Odds Generalized Exponential-Inverse Weibull Distribution: Properties & Estimation

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Abstract

Providing extended and generalized distribution is usually precious for many statisticians. A new distribution, called odds generalized exponential-inverse Weibull distribution (OGE-IW) is suggested for modeling lifetime data. Some structural properties of the new distribution are obtained. Three different estimation procedures, namely; maximum likelihood, percentiles and least squares, are used to estimate the model parameters of subject distribution. The consistency of the parameters of the OGE-IW distribution is demonstrated through a simulation study. A real data application is presented to illustrate the importance of the new distribution compared with some known distributions.

Keywords: T-X family, Inverse Weibull distribution; Maximum likelihood estimators; Least squares estimators, Percentiles estimators.

1. Introduction

The inverse Weibull (IW) distribution has been received some attention in the literature. The IW distribution can be used to a diverse model of failure characteristics, such as infant mortality, age of production, and periods of erosion. The inverse Weibull distribution can also be used to determine the cost-effectiveness and maintenance periods of reliability centered maintenance activities. An early study about IW model has been developed by Erto in 1989. The shapes of the density and failure rate functions for the basic inverse model have been studied by Keller and Kamath (1982). The maximum likelihood and least squares estimators of IW distribution have been studied by Calabria and Pulcini (1990). Bayes two-sample prediction of the IW distribution has been developed by Calabria and Pulcini (1994). Hassan and AL-Thobety (2012) provided an optimal design of failure step stress partially accelerated life tests with type II inverse Weibull data. Hassan *et al*. (2015) studied constant stress partially accelerated life tests with Type II inverse Weibull data using multiple censored data. The probability density function (pdf) and cumulative distribution function (cdf) of IW distribution with shape parameter α and scale parameter β are given, respectively, by

$$g(x;\beta,\alpha)=\alpha\beta x^{-\alpha-1}e^{-\beta x^{-\alpha}}; \qquad \alpha,\beta>0, \quad x>0, \tag{1}$$

$$G(x;\beta,\alpha)=e^{-\beta x^{-\alpha}}. \tag{2}$$

Extended and generalized forms of IW distribution are studied by some authors, among them; Khan (2010) introduced and studied the beta inverse Weibull distribution. de Gusmão *et al.* (2011) introduced three-parameter inverse Weibull distribution, called the generalized inverse Weibull distribution, with unimodal, increasing and decreasing failure rates. Khan and King (2012) proposed four-parameter modified inverse Weibull distribution. Shahbaz *et al.* (2012) suggested the Kumaraswamy inverse Weibull distribution. Elbatal and Muhammed (2014) introduced the exponentiated generalized inverse Weibull distribution. The generalized inverse Weibull distribution including the exponentiated or proportional reverse hazard and Kumaraswamy generalized inverse Weibull distributions have been suggested by Oluyede and Yang (2014). Pararai *et al.* (2014) introduced gamma-inverse Weibull distribution based on gamma generated family. Khan *et al.* (2014) studied characterizations of the transmuted inverse Weibull distribution with an application to bladder cancer remission time's data. Khan and King (2016) introduced the four-parameter new generalized inverse Weibull distribution and investigated its potential usefulness with application to reliability data from engineering studies. Rodrigues *et al.* (2016) introduced exponentiated Kumaraswamy inverse Weibull distribution. Okasha *et al*. (2017) introduced the Marshall–Olkin extended inverse Weibull distribution.

The statistics literature is filled with lots of continuous univariate distributions for describing real data. In recent years, there has been a great interest among statisticians and applied researchers in constructing flexible distribution to facilitate better modeling of lifetime data in various situations. Several methods have been developed for generating new family of lifetime distributions. One approach of generalization was suggested by Marshall and Olkin (1997) by adding one parameter to the survival function $\bar{G}(x)$. In the same trend, Gupta *et al.* (1998) added one parameter to the cdf $G(x)$ of the baseline distribution to define the exponentiated–G class of distribution. Following Gupta’s *et al*. class, Gupta and Kundu (1999) studied the two-parameter generalized exponential distribution as an extension of the exponential distribution. Our interest here with T-X family proposed by Alzaatreh *et al.* (2013), the cdf of T-X family is specified by

$$F(x)=\int_{0}^{W(G(x)}f(t)dt, \tag{3}$$

where, the random variable T called the transformer and $W(G(x))$ be a function of $G(x)$. Based on T-X family; Tahir *et al.* (2015) introduced the odd generalized exponential by using the generalized exponential as generator in (3) and taking the upper limit to be $G(x)/\bar{G}(x)$, the odds function of any distribution. *Maiti and Pramanik (2015) defined a generalized class of any distribution* by taking the exponential distribution as a generator in (3) and taking the upper limit to be $G(x)/\bar{G}(x)$. Further, Alizadeh *et al.* (2017) proposed and studied a new generated family called the generalized odd generalized exponential.

