulation is used to create critical values for the proposed test statistics for a generalized exponential distribution with unknown parameters. The following steps are used in calculating critical values for the proposed test statistics:

Step (1): A random sample X_1, X_2,…,X_n from generalized exponential was generated. Firstly a random sample $U_{(1)}$, $U_{(2)}$,…, $U_{(n)}$ of n order statistics from a uniform (0,1) distribution was generated, then the i-th order statistic from the GE(α, λ) with α=0.5 and λ=1 will be obtained as follows

$$x_{(i)} = (\frac{-1}{\lambda})\ln[1-(U_{(i)})^{\frac{1}{\alpha}}] , \qquad i=1, 2,…,n \qquad (3.1)$$

Step (2): This random sample was used to estimate the unknown parameters by method of maximum likelihood mentioned in **Section** 2.

Step (3): The resulting maximum likelihood estimators of the unknown parameters under each case were then used to determine the hypothesized cumulative distribution function for the generalized exponential distribution.

Step (4): Selected sample size as n = 5(5) 50 and 100. The appropriate test statistics was calculated for the given values of n, as follows

1. The K-S test statistic D_n is

$$D_n = \max\{\max_{1\le i\le n}[\frac{i}{n} - F_0(x_{(i)},\hat{\alpha},\hat{\lambda})], \max_{1\le i\le n}[F_0(x_{(i)},\hat{\alpha},\hat{\lambda}) - \frac{i-1}{n}]\} \quad (3.2)$$

Where $F_0(x_i, \hat{\alpha}, \hat{\lambda})$ is a cumulative distribution function of GE(α , λ) distribution, $\hat{\alpha}$ and $\hat{\lambda}$ are the estimated parameters using maximum likelihood estimators of α and λ ,

2. The C-M statistic W_n^2 is represented by the following formula

$$W_n^2 = \frac{1}{12n} + \sum_{i=1}^{n} [F_0(x_{(i)},\hat{\alpha},\hat{\lambda}) - \frac{2i-1}{2n}]^2 . \quad (3.3)$$

3. The A-D statistic A_n^2 is

$$A_n^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)[\ln F_0(x_{(i)},\hat{\alpha},\hat{\lambda}) + \ln\{1 - F_0(x_{(n-i+1)},\hat{\alpha},\hat{\lambda})\}] \quad (3.4)$$

4. The Watson statistic U_n^2 is

$$U_n^2 = W_n^2 - n\{[\sum_{i=1}^{n}\left(\frac{F_0(x_{(i)},\hat{\alpha},\hat{\lambda})}{n}\right)] - \frac{1}{2}\}^2 . \quad (3.5)$$

5. Liao and Shimokawa L_n, statistic is

$$L_n = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\{\frac{\max[\frac{i}{n} - F_0(x_{(i)},\hat{\alpha},\hat{\lambda}), F_0(x_{(i)},\hat{\alpha},\hat{\lambda}) - \frac{i-1}{n}]}{\sqrt{F_0(x_{(i)},\hat{\alpha},\hat{\lambda})[1 - F_0(x_{(i)},\hat{\alpha},\hat{\lambda})]}}\}. \quad (3.6)$$

Step 5: This procedure was repeated 10000 times, thus generating 10000 independent values of the appropriate test statistics. These 10000 values were then ranked, and the values of these test statistics at seven significance levels, i.e., $\gamma =$ 0.01, 0.025, 0.05, 0.10, 0.15, 0.20, and 0.25 are calculated. These provided the critical values for that particular test under each of the three cases and sample size used.

Tables 1-3 list the critical values for the statistics D_n , W_n^2 , A_n^2 , U_n^2 and L_n and for each case 1, 2 and 3, using Monte Carlo method.

3.2 Method B

Pearson's system technique is used to obtain the sampling distributions of the proposed test statistics. The Pearson system of distributions was originated by Karl Pearson (1895). The criterion for fixing the distribution family is

$$K = \frac{\beta_1 \ (\beta_2 + 3)^2}{4(4\beta_2 - 3)\,(2\beta_2 - 3\beta_1 - 6)} \quad (3.7)$$

Where $\beta_1 = M_3 \big/ M_3^{1.5}$ and $\beta_2 = M_4 \big/ M_2^2$ are the measures of skewness and kurtosis respectively and M_i is the ith moment about mean. Pearson classified the different members of system according to their shapes into a number of types. So for different values of K, there exist different types of distributions. The following steps are used in calculating critical values for the test statistics using Pearson's technique:

Step 1: Repeat the above steps from 1-3 in method A, then mean, variance, skewness, kurtosis and Pearson coefficient are calculated for each test statistic and sample size under each case.

Step 2: The resulting values of equation (3.7) yielded the types of distributions that appear in Tables 4 -6.

Step 3: For any particular distribution, the constants and the parameters of distributions are calculated. The method of moments are used to estimate the parameters of these different types. These provided the critical values for the above test statistics at significance levels, $\gamma =$ 0.01, 0.025, 0.05, 0.10, 0.15, 0.20 and 0.25, for different sample sizes.

