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On the Optimal Design of Failure Step-Stress Partially Accelerated Life Tests for Exponentiated Inverted Weibull with Censoring

Amal S. Hassan

Institute of Statistical Studies and Research (ISSR). Cairo University, Department of Mathematical Statistics.

Abstract: This article deals with the optimal designing of failure step- stress partially accelerated life tests with two stress levels under type-I censoring.The lifetime of the test items is assumed to follow exponentiated inverted Weibull distribution. The point and interval estimates of the model parameters are obtained. Based on type I censoring, optimum test plans for failure step stress partially accelerated test are also developed through the D-optimality criterion. Such method minimizes the generalized asymptotic varianceof the maximum likelihood estimators for the model parameters. Some numerical illustrations are provided to illustrate the proposed procedure.

Key words: Partially accelerated life test; Failure step stress test; Exponentiated inverted Weibull distribution; Type I censoring; Maximum likelihood method; Optimal design; D-optimality.

INTRODUCTION

Today, most manufacturers are doing their best to develop and improve the performance of their products to increase the demand and documenting trust between them and the consumer. The manufacturers facesome challenges while product development, including difficulty in controlling the failure of the product (for estimation of reliability) during the availabletest time. For that, to overcome many of the challenges in standard reliability testing accelerated life testing (ALT) methods may be utilized. It is important to improve the performance of the product, work to develop it and frontispiece thefactors that causes the short life time. Quantitative accelerated life testing involves identifying stress conditions that will accelerate the failure mode so that failures may be observed in a shorter time period. Depending on the product type, the accelerated testing conditions may involve a higher level of temperature, pressure, voltage, load, speed, vibration, etc, and more than one stress may be utilized.

In ALT, acceleration factor is known or there exists a mathematical model which specifies the relationship between lifetime and stress. However, there are some situations where neither acceleration factor is known nor such models exist or hardly assumed. So, in such cases partially accelerated life tests (PALTs) are more suitable test to be performed for itemsthat are subjected to both normal and accelerated conditions. Moreover, it is the most sensible scheme to be used for estimating the acceleration factor and thus extrapolating the accelerated test data to use condition.According to Nelson (1990), the stress can be applied in various ways.One way to accelerate failure is the step-stress, which increases the stress applied to test unit at a specified discrete sequence. DeGroot and Goel (1979) have introduced the concept of step PALT in which item is first run at normal use condition and, if it does not fail for a specified time, then, it is run at accelerated condition until failure.

Concerning the step-stress test method there are two main types, the first one is time step stress (TSS) where this test runs for a specified time at each stress. The second one is failure-step stress (FSS) whereas this test starts to run at a design (normal) until the occurrenceof a specified number of failure items.Then, stress on them is raised and fixed over aspecified time until the test time terminates.

Time step stress partially accelerated life tests (TSS-PALTs) have been studied by several authors for example; Bai and Chung(1992), Bai *et al*. (1993), Abdel-Ghaly *et al*. (2002), Abd-Elfattah, *et al* (2008), Ismail (2006, 2011), Aly and Ismail (2008). On the other hand, there were no more studies done about failure step stress partially accelerated life test (FSS-PALT), unlessIsmail and Aly (2009) who studied the optimum test plans of FSS-PALT for the Weibull distribution under type-II censored sampling. They determined the optimum number of failure items at normal use stress to switch to accelerated stress. The model parameters are estimated using maximum likelihood method. Also, the confidence intervals and the Fisher information matrix of the estimated parameters are obtained. Hassan and Al-Thobety (2012) studied the estimation and optimal design problems forinverted Weibull distribution in FSS-PALT under type II censoring.