Our motivation here is to introduce and study a new extended form for the inverse Weibull distribution with three parameters. We call the new distribution; the odds generalized exponential-inverse Weibull distribution, which is a particular case of T-X family of distributions. The rest of the paper contains the following sections. The new distribution is provided in Section 2. Some statistical properties are given in Section 3. Then, in Section 4, maximum likelihood, least squares and percentiles estimators are obtained. Simulation study and results are presented in Section 5. An application of the OGE-IW model to real data is presented in Section 6. At the end, concluding remarks are addressed in Section 7.

2. Construction of the OGE-IW Distribution

In this section, the pdf, cdf, reliabilty function, hazard rate function (hrf), reversed-hazard rate function and cumulative hazard rate function of OGE-IW distribution are derived. Expansions for its pdf and cdf are also provided.

We obtain the OGE-IW distribution by considering the exponential distribution as transformer in cdf (3); also, taking; $W(G(x)) = G(x)/\bar{G}(x)$, the odds ratio of inverse Weibull distribution defined in (2) as follows

$$F(x;\alpha,\lambda,\beta) = \int_0^{\frac{G(x)}{1-G(x)}} \lambda e^{-\lambda t}\, dt = \int_0^{\frac{e^{-\beta x^{-\alpha}}}{1-e^{-\beta x^{-\alpha}}}} \lambda e^{-\lambda t}\, dt\,.$$

Hence, the cdf of OGE-IW distribution is as follows

$$F(x;\alpha,\lambda,\beta) = 1 - \exp-\left(\frac{\lambda}{e^{\beta x^{-\alpha}} - 1}\right), \quad x > 0 \tag{4}$$

The corresponding pdf is obtained as follows

$$f(x;\alpha,\lambda,\beta) = \alpha\beta\lambda x^{-\alpha-1} e^{-\beta x^{-\alpha}} \left(1 - e^{-\beta x^{-\alpha}}\right)^{-2} \exp-\left(\frac{\lambda}{e^{\beta x^{-\alpha}} - 1}\right). \tag{5}$$

For $\alpha = 2,$ the OGE-IW reduces to a new model named as odds generalized exponential inverse Rayleigh distribuiton. For $\alpha = 1,$ OGE-IW reduces to another new model named as odds generalized exponential inverse exponential distribuiton.

Plots of the pdf of OGE-IW distribution for some selected parameter values are displayed in Figure 1. As seems from this figure, the pdf of OGE-IW distribution can be symmetric, unimodel and right skewed according to the selected values of parameters.