As a result of computer simulation, the following functions are obtained. Each function is defining a specific type of Pearson's curves. In particular, the type I Pearson's curves has the density function

$$f(\mathrm{x}) = k_1 (1 + \frac{\mathrm{x}}{l_1})^{\mathrm{m}_1} (1 - \frac{\mathrm{x}}{l_2})^{\mathrm{m}_2}, \qquad -l_1 < \mathrm{x} < l_2 \tag{3.8}$$

$m_1 > -1$, where l_1, l_2, m_1 and m_2 are the parameters of the family of distributions and k_1 is a constant. While, type IV Pearson's curve has the density

$$f(\mathrm{x}) = k_2 (1 + \frac{\mathrm{x}^2}{a^2})^{-d} \exp[-\psi \tan^{-1}(\frac{\mathrm{x}}{a})], \qquad -\infty < \mathrm{x} < \infty \tag{3.9}$$

where k_2 is a constant, d, a and ψ are the parameters of distributions. The last type of Pearson's curves that fitted to the test statistics is type VI and it has the density function

$$f(\mathrm{x}) = k_3 (\mathrm{x} - \mathrm{p})^{e_1} \mathrm{x}^{-\mathrm{h}_1}, \qquad p \le \mathrm{x} < \infty \tag{3.10}$$

where k_3 is a constant, e_1, h_1 and p are the parameters of distributions.

Tables 4-6 list the critical values for the test statistics and the distribution type using Pearson's system technique. It is clear from these Tables that:

1. When the two parameters are unknown and one of the two parameters is unknown, the sampling distributions for K-S are type I for all sample size.
2. The sampling distributions for C-M, A-D, and Watson statistic are type IV for all sample size and under each case.
3. When the two parameters are unknown the sampling distributions for L_n test are of type VI for small (n=5) and large (n=100) sample sizes. While the sampling distributions for L_n test are of type IV for all sample sizes expect n=5 and n=100.

4. When one of the two parameters is unknown, the sampling distributions of L_n test statistic are of types VI and IV for all sample sizes.

4. Power Study

The power of a goodness-of-fit test is defined as the probability that a statistic will lead to the rejection of the null hypothesis, H_0, when it is false, i.e. when a sample is not from the hypothesised population but an alternative population (Mann *et al* (1974)). Let the complement of the null hypothesis be the alternative hypothesis H_a. The power of a gooness-of-fit test at the significance level γ is denoted by $1-\beta$, where β is the probability of committing a type II error, failling to reject a false null hypothesis.

A power comparison was made among K-S statistic, C-M statistic, A-D statistic, Watson statistic, and L_n test statistic for the generalized exponential distribution with unknown shape and scale parameters. The power was determined by generating 10000 random sample of size n = 5, 15 and 30 from each of seven alternatives for each test. Here n=5, 15 and n= 30 represent small, moderate and fairly large sample sizes respectively. All the alternative distributions are listed below:

1. A standard normal distribution.
2. The Weibull distribution with density $t\, x^{t-1} \exp(-x^{t})$, denoted by W (t).
3. The gamma distribution with density $(\Gamma(t))^{-1} x^{t-1} \exp(-x)$, denoted by $\Gamma(t)$.
4. The exponential distribution with density $t \exp(-tx)$, denoted by exp (t).
5. The chi-square distribution with density $\frac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})} x^{\frac{n}{2}-1} \exp(-\frac{x}{2})$, denoted by χ^2_n.
6. The uniform distribution on the interval [0, 1].

For each test, the appropriate test statistic was calculated and compared to its respective critical values and counted the number of rejections of the null hypothesis. The power results for the tests at the significance level $\gamma = 0.05$ are presented in Table 7.

5. Conclusions

For different significance levels and sample sizes, the change of critical values for all test statistics under case 1 are greater than that the corresponding under case 1 and case 3. As n becomes larger and γ lower, the critical values for test statistics decrease monotonically, for all test statistics in each case.

Power studies using several different distributional forms show that L_n statistic is generally superior to other test statistic. For sample size equal 30, The A-D test statistic is more powerful than The K-S, C-M, and Watson test statistic. The Watson statistic is not appearing to be powerful across this group of different distributions. The power of the test statistic increases as the sample size increases.

Table 1
Critical Points Of Test Statistics Using Method A
Case 1: Both λ And α Unknown