This article considers the problem of optimally designingFSS-PALT using type-Icensoring based onexponentiated inverted Weibulldistribution.This test runs only under two stresses (normal and accelerated). This article can be organized as follows. In Section 2 notations of the model are described. Description of the

Corresponding Author: Amal S. Hassan, Institute of Statistical Studies and Research (ISSR). Cairo University, Department of Mathematical Statistics.

model and test procedure will be introduced in Section 3.In Section4, the maximum likelihood method will be used toobtain the point and interval estimates of the model parameters. The problem of choosing the optimum proportion of items that should befailed at the design stress, $\pi*$ will be addressed using D-optimality criterion in Section 5. For illustrating the theoretical results, Monte Carlo simulation will be carried out inSection 6. Finally, a conclusion is included in Section 7.

Nomenclature

n	total number of test items in FSS-PALT ($n = n_u + n_a + n_c$)
n_u	number of items failed at normal condition ($n_u = \pi n$)
n_a	number of items failed at accelerated condition.
n_c	number of censored items.
λ, θ	shape parameters of exponentiated inverted Weibull ($\lambda, \theta > 0$)
β	accelerated factor $(\beta > 1)$
η	predetermined censoring time
T	lifetime of an item at normal condition
Y	total lifetime of an item in FSS-PALT
π	predetermined proportion of item to be failed under normalcondition,
π_u^*	optimum proportion of test items failed at normal use condition
n_u^*	optimal number of items must fail at normal condition to switch to the accelerated condition
y_{n_u}	observed value of failure time of test items at normal condition
γ	confidence level
δ_{1i}, δ_{2i}	indicator function
$\downarrow (.)$	evaluated at (.)
□	implies a maximum likelihood estimate
F(t)	cumulative distribution function
R(t)	reliability function
F	asymptotic Fisher information matrix

Model and Test Procedure:

The inverted Weibull distribution is one of the most popularprobability distribution to analyze the life time datawith some monotone failure rates. Flaih *et al* (2012) introduced a simple generalization to inverted Weibull distribution by adding a new shape parameter. The exponentiated invertedWeibull(EIW)distributionhas two shape parameters.The EIW distribution has the distribution function of the form

$$F(t) = [\exp(-t^{-\lambda})]^{\theta}, \qquad t, \theta, \lambda > 0 \tag{3.1}$$

with reliability function in the form

$$R(t) = 1 - [\exp(-t^{-\lambda})]^{\theta}, \qquad t, \theta, \lambda > 0 \tag{3.2}$$

where, λ and θ are the shape parameters. For $\theta = 1$ it represents the standard inverted Weibull, and for $\lambda = 1$ it represents thestandard expontiated inverted Weibull. When $\lambda < 1$, the hazard function is continually decreasing which represents early failures. When $\lambda > 1$, the hazard function is continually increasing which represents wear-out failures.Some properties of distribution as hazard function,median and moments are studied by Flaih *et al* (2012). In addition, theyobtained maximum likelihood estimators as well as the least squares estimators and the asymptotic confidence intervals. (for more details about this distribution see, Flaih *et al* (2012)).

3.1 Assumptions:

The following assumptions of FSS-PALT under type I censoring are made:

(1) The failure times $y_i, i = 1...n$ are independent and identically distributed random variables.

(2) The total lifetime of test items denoted by Y pass through two stages, which are the normal and accelerated conditions. Then, the lifetime of the an item under FSS-PALT is defined as

$$Y = \begin{cases} T, & T < y_{n_u} \\ y_{n_u} + \beta^{-1}(T - y_{n_u}) \quad if & T > y_{n_u}, \end{cases} \tag{3.3}$$

where,T is the lifetime of an item at normal condition, and β is theacceleration factor which is the ratio of mean life at normal use condition to that at accelerated condition,usually $\beta > 1$.From the assumptions, the probability density function of a total lifetime$Y$ of test item takes the following forms:

$$f(y) = \begin{cases} f_1(y) = \theta\lambda\, y^{-(\lambda+1)} (e^{-y^{-\lambda}})^{\theta} & 0 < y \le y_{n_u} \\ f_2(y) = \theta\lambda\beta(y_{n_u} + (y - y_{n_u})\beta)^{-(\lambda+1)} (e^{-[(y_{n_u} + (y - y_{n_u})\beta]^{-\lambda}})^{\theta} & if \quad y > y_{n_u} \end{cases} \tag{3.4}$$

where $f_2(y)$ is obtained by the transformation variabletechnique.