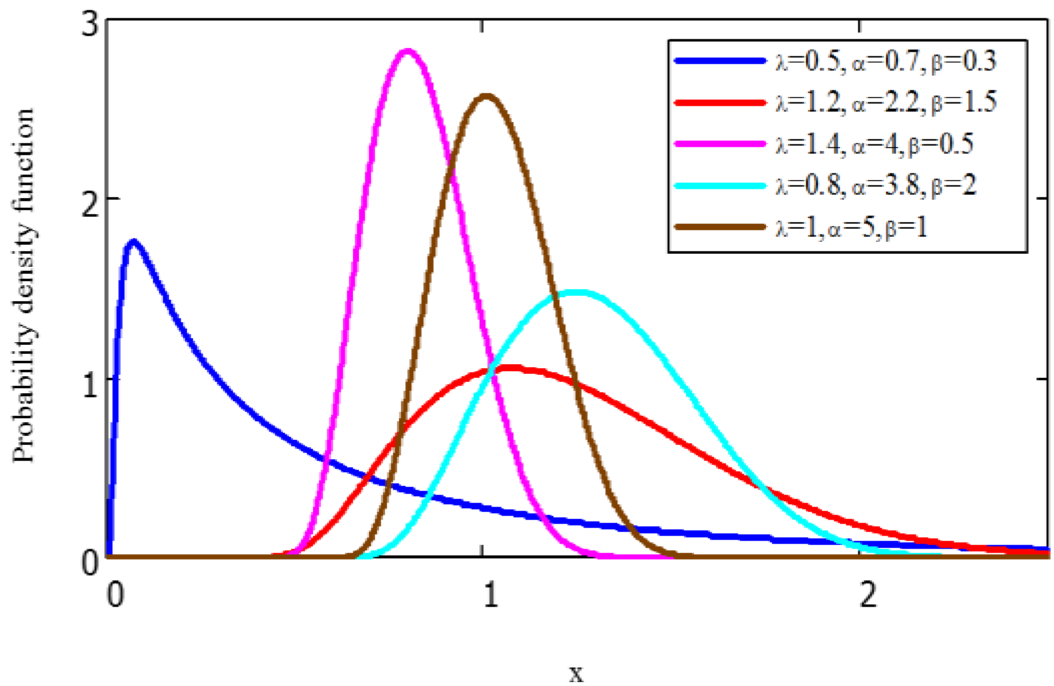

Figure 1: *Plots of the pdf of OGE-IW distribution for selected values of the parameters*

2.1 Expansion for Densities of OGE-IW Distribution

Two useful expansions of OGE-IW pdf and cdf are derived. Since, the pdf (5) can be rewritten as follows

$$f\left(x;\lambda,\alpha,\beta\right)=\alpha\beta\lambda x^{-\alpha-1}e^{-\beta x^{-\alpha}}\left(1-e^{-\beta x^{-\alpha}}\right)^{-2}\exp-\lambda\left(\frac{e^{-\beta x^{-\alpha}}}{1-e^{-\beta x^{-\alpha}}}\right). \tag{6}$$

Then, by using the exponential expansion for the last term in (6) and further the binomial expansion for a positive real power yields

$$f\left(x;\lambda,\alpha,\beta\right)=\sum_{j,i=0}^{\infty}\left(-1\right)^{j}\frac{\alpha\beta\lambda^{j+1}}{j!}\frac{\Gamma\left(j+2+i\right)}{\Gamma\left(j+2\right)i!}x^{-\alpha-1}e^{-\beta(j+i+1)x^{-\alpha}}. \tag{7}$$

Then the pdf (7) can be formed as follows

$$f\left(x;\lambda,\alpha,\beta\right)=\sum_{j,i=0}^{\infty}c_{j,i}\,g_{\beta(j+i+1)}\left(x\right), \tag{8}$$

where, denotes the pdf of IW distribution $g_{\beta(j+i+1)}(x)$ and $c_{j,i}=\left(-1\right)^{j}\frac{\lambda^{j+1}}{j!}\frac{\Gamma\left(j+i+1\right)}{\Gamma\left(j+2\right)i!}$ with parameters $\beta(j+i+1)$ and α.

Further, an expansion for $\left[F(x;\lambda,\alpha,\beta)\right]^{t}$, for t a positive real power is derived as follows

$$\left[F(x;\lambda,\alpha,\beta)\right]^{t}=\sum_{m=0}^{t}\sum_{l,p=0}^{\infty}\omega_{m,l,p}G_{\beta(l+p)}(x), \tag{9}$$

where, $\omega_{m,l,p}=(-1)^{m+l}\binom{t}{m}\frac{(\lambda m)^{l}\,\Gamma(l+p)}{p!\,l!\,\Gamma(l)}$ and $G_{\beta(l+p)}(x)$ is the cdf of IW with parameters $\beta(l+p)$ and α.

2.2 Reliability Analysis

This subsection gives expressions for the reliability function, hazard function, and reversed hazard function.

The survival function and hrf of the OGE-IW distribution are respectively given by

$$\bar{F}(x;\alpha,\lambda,\beta)=\exp-\left(\frac{\lambda}{e^{\beta x^{-\alpha}}-1}\right),$$

$$h(x;\alpha,\lambda,\beta)=\lambda\alpha\beta x^{-\alpha-1}\left(1-e^{-\beta x^{-\alpha}}\right)^{-2}.$$

Figure 2 gives the plots of the hrf of OGE-IW distribution for some selected parameter values. Figure 2 indicates that OGE-IW hrfs can have increasing, decreasing and constant. This fact implies that the OGE-IW can be very useful for fitting data sets with various shapes.