Sample Size n	Test Statistics	Significance level γ						
		0.01	0.025	0.05	0.10	0.15	0.20	0.25
5	D_n	0.488	0.365	0.341	0.311	0.286	0.269	0.258
	W^2_n	0.234	0.159	0.133	0.110	0.096	0.087	0.079
	A^2_n	1.217	0.910	0.776	0.646	0.636	0.519	0.479
	U^2_n	0.200	0.140	0.118	0.098	0.084	0.079	0.073
	L_n	1.731	1.460	1.372	1.287	1.235	1.194	1.159
10	D_n	0.294	0.268	0.247	0.223	0.209	0.189	0.187
	W^2_n	0.195	0.157	0.133	0.109	0.095	0.083	0.076
	A^2_n	0.983	0.902	0.776	0.646	0.576	0.519	0.478
	U^2_n	0.170	0.140	0.118	0.099	0.084	0.078	0.070
	L_n	1.415	1.296	1.214	1.133	1.080	1.042	1.011
15	D_n	0.249	0.226	0.208	0.188	0.175	0.166	0.158
	W^2_n	0.188	0.157	0.132	0.107	0.094	0.084	0.077
	A^2_n	1.096	0.891	0.774	0.646	0.572	0.518	0.478
	U^2_n	0.166	0.138	0.117	0.097	0.085	0.077	0.070
	L_n	1.364	1.124	1.155	1.061	1.015	0.977	0.945
20	D_n	0.216	0.199	0.182	0.165	0.154	0.146	0.139
	W^2_n	0.188	0.155	0.132	0.107	0.095	0.084	0.077
	A^2_n	1.080	0.889	0.771	0.643	0.571	0.518	0.478
	U^2_n	0.166	0.137	0.118	0.097	0.097	0.077	0.070
	L_n	1.291	1.174	1.097	1.016	0.968	0.928	0.899
25	D_n	0.196	0.181	0.166	0.150	0.140	0.133	0.126
	W^2_n	0.187	0.155	0.131	0.108	0.094	0.084	0.076
	A^2_n	1.067	0.889	0.764	0.643	0.571	0.517	0.475
	U^2_n	0.166	0.137	0.118	0.097	0.085	0.077	0.070
	L_n	1.270	1.154	1.075	0.989	0.933	0.899	0.869
30	D_n	0.181	0.164	0.153	0.138	0.129	0.122	0.116
	W^2_n	0.185	0.155	0.130	0.109	0.094	0.085	0.077
	A^2_n	1.062	0.887	0.763	0.641	0.569	0.515	0.475
	U^2_n	0.166	0.136	0.118	0.098	0.086	0.077	0.071
	L_n	1.226	1.120	1.044	0.963	0.913	0.878	0.847
35	D_n	0.169	0.153	0.142	0.129	0.121	0.114	0.109
	W^2_n	0.185	0.154	0.130	0.108	0.094	0.085	0.077
	A^2_n	1.050	0.881	0.759	0.638	0.568	0.514	0.474
	U^2_n	0.165	0.136	0.118	0.097	0.086	0.077	0.071
	L_n	1.195	1.090	1.015	0.940	0.890	0.855	0.827
40	D_n	0.156	0.143	0.132	0.121	0.113	0.108	0.103
	W^2_n	0.184	0.153	0.130	0.107	0.094	0.085	0.077
	A^2_n	1.051	0.877	0.753	0.636	0.565	0.513	0.473
	U^2_n	0.164	0.136	0.118	0.097	0.085	0.077	0.070
	L_n	1.167	1.068	0.995	0.942	0.878	0.842	0.814
45	D_n	0.151	0.136	0.126	0.115	0.107	0.102	0.097
	W^2_n	0.184	0.153	0.130	0.107	0.093	0.084	0.076
	A^2_n	1.048	0.870	0.749	0.636	0.564	0.512	0.472
	U^2_n	0.164	0.136	0.117	0.097	0.084	0.076	0.070
	L_n	1.163	1.052	0.987	0.906	0.859	0.824	0.796
50	D_n	0.144	0.130	0.121	0.110	0.102	0.097	0.093
	W^2_n	0.182	0.148	0.129	0.107	0.094	0.084	0.077
	A^2_n	1.042	0.829	0.723	0.618	0.551	0.504	0.466
	U^2_n	0.163	0.131	0.113	0.096	0.085	0.077	0.070
	L_n	1.142	1.038	0.970	0.897	0.848	0.812	0.787
100	D_n	0.102	0.094	0.087	0.080	0.075	0.071	0.068
	W^2_n	0.175	0.146	0.125	0.104	0.092	0.085	0.077
	A^2_n	0.983	0.808	0.710	0.616	0.551	0.503	0.462
	U^2_n	0.162	0.128	0.113	0.095	0.086	0.076	0.070
	L_n	1.065	0.979	0.905	0.832	0.786	0.753	0.724

Table 2
Critical Points Of Test Statistics Using Method A
Case 2: Scale Parameter λ Unknown