3.2 Test Procedure:

1. The test is conducted as follows, a random sample of n independent and identically items firstly tested under normal condition and run until time y_{n_u}, when exactly n_u failures are observed.

2. After time y_{n_u}, the surviving $(n - n_u)$ itemsare subjected to accelerated condition and continued until predetermined censoring time $\eta$is reached. Therefore, the number of failure items are n_a and the number of censoring items are n_c ($n_c = n - n_u - n_a$). The effect of this switch is to multiply the remaining lifetime of items by the inverse of the accelerator factor β. In this case, the switching to the higher stress will shorten the life of test items.

3. Let $\pi = n_u / n$ be the proportion of the items failed under normal condition and predetermined. Therefore, $n_a = n(1 - \pi) - n_c$ and $n_c = n(1 - \pi) - n_a$.

Point and Interval Estimates:

In this Section, the point and interval estimates of the model parameters areintroduced using the maximum likelihood method.

4.1 Point Estimates:

In type I censoring, the test applied to n identical items will terminate when all items fail or censoring time η, $(0 < y_{(n_u)} < \eta < \infty)$ is reached.Hence, the observed values of the total lifetime Y are $y_{(1)} \le ... \le y_{(n_u - 1)} \le y_{(n_u)} \le y_{(n_u + 1)} \le ... \le y_{(n_u + n_a)} \le \eta$, where n_u and n_a are the number of failure items that occur before y_{n_u} and the number of failures that occur before η at normal and accelerated conditions, respectively.Let δ_{1i} and δ_{2i} be indicator functions such that:

$$\delta_{1i} = \begin{cases} 1 & y_{(i)} \le y_{(n_u)} \\ 0 & otherwise \end{cases}, \qquad i = 1,2,...,n$$

and,

$$\delta_{2i} = \begin{cases} 1 & y_{n_u} \le y_{(i)} \le \eta \\ 0 & otherwise \end{cases}, \qquad i = 1,2,...,n$$

For simplifying $y_{(i)}$ can be expressed by y_i. Since the lifetimes $y_1, ..., y_n$ of n items areindependent and identically distributed random variables, then their likelihood function put on FSS-PALT under type I censoring can be written as

$$L(\theta,\lambda,\beta)\,\alpha\prod_{i=1}^{n}[f_1(y_i)]^{\delta_{1i}}[f_2(y_i)]^{\delta_{2i}}[R(\eta)]^{\bar{\delta}_{1i}\bar{\delta}_{2i}}$$

$$L(\theta,\lambda,\beta)\,\alpha\prod_{i=1}^{n}[\theta\lambda y_i^{-(\lambda+1)}(e^{-y_i^{-\lambda}})^{\theta}]^{\delta_{1i}}\{\theta\lambda\beta(y_{n_u}+(y_i-y_{n_u})\beta)^{-(\lambda+1)}(e^{-[(y_{n_u}+(y_i-y_{n_u})\beta]^{-\lambda}})^{\theta}\}^{\delta_{2i}}$$
$$\times[1-(e^{-[(y_{n_u}+(\eta-y_{n_u})\beta]^{-\lambda}})^{\theta}\quad]^{\bar{\delta}_{1i}\bar{\delta}_{2i}} \tag{4.1}$$

where, $\bar{\delta}_{1i}=1-\delta_{1i}$ and $\bar{\delta}_{2i}=1-\delta_{2i}$.