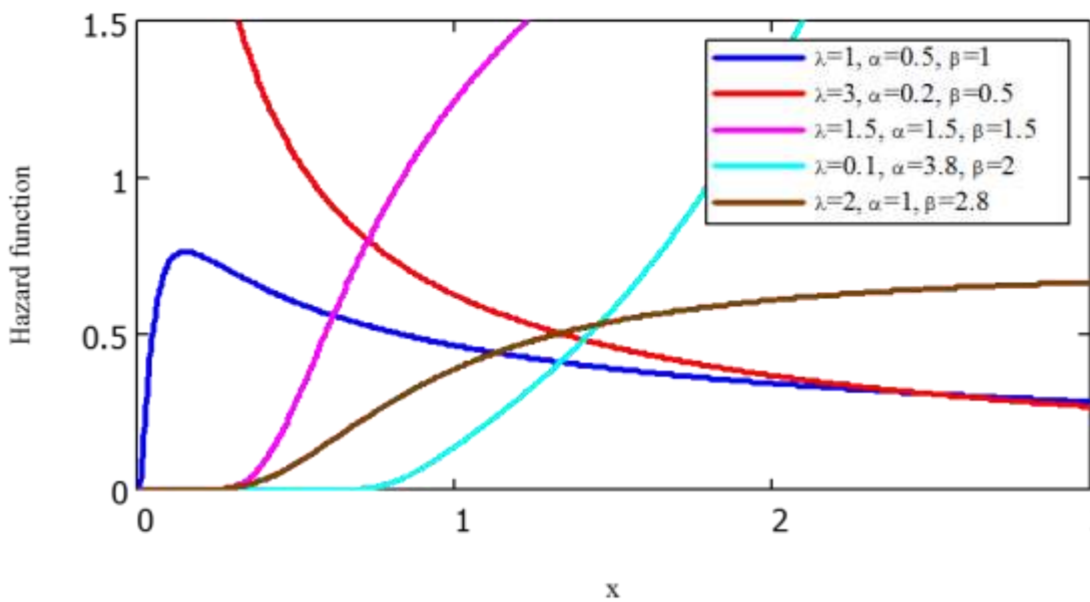

Figure 2: *Plots of hrf of OGE-IW distribution for selected values of the parameters*

The reversed-hazard rate function of the OGE-IW distribution is as follows

$$\upsilon(x;\alpha,\lambda,\beta)=\frac{\lambda\alpha\beta x^{-\alpha-1}e^{-\beta x^{-\alpha}}}{\left(1-e^{-\beta x^{-\alpha}}\right)^{2}\left[\exp-\left(\frac{\lambda}{e^{\beta x^{-\alpha}}-1}\right)-1\right]},$$

Additionaly, the cumulative hazard rate function of the OGE-IW is given by

$$H\left(x;\alpha,\lambda,\beta\right)=-\ln\left|\exp-\left(\frac{\lambda}{e^{\beta x^{-\alpha}}-1}\right)\right|.$$

3. Some Mathematical Properties

In this section, some mathematical properties of the OGE-IW distribution, including, moments, probability weighted moments, incomplete moments, order statistics and entropy measure are derived.

3.1 Moments

The rth moment of OGE-IW is derived by using pdf (8) as follows

$$\mu_r^{'}=\sum_{j,i=0}^{\infty}c_{j,i}\int_0^{\infty}x^r g_{\beta(j+i+1)}\left(x\right)dx=\sum_{j,i=0}^{\infty}c_{j,i}\left[\beta\left(j+i+1\right)\right]^{\frac{r}{\alpha}}\Gamma\left(1-\frac{r}{\alpha}\right),r=1,2,3,... \qquad (10)$$

In particular, the mean and variance of the OGE-IW distribution are given by

$$E(X)=\sum_{j,i=0}^{\infty}c_{j,i}\left[\beta\left(j+i+1\right)\right]^{\frac{1}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right),$$

$$\text{var}(X)=\sum_{j,i=0}^{\infty}c_{j,i}\left[\beta\left(j+i+1\right)\right]^{\frac{1}{\alpha}}\Gamma\left(1-\frac{2}{\alpha}\right)-\left[\sum_{j,i=0}^{\infty}c_{j,i}\left[\beta\left(j+i+1\right)\right]^{\frac{1}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right)\right]^{2}$$

The skewness $\left(g_1\right)$ of the OGE-IW distribution is given by

$$g_1=\frac{\mu_3^{2}}{\mu_2^{3}}=\frac{(\mu_3^{'}-3\mu_2^{'}\mu_1^{'}+2\mu_1^{'3})^2}{(\mu_2^{'}-\mu_1^{'2})^3}.$$

The kurtosis $\left(g_2\right)$ of the OGE-IW is given by

$$g_2=\frac{\mu_4}{\mu_2^{2}}=\frac{\mu_4^{'}-4\mu_3^{'}\mu_1^{'}+6\mu_2^{'}\mu_1^{'2}-3\mu_1^{'4}}{(\mu_2^{'}-\mu_1^{'2})^2}.$$

Table (1) provides the mean (μ) and variance (var) of the OGE-IW distribution for various parameter values. From Table (1), we notice that both values of the mean and variance of the OGE-IW decrease as the values of α and λ increases. Also, the values of the mean and variance increase as the values of β increase. Table (2) contains the skewness and kurtosis of the OGE-IW distribution for various values of parameters α and λ.We notice that both the skewness and the kurtosis are decreasing functions of α and λ.