Sample Size n	Test Statistics	Significance level γ						
		0.01	0.025	0.05	0.10	0.15	0.20	0.25
5	D_n	0.791	0.609	0.529	0.455	0.405	0.369	0.342
	W^2_n	1.876	1.110	0.848	0.571	0.424	0.335	0.273
	A^2_n	12.39	7.065	4.932	3.192	2.408	1.909	1.574
	U^2_n	0.424	0.253	0.195	0.149	0.124	0.109	0.098
	L_n	7.877	5.319	4.029	2.989	2.510	2.231	2.010
10	D_n	0.606	0.433	0.387	0.340	0.306	0.283	0.264
	W^2_n	1.848	0.879	0.647	0.454	0.351	0.287	0.241
	A^2_n	10.876	4.708	3.463	2.437	1.937	1.597	1.364
	U^2_n	0.409	0.216	0.178	0.141	0.121	0.106	0.096
	L_n	5.607	2.991	2.534	2.073	1.853	1.685	1.565
15	D_n	0.524	0.358	0.322	0.283	0.257	0.238	0.223
	W^2_n	1.697	0.808	0.610	0.423	0.332	0.272	0.231
	A^2_n	9.917	4.223	3.189	2.306	1.828	1.510	1.289
	U^2_n	0.395	0.212	0.173	0.138	0.118	0.105	0.094
	L_n	4.285	2.521	2.197	1.877	1.673	1.531	1.426
20	D_n	0.437	0.316	0.284	0.250	0.228	0.210	0.197
	W^2_n	1.644	0.774	0.601	0.416	0.322	0.266	0.225
	A^2_n	8.801	3.992	3.188	2.249	1.759	1.491	1.264
	U^2_n	0.387	0.207	0.171	0.138	0.118	0.104	0.093
	L_n	3.937	2.392	2.098	1.876	1.565	1.448	1.353
25	D_n	0.407	0.282	0.255	0.226	0.208	0.193	0.181
	W^2_n	1.636	0.717	0.562	0.409	0.323	0.266	0.225
	A^2_n	8.341	3.778	2.956	2.167	1.753	1.483	1.273
	U^2_n	0.383	0.208	0.173	0.137	0.117	0.103	0.094
	L_n	3.682	2.237	1.947	1.688	1.529	1.410	1.317
30	D_n	0.356	0.261	0.239	0.207	0.189	0.176	0.165
	W^2_n	1.635	0.716	0.564	0.398	0.317	0.261	0.219
	A^2_n	8.177	3.752	2.964	2.130	1.719	1.464	1.244
	U^2_n	0.397	0.205	0.175	0.137	0.117	0.103	0.093
	L_n	3.389	2.136	1.921	1.633	1.470	1.356	1.268
35	D_n	0.339	0.242	0.217	0.192	0.176	0.164	0.154
	W^2_n	1.621	0.707	0.555	0.396	0.316	0.260	0.218
	A^2_n	7.846	3.605	2.798	2.076	1.665	1.455	1.196
	U^2_n	0.397	0.203	0.166	0.136	0.117	0.104	0.092
	L_n	3.169	2.082	1.835	1.579	1.421	1.314	1.218
40	D_n	0.311	0.228	0.205	0.181	0.167	0.156	0.147
	W^2_n	1.560	0.704	0.545	0.392	0.315	0.261	0.220
	A^2_n	7.809	3.723	2.863	2.112	1.714	1.441	1.248
	U^2_n	0.378	0.203	0.167	0.136	0.117	0.103	0.094
	L_n	2.926	2.078	1.828	1.575	1.421	1.318	1.227
45	D_n	0.295	0.216	0.196	0.173	0.159	0.148	0.139
	W^2_n	1.515	0.704	0.540	0.389	0.313	0.285	0.216
	A^2_n	7.642	3.688	2.835	2.119	1.708	1.435	1.236
	U^2_n	0.374	0.205	0.171	0.135	0.116	0.103	0.094
	L_n	2.913	2.043	1.802	1.563	1.403	1.297	1.209
50	D_n	0.287	0.205	0.187	0.163	0.150	0.140	0.132
	W^2_n	1.486	0.692	0.532	0.387	0.307	0.251	0.214
	A^2_n	7.595	3.671	2.866	2.121	1.714	1.404	1.231
	U^2_n	0.360	0.204	0.170	0.135	0.116	0.103	0.092
	L_n	2.906	2.029	1.797	1.545	1.393	1.279	1.187
100	D_n	0.191	0.148	0.133	0.119	0.110	0.102	0.096
	W^2_n	1.403	0.649	0.516	0.385	0.302	0.252	0.213
	A^2_n	7.197	3.364	2.743	2.077	1.684	1.349	1.201
	U^2_n	0.352	0.197	0.168	0.133	0.115	0.102	0.092
	L_n	2.583	1.865	1.665	1.447	1.305	1.194	1.110

Table 3
Critical Points Of Test Statistics Using Method A
Case 3 : Shape Parameter α Unknown