Taking the natural logarithm of the likelihood function, and write $\ln L$ instead of $\ln L(\theta,\lambda,\beta)$,

$$\ln L=(n_u+n_a)(\ln\theta+\ln\lambda)+n_a\ln\beta-(\lambda+1)\sum_{i=1}^{n}[\delta_{1i}\ln y_i+\delta_{2i}\ln(A_1)]-\theta\sum_{i=1}^{n}\delta_{1i}y_i^{-\lambda}$$
$$-\theta\sum_{i=1}^{n}\delta_{2i}A_1^{-\lambda}+n_c\ln(1-A_3^{\theta}), \tag{4.2}$$

where,

$$A_1=y_{n_u}+\beta(y_i-y_{n_u}),\ A_2=y_{n_u}+\beta(\eta-y_{n_u}),\ A_3=\exp(-A_2^{-\lambda}),$$

$$\sum_{i=1}^{n}\delta_{1i}=n_u,\ \sum_{i=1}^{n}\delta_{2i}=n_a,\text{ and }\sum_{i=1}^{n}\overline{\delta_{1i}}\ \overline{\delta_{2i}}=n_c.$$

Maximum likelihood estimators (MLEs) $\hat{\theta},\hat{\lambda}$ and $\hat{\beta}$ of θ,λ and β are the solution of the following system of equations

$$\frac{\partial\ln L}{\partial\theta}=\frac{n_u+n_a}{\theta}-\sum_{i=1}^{n}[\delta_{1i}y_i^{-\lambda}+\delta_{2i}A_1^{-\lambda}]+\frac{n_cA_2^{-\lambda}}{A_3^{-\theta}-1}, \tag{4.3}$$

$$\frac{\partial\ln L}{\partial\lambda}=\frac{n_u+n_a}{\lambda}+\theta\sum_{i=1}^{n}[\delta_{1i}y_i^{-\lambda}\ln y_i+\delta_{2i}A_1^{-\lambda}\ln A_1]-\sum_{i=1}^{n}[\delta_{1i}\ln y_i+\delta_{2i}\ln A_1]$$
$$-\frac{n_c\theta A_2^{-\lambda}\ln A_2}{A_3^{-\theta}-1}, \tag{4.4}$$

$$\frac{\partial\ln L}{\partial\beta}=\frac{n_a}{\beta}-(\lambda+1)[\sum_{i=1}^{n}\frac{\delta_{2i}(y_i-y_{n_u})}{A_1}]+\theta\lambda[\sum_{i=1}^{n}\frac{\delta_{2i}(y_i-y_{n_u})}{A_1^{\lambda+1}}]-\frac{n_c\theta\lambda(\eta-y_{n_u})}{A_2^{\lambda+1}(A_3^{-\theta}-1)}. \tag{4.5}$$

These equations do not admit explicit solutions. So, Newton-Raphson method is used to solve these equations numericallyvia MathCAD "14" to obtain $\hat{\lambda}$, $\hat{\theta}$ and $\hat{\beta}$.

The asymptotic variances and covariance matrix of the MLE of the parameters can be approximated by numerically inverting the asymptotic Fisher-information matrix (*F*). The asymptotic Fisher information matrix *F* can be written as follows:

$$F=\begin{bmatrix}f_{11}&f_{12}&f_{13}\\f_{21}&f_{22}&f_{23}\\f_{31}&f_{32}&f_{33}\end{bmatrix}=\begin{bmatrix}\frac{-\partial^2\ln L}{\partial\theta^2}&\frac{-\partial^2\ln L}{\partial\theta\partial\lambda}&\frac{-\partial^2\ln L}{\partial\theta\partial\beta}\\\frac{-\partial^2\ln L}{\partial\lambda\partial\theta}&\frac{-\partial^2\ln L}{\partial\lambda^2}&\frac{-\partial^2\ln L}{\partial\lambda\partial\beta}\\\frac{-\partial^2\ln L}{\partial\beta\partial\theta}&\frac{-\partial^2\ln L}{\partial\beta\partial\lambda}&\frac{-\partial^2\ln L}{\partial\beta^2}\end{bmatrix}\downarrow(\hat{\theta},\hat{\lambda},\hat{\beta})$$