Table 1: Mean and variance of OGE-IW distribution for various values of α, λ and β

		$\beta = 0.5$		$\beta = 1$		$\beta = 2.5$	
α	λ	μ	var	μ	var	μ	var
2	1	0.7900	0.0800	1.1180	0.1600	1.7670	0.4000
	2	0.6330	0.0370	0.8960	0.0470	1.4160	0.1840
	3	0.5660	0.0240	0.8010	0.0470	1.2660	0.1180
4.5	1	0.8870	0.0200	1.0350	0.0270	1.2680	0.0400
	2	0.8070	0.0120	0.9420	0.0160	1.1540	0.0240
	3	0.7700	0.0085	0.8980	0.0740	1.1010	0.0170
5	1	0.8970	0.0160	1.0300	0.0220	1.2370	0.0310
	2	0.8240	0.0098	0.9470	0.0130	1.1370	0.0190
	3	0.7900	0.0072	0.9070	0.0096	1.0890	0.0140

Table 2: Skewness and kurtosis of OGE-IW distribution for various values of α and λ

α	λ	g_1	g_2
2	1	0.7910	3.6220
	2	0.7530	3.6210
	3	0.7040	3.5700
4.5	1	0.2940	2.7570
	2	0.3100	2.8560
	3	0.2950	2.8980
5	1	0.2550	2.7230
	2	0.2750	2.8240
	3	0.2630	2.8690

Furthermore, the moment generating function of OGE-IW can be obtained as follows

$$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} E\left(X^r\right) = \sum_{r=0}^{\infty} \sum_{j,i=0}^{\infty} c_{j,i} \frac{t^r \left[\beta(j+i+1)\right]^{\frac{r}{\alpha}} \Gamma\left(1 - \frac{r}{\alpha}\right)}{r!}, r = 1, 2, \ldots$$

3.2 Probability Weighted Moments (PWMs)

Greenwood *et al.* (1979) introduced the probability weighted moments to derive estimators of the parameters and quantiles of distributions. The PWMs of OGE-IW distribution is defined by

$$\tau_{r,t} = E\left\{X^r \left[F(x)\right]^t\right\} = \int_{-\infty}^{\infty} x^r \left[F(x)\right]^t f(x) dx, \tag{11}$$

where, t and r are positive integers. Inserting pdf (8) and cdf (9) in (11), then the PWMs of the OGE-IW distribution is obtained as follows

$$\tau_{r,t} = \sum_{m=0}^{t} \sum_{l,p,j,i=0}^{\infty} \omega_{m,l,p} c_{j,i} \frac{\beta(j+i+1)\Gamma\left(1-\frac{r}{\alpha}\right)}{\left[\beta(j+i+p+l+1)\right]^{1-\frac{r}{\alpha}}}.$$

3.3 Incomplete Moments

Theis defined by $£_s(a)$, moment, sayincomplete sth

$$£_s(a) = \int_{-\infty}^{a} x^s f(x) dx. \tag{12}$$

Hence, the sth moment of OGE-IW is derived by inserting (8) in (12) as follows

$$£_s(a) = \sum_{j,i=0}^{\infty} c_{j,i} \beta(j+i+1)^{\frac{s}{\alpha}} \Gamma\left(1-\frac{s}{\alpha}, \beta(j+i+1)a^{-\alpha}\right). \tag{13}$$

where, $\Gamma\left(1-\frac{s}{\alpha}, \beta(j+i+1)a^{-\alpha}\right)$ is the upper incomplete gamma function. In particular, the first incomplete moments of the OGE-IW distribution can be obtained by putting $s=1$ in (13), as follows

$$£_1(a) = \sum_{j,i=0}^{\infty} c_{j,i} \beta(j+i+1)^{\frac{1}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}, \beta(j+i+1)a^{-\alpha}\right). \tag{14}$$