Sample Size n	Test Statistics	Significance level γ						
		0.01	0.025	0.05	0.10	0.15	0.20	0.25
5	D_n	0.555	0.463	0.430	0.388	0.362	0.341	0.324
	W^2_n	0.505	0.273	0.223	0.176	0.151	0.130	0.118
	A^2_n	3.218	1.593	1.337	1.071	1.051	0.824	0.744
	U^2_n	0.289	0.191	0.160	0.138	0.113	0.102	0.094
	L_n	6.018	2.390	1.974	1.603	1.418	1.314	1.250
10	D_n	0.426	0.332	0.306	0.278	0.258	0.243	0.230
	W^2_n	0.460	0.255	0.222	0.175	0.148	0.127	0.116
	A^2_n	3.155	1.584	1.318	1.071	0.915	0.816	0.735
	U^2_n	0.312	0.189	0.159	0.138	0.111	0.101	0.090
	L_n	4.689	1.948	1.656	1.400	1.271	1.198	1.141
15	D_n	0.376	0.276	0.252	0.229	0.212	0.199	0.189
	W^2_n	0.505	0.266	0.222	0.174	0.147	0.130	0.118
	A^2_n	3.123	1.580	1.316	1.063	0.915	0.811	0.734
	U^2_n	0.357	0.189	0.159	0.130	0.111	0.099	0.089
	L_n	3.485	1.731	1.501	1.318	1.212	1.141	1.090
20	D_n	0.314	0.242	0.221	0.200	0.187	0.176	0.167
	W^2_n	0.505	0.266	0.219	0.173	0.148	0.130	0.118
	A^2_n	3.018	1.575	1.309	1.057	0.910	0.809	0.732
	U^2_n	0.357	0.187	0.159	0.130	0.113	0.101	0.091
	L_n	2.758	1.597	1.413	1.250	1.158	1.094	1.045
25	D_n	0.284	0.217	0.200	0.181	0.168	0.158	0.150
	W^2_n	0.505	0.266	0.219	0.173	0.147	0.130	0.115
	A^2_n	2.956	1.567	1.303	1.055	0.908	0.805	0.730
	U^2_n	0.338	0.186	0.159	0.129	0.112	0.101	0.091
	L_n	2.391	1.506	1.368	1.207	1.118	1.058	1.011
30	D_n	0.285	0.200	0.184	0.165	0.154	0.145	0.137
	W^2_n	0.505	0.269	0.219	0.173	0.147	0.130	0.115
	A^2_n	2.836	1.580	1.301	1.051	0.905	0.804	0.729
	U^2_n	0.332	0.186	0.157	0.129	0.111	0.099	0.089
	L_n	2.403	1.482	1.320	1.168	1.132	1.030	0.983
35	D_n	0.240	0.186	0.173	0.155	0.145	0.136	0.129
	W^2_n	0.497	0.266	0.218	0.173	0.148	0.132	0.116
	A^2_n	2.816	1.566	1.284	1.049	0.897	0.804	0.728
	U^2_n	0.328	0.187	0.159	0.130	0.113	0.100	0.091
	L_n	2.146	1.445	1.307	1.158	1.092	1.018	0.974
40	D_n	0.228	0.174	0.160	0.143	0.135	0.127	0.121
	W^2_n	0.495	0.266	0.216	0.174	0.148	0.129	0.116
	A^2_n	2.874	1.584	1.283	1.040	0.895	0.794	0.723
	U^2_n	0.321	0.187	0.159	0.128	0.112	0.100	0.091
	L_n	2.070	1.390	1.264	1.132	1.074	1.004	0.957
45	D_n	0.213	0.165	0.152	0.137	0.128	0.121	0.115
	W^2_n	0.492	0.267	0.218	0.173	0.147	0.130	0.115
	A^2_n	2.791	1.566	1.279	1.030	0.886	0.775	0.719
	U^2_n	0.323	0.186	0.157	0.128	0.112	0.100	0.091
	L_n	2.061	1.379	1.252	1.119	1.042	0.985	0.938
50	D_n	0.200	0.158	0.145	0.130	0.121	0.115	0.109
	W^2_n	0.479	0.264	0.216	0.172	0.147	0.130	0.115
	A^2_n	2.728	1.562	1.251	1.018	0.876	0.777	0.702
	U^2_n	0.310	0.186	0.156	0.128	0.111	0.099	0.090
	L_n	2.047	1.359	1.237	1.105	1.029	0.977	0.933
100	D_n	0.144	0.112	0.103	0.093	0.087	0.082	0.079
	W^2_n	0.457	0.263	0.216	0.172	0.147	0.128	0.113
	A^2_n	2.650	1.545	1.222	0.999	0.864	0.777	0.699
	U^2_n	0.307	0.183	0.155	0.126	0.110	0.099	0.089
	L_n	1.724	1.250	1.139	1.027	0.958	0.902	0.861