4.2 Interval Estimation:

For large sample size n, the MLEs, $\hat{\phi}$ under appropriate regularity conditions are consistent and asymptotically normally distributed with means ϕ and variances $\sigma_n^2(\hat{\phi})$. Therefore, the two-sided approximate $\gamma 100$ percent confidence limits for the maximum likelihood estimatesof apopulation parameter θ, λ and β can be obtained, respectively, as follows:

$$L_\theta = \hat{\theta} - z\sigma(\hat{\theta}),\ \ U_\theta = \hat{\theta} + z\sigma(\hat{\theta}),$$

$$L_\lambda = \hat{\lambda} - z\sigma(\hat{\lambda}),\ \ U_\lambda = \hat{\lambda} + z\sigma(\hat{\lambda}), \tag{4.6}$$

and,

$$L_\beta = \hat{\beta} - z\sigma(\hat{\beta}),\ \ U_\beta = \hat{\beta} + z\sigma(\hat{\beta}),$$

where, z is the $[\frac{100(1-\gamma)}{2}]^{th}$ standard normal percentile and $\sigma(.)$ is the standard deviation for the maximum likelihood estimates. Therefore, the two-sided approximate confidence limits for α, β and λ will be constructed with confidence levels 95 % as shown in Table 1.

Optimum Test Plan:

The optimal plans for FSS-PALT consider the optimum proportion of test items that must fail at normal stress according to a certain optimality criterion which is ageneralized asymptotic variance(GAV) of the MLE of the model parameters. The GAV for the MLE of the model parameters is the reciprocal of the determination of the Fisher information matrix (Bai *et al*. 1993). That is,

$$GAV(\hat{\theta}, \hat{\lambda}, \hat{\beta}) = \frac{1}{|F|} \tag{5.1}$$

The optimum failure step stress test plan for products having EIW lifetime distribution is to find the optimum proportion of test items failing at normal stress π^*such that the GAV is minimized. The minimization of GAV over π can be achieved by solving the following equation:

$$\frac{\partial |F|}{\partial \pi} = 0\ , \tag{5.2}$$

In general, the solution to equation (5.2) has no explicit solution. So, Newton-Raphson method is applied to obtain the optimal stress change point π_u^* which minimizing the GAV. Thus, the optimal numbers of items must fail at normal condition n_u^* for switching to the accelerated condition is:

$$n_u^* = \pi^* n, \tag{5.3}$$

where n is the sample size.

6. Simulation Procedure:

This Section presents the numerical solutions to obtain the MLEs of the unknownparameters θ, λ and β, their mean squared errors (MSEs), absolute relative biases(ARBs), lower confidence bounds (LCBs) andupper confidence bounds (UCBs). Also, it presents thenumerical solutions to determine theoretical results of the optimal design.Simulation procedures are described through the following two algorithmswhich have beenconducted usingMathCAD "14".

6.1 First Algorithm:

The following algorithm is designed to illustrate the statistical inference methods:

Step 1: A random sampleof sizes 100 (100) 500 and 700are generated from EIW distribution under type I censoring using different combinations of the parameter values for θ, λ and β

Step 2: Under type I censoring, choose a proportion of test items failing at normal condition to be $\pi = 40\%$ and censoring time of a FSS-PALT to be $\eta = 8$.

Step 3: For each sample, the acceleration factor and the parameters of the distribution are estimated in failure step stress PALT under type I censored sample.

Step 4: Repeat the previous steps from 1 to 3 *N* times representing *N* different samples, where *N*=1000.

Step 5: Newton-Raphson method is used for solving the three nonlinear equations (4.3), (4.4) and (4.5) respectively, to obtain the estimators of θ, λ and β.

Step 6: The ARBs, MSEsof θ, λ and β over the 1000 samples are obtained.

Step 7: Calculate the Fisher-information matrix then inverted to get the asymptotic variance and covariance matrix of the estimators for different sample sizes. The LCBs and the UCBs for each parameter are obtained with confidence level $\gamma = 0.95$

Table 1 representsthe ARBs, MSEs,LCBsand UCBs for the parameters θ, λ and β for the fourcases of differentcombinations of the initial values of the parameters.