Bonferroni and Lorenz curves are useful applications to first incomplete moments. These curves are very useful in economics, reliability, demography, insurance and medicine. The Lorenz and Bonferroni curves are obtained, respectively, as follows

$$L_F(x) = \frac{1}{E(X)} \int_0^x af(a)da = \frac{\sum_{j,i=0}^{\infty} c_{j,i} \beta(j+i+1)^{\frac{1}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}, \beta(j+i+1)x^{-\alpha}\right)}{\sum_{j,i=0}^{\infty} c_{j,i} \left[\beta(j+i+1)\right]^{\frac{1}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right)},$$

and

$$B_F(x) = \frac{L_F(x)}{F(x)} = \frac{\sum_{j,i=0}^{\infty} c_{j,i} \beta(j+i+1)^{\frac{1}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}, \beta(j+i+1)x^{-\alpha}\right)}{\left[1-\exp-\left(\frac{\lambda}{e^{\beta x^{-\alpha}}-1}\right)\right] \sum_{j,i=0}^{\infty} c_{j,i} \left[\beta(j+i+1)\right]^{\frac{1}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right)}.$$

Another application of the first incomplete moments refers to the mean deviations which provide useful information about the characteristics of a population. Indeed, the amount of dispersion in a population may be measured to some extent by the totality of the deviations from the mean and median. The mean deviations of X about the meanand μ about the median m can be calculated from the following relations

$$\delta_1 = 2\mu F(\mu) - 2T(\mu) \qquad \text{and} \qquad \delta_2 = \mu - 2T(m),$$

where, $T(q)=\int_0^q xf(x)dx$ which is the first incomplete moment, then from (14) $T(\mu)$ and $T(m)$ are obtained, respectively, as follows

$$T(\mu)=\int_0^{\mu} xf(x)dx=\sum_{j,i=0}^{\infty} c_{j,i}\,\beta\left(j+i+1\right)^{\frac{1}{\alpha}}\Gamma\left(1-\frac{1}{\alpha},\beta\left(j+i+1\right)\mu^{-\alpha}\right),$$

$$T(m)=\int_0^{m} xf(x)dx=\sum_{j,i=0}^{\infty} c_{j,i}\,\beta\left(j+i+1\right)^{\frac{1}{\alpha}}\Gamma\left(1-\frac{1}{\alpha},\beta\left(j+i+1\right)m^{-\alpha}\right).$$

3.4 Rényi Entropy

The entropy of a random variable X with density function $f(x)$ is a measure of the uncertainty variation. The Rényi entropy is defined as

$$I_R(\delta)=\frac{1}{1-\delta}ln\left\{\int_{-\infty}^{\infty} f^{\delta}(x)dx\right\}, \tag{15}$$

where $\delta>0$ and $\delta\neq 1$. Applying the exponential and binomial expansions, then $f^{\delta}(x;\lambda,\alpha,\beta)$ can be expressed as follows

$$f^{\delta}(x;\lambda,\alpha,\beta)=\sum_{j=0}^{\infty}(-1)^j\frac{(\alpha\beta)^{\delta}\delta^j\lambda^{j+\delta}x^{-\delta(\alpha+1)}\Gamma\left(j+i+2\delta\right)e^{-\beta(j+\delta+i)x^{-\alpha}}}{j!\Gamma\left(j+2\delta\right)i!}\left(1-e^{-\beta x^{-\alpha}}\right)^{-(j+2\delta)} \tag{16}$$

Inserting (16) in (15), then the Rényi entropy of OGE-IW distribution becomes

$$I_R(\delta)=\frac{1}{1-\delta}ln\left\{\sum_{j,i=0}^{\infty}(-1)^j\frac{(\beta)^{\delta}\alpha^{\delta-1}\delta^j\lambda^{j+\delta}\Gamma\left(j+2\delta+i\right)}{j!i!\Gamma\left(j+2\delta\right)\left[\beta\left(j+\delta+i\right)\right]^{\frac{\delta(\alpha+1)-1}{\alpha}}}\Gamma\left(\frac{\delta\left(\alpha+1\right)-1}{\alpha}\right)\right\}.$$

3.5 Order Statistics

Let $X_{1:n}<X_{2:n}<...<X_{n:n}$ denote the order statistics for a random sample $X_1, X_1,..., X_n$ from OGE-IW distribution with pdf (8) and cdf (9). The pdf of rth order statistics is defined by

$$f_{r:n}(x)=\frac{1}{B\left(r,n-r+1\right)}\sum_{k=0}^{n-r}\binom{n-r}{k}(-1)^k\left[F(x)\right]^{k+r-1}f(x)\,. \tag{17}$$