Table4

Critical Points Of Test statistics Using Method B

Case 1: Both λ And α Unknown

Sample Size n	Test Statistics	Distribution Type	Significance level γ						
			0.01	0.025	0.05	0.10	0.15	0.20	0.25
5	D_n	I	0.286	0.270	0.255	0.236	0.222	0.212	0.203
	W^2_n	IV	0.449	0.380	0.333	0.292	0.270	0.255	0.243
	A^2_n	IV	1.967	1.926	1.709	1.505	1.396	1.323	1.267
	U^2_n	IV	0.347	0.327	0.307	0.305	0.295	0.283	0.273
	L_n	VI	1.589	1.345	1.261	1.179	1.132	1.099	1.074
10	D_n	I	0.238	0.215	0.197	0.178	0.166	0.157	0.149
	W^2_n	IV	0.388	0.334	0.296	0.262	0.244	0.231	0.221
	A^2_n	IV	1.950	1.714	1.539	1.377	1.370	1.226	1.180
	U^2_n	IV	0.302	0.295	0.261	0.230	0.214	0.201	0.193
	L_n	IV	1.460	1.272	1.116	1.096	1.047	1.012	0.985
15	D_n	I	0.200	0.184	0.170	0.156	0.147	0.140	0.134
	W^2_n	IV	0.386	0.329	0.290	0.254	0.235	0.222	0.212
	A^2_n	IV	1.926	1.676	1.489	1.317	1.223	1.159	1.111
	U^2_n	IV	0.292	0.265	0.239	0.214	0.213	0.192	0.185
	L_n	IV	1.391	1.264	1.096	0.977	0.924	0.877	0.865
20	D_n	I	0.188	0.171	0.157	0.143	0.134	0.128	0.123
	W^2_n	IV	0.377	0.321	0.284	0.250	0.231	0.219	0.209
	A^2_n	IV	1.913	1.652	1.474	1.309	1.218	1.156	1.110
	U^2_n	IV	0.287	0.264	0.239	0.215	0.201	0.192	0.185
	L_n	IV	1.321	1.163	1.068	0.933	0.924	0.860	0.848
25	D_n	I	0.165	0.153	0.143	0.132	0.125	0.120	0.116
	W^2_n	IV	0.365	0.309	0.272	0.238	0.219	0.207	0.191
	A^2_n	IV	1.860	1.594	1.396	1.247	1.156	1.112	1.096
	U^2_n	IV	0.279	0.255	0.230	0.206	0.193	0.184	0.162
	L_n	IV	1.288	1.129	1.055	0.930	0.912	0.828	0.800
30	D_n	I	0.151	0.140	0.131	0.121	0.115	0.111	0.107
	W^2_n	IV	0.354	0.303	0.268	0.236	0.219	0.207	0.198
	A^2_n	IV	1.807	1.564	1.381	1.240	1.155	1.094	1.051
	U^2_n	IV	0.279	0.249	0.216	0.198	0.184	0.177	0.175
	L_n	IV	1.296	1.128	1.016	0.919	0.889	0.814	0.799
35	D_n	I	0.146	0.135	0.126	0.117	0.111	0.107	0.103
	W^2_n	IV	0.346	0.295	0.261	0.229	0.212	0.200	0.198
	A^2_n	IV	1.778	1.543	1.277	1.230	1.146	1.088	1.051
	U^2_n	IV	0.287	0.242	0.214	0.193	0.180	0.171	0.168
	L_n	IV	1.288	1.109	1.012	0.912	0.865	0.802	0.784
40	D_n	I	0.139	0.129	0.121	0.112	0.107	0.103	0.100
	W^2_n	IV	0.329	0.275	0.240	0.208	0.191	0.190	0.178
	A^2_n	IV	1.522	1.380	1.247	1.178	1.121	1.081	1.045
	U^2_n	IV	0.275	0.239	0.198	0.190	0.178	0.169	0.164
	L_n	IV	1.252	1.101	0.996	0.896	0.840	0.782	0.773
45	D_n	I	0.132	0.123	0.116	0.108	0.103	0.099	0.096
	W^2_n	IV	0.296	0.251	0.221	0.193	0.178	0.168	0.160
	A^2_n	IV	1.520	1.303	1.153	1.015	0.938	0.894	0.864
	U^2_n	IV	0.256	0.220	0.195	0.172	0.162	0.159	0.151
	L_n	IV	1.243	1.057	0.945	0.877	0.785	0.728	0.700
50	D_n	I	0.123	0.115	0.108	0.101	0.097	0.093	0.091
	W^2_n	IV	0.275	0.232	0.203	0.176	0.161	0.151	0.144
	A^2_n	IV	1.422	1.209	1.071	0.987	0.932	0.886	0.846
	U^2_n	IV	0.241	0.209	0.187	0.166	0.154	0.146	0.140
	L_n	IV	1.158	1.016	0.938	0.828	0.767	0.725	0.684
100	D_n	I	0.089	0.084	0.080	0.076	0.073	0.071	0.069
	W^2_n	IV	0.235	0.194	0.168	0.165	0.157	0.149	0.143
	A^2_n	IV	1.310	1.178	0.987	0.938	0.865	0.814	0.776
	U^2_n	IV	0.226	0.194	0.173	0.152	0.141	0.133	0.127
	L_n	VI	1.123	1.005	0.915	0.819	0.766	0.724	0.680

Table 5
Critical Points Of Test Statistics Using Method B
Case 2: Scale Parameter λ Unknown