Table 1: The MSEs, ARBs,LCBs and UCBsof the estimators of the parameters (λ, θ, β) for different sized samples under type I censoring for FSS-PALT.

Sample size n	Parameter (λ, θ, β)	Case 1 $(\lambda = 1.5, \theta = 0.5, \beta = 1.1)$				Case 2 $(\lambda = 2.5, \theta = 1.5, \beta = 1.2)$			
		MSE	ARB	LCB	UCB	MSE	ARB	LCB	UCB
100	λ	0.042	0.041	1.057	1.819	0.588	0.226	2.049	4.081
	θ	0.005	0.084	0.444	0.640	0.018	0.058	1.211	1.614
	β	0.331	0.102	0.107	2.318	0.452	0.516	0.064	1.019
200	λ	0.013	0.031	1.298	1.660	0.255	0.148	2.197	3.544
	θ	0.001	0.051	0.470	0.581	0.009	0.039	1.289	1.593
	β	0.081	0.024	0.571	1.683	0.315	0.430	0.252	1.115
300	λ	0.009	0.023	1.288	1.642	0.154	0.121	2.311	3.293
	θ	0.001	0.046	0.478	0.568	0.006	0.031	1.329	1.576
	β	0.053	0.021	0.650	1.555	0.262	0.400	0.371	1.068
400	λ	0.007	0.020	1.309	1.631	0.122	0.107	2.329	3.206
	θ	0.000	0.034	0.473	0.561	0.007	0.035	1.328	1.557
	β	0.041	0.021	0.680	1.473	0.233	0.377	0.417	1.078
500	λ	0.005	0.011	1.343	1.642	0.105	0.106	2.402	3.129
	θ	0.000	0.024	0.475	0.548	0.005	0.031	1.356	1.550
	β	0.030	0.043	0.726	1.374	0.211	0.363	0.476	1.054
700	λ	0.003	0.013	1.369	1.590	0.076	0.094	2.453	3.017
	θ	0.000	0.028	0.484	0.544	0.004	0.027	1.376	1.546
	β	0.017	0.034	0.817	1.307	0.197	0.357	0.538	1.006
Sample size n	Parameter (λ, θ, β)	Case 3 $(\lambda = 1.8, \theta = 0.8, \beta = 1.3)$				Case 4 $(\lambda = 1.75, \theta = 1, \beta = 1.2)$			
		MSE	ARB	LCB	UCB	MSE	ARB	LCB	UCB
100	λ	0.063	0.029	1.371	2.334	0.092	0.086	1.386	2.416
	θ	0.001	0.003	0.733	0.862	0.003	0.044	0.889	1.036
	β	0.257	0.256	0.218	1.716	0.267	0.330	1.550	1.453
200	λ	0.019	0.023	1.585	2.097	0.038	0.070	1.575	2.171
	θ	0.000	0.007	0.743	0.844	0.002	0.041	0.906	1.011
		0.171	0.267	0.515	1.390	0.181	0.307	0.419	1.240

	β								
300	λ	0.014	0.023	1.624	2.059	0.026	0.061	1.618	2.095
	θ	0.000	0.007	0.745	0.821	0.002	0.041	0.912	1.007
	β	0.167	0.267	0.562	1.315	0.155	0.309	0.531	1.148
400	λ	0.011	0.024	1.651	2.034	0.022	0.063	1.635	2.062
	θ	0.000	0.016	0.753	0.822	0.002	0.042	0.921	0.996
	β	0.152	0.277	0.645	1.236	0.157	0.303	0.503	1.186
500	λ	0.011	0.027	1.669	2.028	0.022	0.063	1.674	2.049
	θ	0.000	0.021	0.753	0.815	0.002	0.037	0.921	0.992
	β	0.150	0.277	0.661	1.220	0.154	0.300	0.568	1.096
700	λ	0.008	0.024	1.691	1.995	0.019	0.057	1.704	2.018
	θ	0.000	0.015	0.764	0.813	0.002	0.038	0.931	0.994
	β	0.144	0.277	0.719	1.156	0.146	0.296	0.608	1.065

As shown from the above Table, one can observe that the numerical resultssupport the theoretical findings. That is, the MLEs have good statisticalproperties. As the sample size increases, ARBs and MSEs of the MLEs decrease and the confidencebounds of the parameters are much narrow.