Using the binomial expansion for $\left[F(x)\right]^{k+r-1}$, replacing t in (9) with $k+r-1$. Hence the pdf (17) becomes

$$f_{r:n}(x)=\frac{1}{B\left(r,n-r+1\right)}\sum_{k=0}^{n-r}\sum_{m=0}^{k+r-1}\sum_{j,i,l,p=0}^{\infty}\eta_{k,j,i,m,l,p}\,x^{-\alpha-1}\,e^{-\beta(j+i+l+p+1)x^{-\alpha}}, \tag{18}$$

where

$$\eta_{k,j,i,m,l,p}=(-1)^{k+j+m+l}\binom{n-r}{k}\binom{k+r-1}{m}\frac{\alpha\beta\lambda^{j+1}\Gamma\left(j+2+i\right)\left(\lambda m\right)^{l}\Gamma\left(p+l\right)}{\Gamma\left(j+2\right)\Gamma\left(l\right)j!i!l!p!}\,.$$

In particular, the pdf of the smallest order statistics is obtained by substituting $r=1$ in (18) as follows

$$f_{1:n}(x)=n\sum_{k=0}^{n-1}\sum_{m=0}^{k+r-1}\sum_{j,i,l,p=0}^{\infty}\pi_{k,j,i,m,l,p}x^{-\alpha-1}\ e^{-\beta(j+i+l+p+1)x^{-\alpha}},$$

where

$$\pi_{k,j,i,m,l,p}=(-1)^{j+m+l}\binom{n-1}{k}\binom{k}{m}\alpha\beta\lambda^{j+1}\frac{\Gamma(j+2+i)(\lambda m)^{l}\,\Gamma(p+l)}{\Gamma(j+2)\Gamma(l)\,j!i!l!p!}.$$

Also, the pdf of largest order statistics is obtained by substituting $r=n$ in (18) as follows

$$f_{n:n}(x)=n\sum_{m=0}^{k+r-1}\sum_{j,i,l,p=0}^{\infty}\nu_{k,j,i,m,l,p}x^{-\alpha-1}\ e^{-\beta(j+i+l+p+1)x^{-\alpha}},$$

where

$$\nu_{k,j,i,m,l,p}=(-1)^{j+m+l}\binom{k+n-1}{m}\alpha\beta\lambda^{j+1}\frac{\Gamma(j+2+i)(\lambda m)^{l}\,\Gamma(p+l)}{\Gamma(j+2)\Gamma(l)\,j!i!l!p!}.$$

4. Parameter Estimation

In this section, the parameter estimators of the OGE-IW model parameters are obtained based on maximum likelihood (ML), least squares (LS) and percentiles methods.

4.1 Maximum Likelihood Estimators

In this subsection, the estimation of the unknown parameters of the OGE-IW distribution is considered using the ML method. Let $X_1,...,X_n$ be observed values from the OGE-IW distribution. The total log-likelihood function, denoted by lnL, for the parameters λ,α and β in complete sample is as follows

$$\ln L=\mathrm{n}\ln\alpha+\mathrm{n}\ln\beta+\mathrm{n}\ln\lambda-(\alpha+1)\sum_{i=1}^{n}\ln x_i-\beta\sum_{i=1}^{n}x_i^{-\alpha}-\sum_{i=1}^{n}\left(\frac{\lambda}{e^{\beta x_i^{-\alpha}}-1}\right)-2\sum_{i=1}^{n}\ln\left(1-e^{-\beta x_i^{-\alpha}}\right).$$

The partial derivatives of the log-likelihood function with respect to λ,α and β components of the score vector $U_L=(U_\lambda,U_\alpha,U_\beta)^T$ can be obtained as follows

$$U_\lambda=\frac{\partial\ln L}{\partial\lambda}=\frac{n}{\lambda}-\sum_{i=1}^{n}\left(\frac{1}{e^{\beta x_i^{-\alpha}}-1}\right),$$

$$U_\alpha=\frac{\partial\ln L}{\partial\alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n}\ln x_i+\beta\sum_{i=1}^{n}x_i^{-\alpha}\ln x_i-\sum_{i=1}^{n}\frac{\lambda\beta x_i^{-\alpha}\ln x_i e^{\beta x_i^{-\alpha}}}{\left(e^{\beta x_i^{-\alpha}}-1\right)^2}+2\sum_{i=1}^{n}\frac{\beta x_i^{-\alpha}\ln x_i e^{-\beta x_i^{-\alpha}}}{\left(1-e^{-\beta x_i^{-\alpha}}\right)},$$

$$U_\beta=\frac{\partial\ln L}{\partial\beta}=\frac{n}{\beta}-\sum_{i=1}^{n}x_i^{-\alpha}+\sum_{i=1}^{n}\frac{\lambda x_i^{-\alpha}e^{\beta x_i^{-\alpha}}}{\left(e^{\beta x_i^{-\alpha}}-1\right)^2}-2\sum_{i=1}^{n}\frac{x_i^{-\alpha}}{\left(e^{\beta x_i^{-\alpha}}-1\right)}.$$

Then the maximum likelihood estimates (MLEs) of the parameters, denoted by $\hat{\lambda}$, $\hat{\alpha}$ and $\hat{\beta}$ are obtained by setting U_{λ}, U_{α} and U_{β} to be zero and solving them numerically.