Sample Size n	Test Statistics	Distribution Type	Significance level γ						
			0.01	0.025	0.05	0.10	0.15	0.20	0.25
5	D_n	I	0.622	0.556	0.501	0.501	0.442	0.404	0. 239
	W^2_n	IV	1.959	1.622	1.403	1.362	1.184	1.085	0.988
	A^2_n	IV	10.82	9.832	8.438	6.203	5.758	5.650	5.043
	U^2_n	IV	0.485	0.420	0.375	0.330	0.312	0.297	0.286
	L_n	VI	4.856	3.742	3.317	3.076	2.939	2.843	2.770
10	D_n	I	0.465	0.414	0.374	0.332	0.307	0.288	0.218
	W^2_n	IV	1.861	1.554	1.362	1.122	1.107	1.038	0.905
	A^2_n	IV	9.962	8.260	7.148	6.167	5.722	5.300	4.609
	U^2_n	IV	0.473	0.396	0.345	0.300	0.275	0.259	0.246
	L_n	IV	4.769	3.739	3.273	3.033	2.896	2.801	2.728
15	D_n	I	0.387	0.347	0.316	0.284	0.264	0.250	0.208
	W^2_n	IV	1.832	1.528	1.312	1.116	1.021	0.954	0.904
	A^2_n	IV	9.348	7.697	6.622	5.832	5.174	4.839	4.590
	U^2_n	IV	0.450	0.382	0.336	0.295	0.272	0.257	0.245
	L_n	IV	4.084	3.567	3.211	2.762	2.619	2.522	2.447
20	D_n	I	0.341	0.309	0.283	0.256	0.239	0.227	0.191
	W^2_n	IV	1.830	1.511	1.301	1.109	1.018	0.952	0.870
	A^2_n	IV	9.233	7.619	6.565	5.675	5.139	4.808	4.563
	U^2_n	IV	0.431	0.369	0.327	0.273	0.267	0.253	0.242
	L_n	IV	3.913	3.521	3.101	2.745	2.618	2.519	2.442
25	D_n	I	0.320	0.290	0.266	0.242	0.227	0.216	0.181
	W^2_n	IV	1.796	1.458	1.255	1.076	0.981	0.917	0.835
	A^2_n	IV	9.115	7.388	6.370	5.633	4.989	4.669	4.430
	U^2_n	IV	0.428	0.358	0.313	0.269	0.251	0.236	0.226
	L_n	IV	3.866	3.286	3.092	2.675	2.469	2.263	2.093
30	D_n	I	0.288	0.262	0.242	0.221	0.208	0.199	0.180
	W^2_n	IV	1.768	1.453	1.232	1.040	0.940	0.876	0.828
	A^2_n	IV	8.831	7.364	6.246	5.271	4.766	4.417	4.277
	U^2_n	IV	0.425	0.357	0.313	0.264	0.242	0.236	0.225
	L_n	IV	3.780	3.224	3.018	2.537	2.284	2.249	2.092
35	D_n	I	0.280	0.243	0.226	0.199	0.196	0.188	0.180
	W^2_n	IV	1.760	1.430	1.221	1.037	0.938	0.876	0.821
	A^2_n	IV	9.102	7.352	6.209	5.253	4.758	4.412	4.201
	U^2_n	IV	0.421	0.352	0.306	0.252	0.237	0.227	0.216
	L_n	IV	3.685	3.186	2.727	2.442	2.249	2.175	2.014
40	D_n	I	0.245	0.226	0.212	0.196	0.186	0.179	0.173
	W^2_n	IV	1.753	1.415	1.215	0.992	0.895	0.871	0.781
	A^2_n	IV	8.946	7.223	6.207	5.203	4.548	4.412	4.164
	U^2_n	IV	0.384	0.321	0.279	0.241	0.221	0.208	0.197
	L_n	IV	3.664	3.059	2.722	2.394	2.222	2.175	1.930
45	D_n	I	0.236	0.218	0.204	0.189	0.179	0.172	0.167
	W^2_n	IV	1.711	1.387	1.177	0.990	0.893	0.829	0.778
	A^2_n	IV	8.907	7.218	6.172	5.055	4.514	4.313	4.047
	U^2_n	IV	0.381	0.310	0.274	0.235	0.214	0.207	0.189
	L_n	IV	3.532	3.043	2.706	2.282	2.174	2.104	1.918
50	D_n	I	0.237	0.217	0.202	0.186	0.176	0.169	0.163
	W^2_n	IV	1.693	1.379	1.173	0.984	0.889	0.827	0.778
	A^2_n	IV	8.831	7.113	6.016	4.992	4.506	4.182	3.959
	U^2_n	IV	0.368	0.308	0.267	0.225	0.204	0.200	0.183
	L_n	IV	3.483	3.031	2.684	2.268	2.066	1.929	1.827
100	D_n	I	0.154	0.147	0.140	0.133	0.129	0.125	0.123
	W^2_n	IV	1.688	1.370	1.164	0.944	0.881	0.825	0.778
	A^2_n	IV	8.814	6.964	5.913	4.832	4.313	4.210	3.941
	U^2_n	IV	0.367	0.305	0.263	0.225	0.204	0.190	0.180
	L_n	VI	3.468	3.025	2.642	2.223	1.985	1.829	1.714

Table6

Critical Points Of Test statistics Using Method B
Case 3: Shape Parameter α Unknown