6.2 Second Algorithm:

Thefollowing algorithm is designed to get theoptimum proportion of test items failing at normal stress π^*.

Step 1: Using each combination of the MLEs of the model parameters obtained by using the above steps as initial values, samples of different sizes are generated from GIW distribution.

Step 2: For each sample, equation (5.2) is computed and put equal to zero. By solving this equation, optimum proportion of items π^* that must fail at normalstress is obtained.

Step 3: The value of n_u^* (the optimum number of failure items at normal condition) is determined.

Table 2 gives the optimum stress-change point π^* the values of n_u^* and GAV.

Table 2: The results of optimal design of FSS-PALTs.

Sample size n	Case 1 $(\lambda = 1.5, \theta = 0.5, \beta = 1.1)$			Case 2 $(\lambda = 2.5, \theta = 1.5, \beta = 1.2)$		
	π^*	n_u^*	GAV	π^*	n_u^*	GAV
100	0.580	58	0.000001	0.432	43	0.00002
200	0.594	119	0.000000	0.418	84	0.00257
300	0.600	180	0.000003	0.416	124	0.00000
400	0.603	241	0.000001	0.432	165	0.00000
500	0.601	301	0.000000	0.418	205	0.00000
700	0.601	420	0.000000	0.409	286	0.00000
Sample size n	Case 3 $(\lambda = 1.8, \theta = 0.8, \beta = 1.3)$			Case 4 $(\lambda = 1.75, \theta = 1, \beta = 1.2)$		
	π^*	n_u^*	GAV	π^*	n_u^*	GAV
100	0.478	48	0.00000	0.423	42	0.00000
200	0.479	96	0.00000	0.415	83	0.00000
300	0.477	143	0.00000	0.417	125	0.00000
400	0.472	188	0.00000	0.414	166	0.00000
500	0.418	205	0.00000	0.414	207	0.00000
700	0.409	286	0.00000	0.413	289	0.00000

It seems from Table 2that, the optimum value of π^* for parameters cases 2, 3 and 4, is approximately 40%. This means that less than half of the observations will fail under the normal conditions. In other words, the optimum number of items that should fail at normal condition to switch to the accelerated condition is $n_u^* = 0.4 \times n.$ While the optimum value of π^* for the case parameter $(\lambda = 1.5, \theta = 0.5, \beta = 1.1)$ is approximately 60%, that means, the optimum number of items that should fail at normal condition to switch to the accelerated condition is $n_u^* = 0.6 \times n.$ As shown via the optimalvalue of π, the partial accelerating is important and needed. That is, testingwill be run not only at normal condition but also at accelerated condition.

Conclusion:

FSS- PALT requires constant monitoring of the items under test and may not be convenient. But it is more appropriate than TSS-PALT, where it enables the experimenter to collect sufficient information and to make a good statistical inference about the population parameters. This makes the prediction reliability with highly level of accuracy.

This articleis concerned with the problem of optimally designing simple failure step stress PALT plans under type-I censoring. The test itemis assumed to follow GIW distribution. The point and interval estimates of the model parametersare studied togetherwith some further properties.

Optimal test plans are important to improve the accuracy of parameter estimation thereby improving the quality of inference. For this, the optimum of these plans is more useful and efficient to estimate the life distribution at design stress.Optimum plans for the FS-PALT are obtained numerically usingthe D-optimality via a simulation study. The optimal test plans are obtained by minimizing the GAV of the MLEs of the model parameters. It is obvious via the optimal value of π^*that the PALT model is more appropriate.

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