Sample Size n	Test Statistics	Distribution Type	Significance level γ						
			0.01	0.025	0.05	0.10	0.15	0.20	0.25
5	D_n	I	0.368	0.348	0.329	0.305	0.288	0.275	0.264
	W^2_n	IV	0.673	0.588	0.576	0.450	0.418	0.396	0.379
	A^2_n	IV	3.749	3.241	2.894	2.577	2.403	2.285	2.141
	U^2_n	IV	0.442	0.386	0.347	0.312	0.292	0.278	0.268
	L_n	IV	3.271	2.547	2.089	1.694	1.574	1.525	1.448
10	D_n	I	0.272	0.252	0.235	0.216	0.203	0.194	0.186
	W^2_n	IV	0.598	0.527	0.510	0.404	0.403	0.349	0.330
	A^2_n	IV	3.520	2.961	2.588	2.253	2.073	1.952	1.861
	U^2_n	IV	0.410	0.352	0.313	0.278	0.260	0.247	0.238
	L_n	IV	3.003	2.359	1.954	1.609	1.489	1.352	1.252
15	D_n	I	0.247	0.224	0.206	0.187	0.175	0.166	0.159
	W^2_n	IV	0.578	0.509	0.461	0.395	0.365	0.345	0.330
	A^2_n	IV	3.341	2.876	2.519	2.197	2.024	1.918	1.837
	U^2_n	IV	0.398	0.347	0.311	0.277	0.257	0.244	0.233
	L_n	IV	2.988	2.345	1.939	1.592	1.431	1.314	1.228
20	D_n	I	0.211	0.199	0.184	0.169	0.160	0.153	0.147
	W^2_n	IV	0.573	0.492	0.449	0.382	0.353	0.334	0.320
	A^2_n	IV	3.313	2.820	2.486	2.183	2.023	1.906	1.822
	U^2_n	IV	0.385	0.337	0.303	0.271	0.253	0.241	0.232
	L_n	IV	2.175	2.140	1.766	1.446	1.411	1.292	1.204
25	D_n	I	0.191	0.178	0.167	0.155	0.148	0.142	0.138
	W^2_n	IV	0.555	0.490	0.434	0.382	0.323	0.304	0.290
	A^2_n	IV	3.220	2.777	2.238	2.181	2.018	1.906	1.818
	U^2_n	IV	0.376	0.334	0.299	0.266	0.236	0.233	0.226
	L_n	IV	2.119	2.080	1.513	1.350	1.261	1.199	1.153
30	D_n	I	0.179	0.166	0.155	0.144	0.137	0.131	0.127
	W^2_n	IV	0.532	0.465	0.434	0.352	0.322	0.304	0.288
	A^2_n	IV	3.005	2.663	2.466	1.971	1.803	1.699	1.693
	U^2_n	IV	0.361	0.324	0.287	0.259	0.248	0.224	0.214
	L_n	IV	2.070	1.749	1.482	1.350	1.214	1.149	1.100
35	D_n	I	0.167	0.157	0.147	0.138	0.131	0.127	0.123
	W^2_n	IV	0.521	0.453	0.399	0.354	0.349	0.303	0.275
	A^2_n	IV	3.231	2.453	2.296	1.957	1.797	1.690	1.620
	U^2_n	IV	0.386	0.313	0.280	0.253	0.236	0.220	0.211
	L_n	IV	2.064	1.709	1.475	1.309	1.214	1.131	1.082
40	D_n	I	0.162	0.150	0.140	0.130	0.123	0.118	0.115
	W^2_n	IV	0.498	0.447	0.374	0.322	0.294	0.275	0.261
	A^2_n	IV	2.940	2.442	2.112	1.817	1.657	1.550	1.470
	U^2_n	IV	0.349	0.295	0.266	0.242	0.232	0.195	0.186
	L_n	IV	2.016	1.688	1.468	1.309	1.113	1.030	0.968
45	D_n	I	0.149	0.140	0.132	0.123	0.117	0.113	0.110
	W^2_n	IV	0.439	0.432	0.351	0.300	0.272	0.253	0.239
	A^2_n	IV	3.341	2.345	1.991	1.679	1.513	1.402	1.320
	U^2_n	IV	0.398	0.290	0.259	0.226	0.208	0.179	0.169
	L_n	IV	1.946	1.667	1.446	1.238	1.113	1.018	0.960
50	D_n	I	0.147	0.137	0.129	0.120	0.115	0.111	0.107
	W^2_n	IV	0.370	0.360	0.323	0.280	0.256	0.240	0.229
	A^2_n	IV	2.506	2.040	1.735	1.464	1.319	1.221	1.148
	U^2_n	IV	0.324	0.277	0.239	0.204	0.186	0.175	0.167
	L_n	IV	1.945	1.661	1.413	1.192	1.098	1.000	0.944
100	D_n	I	0.109	0.102	0.096	0.090	0.086	0.084	0.081
	W^2_n	IV	0.347	0.321	0.309	0.274	0.254	0.240	0.228
	A^2_n	IV	2.052	1.663	1.406	1.175	1.051	0.966	0.902
	U^2_n	IV	0.304	0.260	0.230	0.202	0.185	0.173	0.164
	L_n	VI	1.942	1.624	1.400	1.187	1.074	0.999	0.943

Table 7

Power Of Tests For Generalized
Exponential Distribution
Level Of Significance $\gamma = 0.05$

Sample Size n	Test Statistics	Alternatives						
		Normal	Exp(1)	Exp(3)	W(2)	$\Gamma(3)$	χ_1^2	Uniform
5	D_n	.410	.409	.412	.405	.403	.411	.411
	W^2_n	.305	.305	.305	.295	.301	.295	.312
	A^2_n	.329	.350	.356	.347	.350	.346	.354
	U^2_n	.105	.106	.109	.102	.106	.107	.106
	L_n	.500	.489	.492	.401	.487	.489	.496
15	D_n	.420	.415	.414	.408	.410	.413	.413
	W^2_n	.338	.350	.344	.347	.339	.365	.340
	A^2_n	.394	.404	.398	.406	.400	.405	.396
	U^2_n	.109	.107	.110	.107	.107	.106	.111
	L_n	.501	.582	.597	.486	.603	.612	.602
30	D_n	.422	.428	.423	.428	.422	.428	.419
	W^2_n	.407	.411	.407	.406	.414	.413	.385
	A^2_n	.504	.454	.449	.447	.452	.456	.454
	U^2_n	.216	.124	.122	.126	.121	.120	.120
	L_n	.667	.675	.678	.715	.689	.678	.676

Entries are probability of rejecting H_0 when the random sample is actually from the stated alternatives distributions.

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