4.2. Least Squares Estimator

Suppose that $X_1, X_2, ..., X_n$ is a random sample of size n from the OGE-IW distribution and suppose $X_{1:n} < X_{2:n} < ... < X_{n:n}$ denotes the corresponding ordered sample. The LS estimators of the unknown parameters λ, α and β denoted by $\tilde{\lambda}, \tilde{\alpha}$ and $\tilde{\beta}$ of the OGE-IW distribution can be obtained by minimizing the sum of squares errors with respect to λ, α and β,

$$\sum_{i=1}^{n}\left[F_{i:n}(x) - \frac{i}{n+1}\right]^2 .$$

So the LS estimators $\tilde{\lambda}, \tilde{\alpha}$ and $\tilde{\beta}$ of the OGE-IW model can be obtained by minimizing the following quantity

$$\sum_{i=1}^{n}\left[\left[1 - \exp-\left(\frac{\lambda}{e^{\beta x_i^{-\alpha}} - 1}\right)\right] - \frac{i}{n+1}\right]^2 ,$$

with respect to λ, α and β respectively**.**

4.3. Percentiles Estimator

Let $X_1, X_2, ..., X_n$ be a random sample from the OGE-IW, let $X_{i:n}$ denotes the *ith* order statistic, i.e, $X_{1:n} < X_{2:n} < ... < X_{n:n}$. If p_i denotes some estimates of $F(x_{i:n}; \lambda, \alpha, \beta)$, then the estimator of unknown parameters, denoted by $\bar{\lambda}$, $\bar{\alpha}$ and $\bar{\beta}$, can be obtained by minimizing the following equation with respect to λ, α and β

$$\sum_{i=1}^{n}\left[\ln(p_i) - \ln\left[1 - \exp-\left(\frac{\lambda}{e^{\beta x_i^{-\alpha}} - 1}\right)\right]\right]^2 .$$

In percentiles method (PM) of estimate, p_i takes a several possible choice as estimates for $F(x_{i:n}; \lambda, \alpha, \beta)$, in this study, the formula $p_i = \frac{i}{n+1}$, is the expected value of the OGE-IW distribution and will be used.

5. Numerical Study

In this section, numerical study is performed to evaluate and compare the performance of the estimates with respect to their biases, and mean square errors (MSEs) for different sample sizes and for different parameter values. The numerical procedures are described through the following algorithm.

Step(1): A random sample $X_1,...,X_n$ of sizes n=(10,20,30,50,100) are selected, these random samples are generated from the OGE-IW distribution by using the following transformation

$x_i = \left[\frac{1}{\beta} ln \left[\frac{-\lambda}{ln(1-u_i)} + 1 \right] \right]^{\frac{-1}{\alpha}}$, $i = 1,2,...,n$ and u_i are random sample from uniform(0,1).

Step(2): Eight different set values of the parameters are selected as,

$set1 \equiv (\lambda=0.2, \alpha=0.5, \beta=0.1), set2 \equiv (\lambda=0.2, \alpha=0.5, \beta=0.3), set3 \equiv (\lambda=0.2, \alpha=0.5, \beta=0.5),$

$set4 \equiv (\lambda=0.2, \alpha=0.5, \beta=0.7), set5 \equiv (\lambda=0.2, \alpha=0.75, \beta=0.3), set6 \equiv (\lambda=0.2, \alpha=1, \beta=0.3),$

$set7 \equiv (\lambda=0.2, \alpha=1.25, \beta=0.3)$ and $set8 \equiv (\lambda=0.2, \alpha=1.5, \beta=0.3)$.

Step(3): For each model parameters and for each sample size, the MLEs, LS estimates and percentiles estimates (PEs) of λ,α and β are computed.

Step(4): Steps from 1 to 3 are repeated 1000 times for each sample size and for selected sets of parameters. Then, the biases and MSEs of the estimates of the unknown parameters are computed.

Numerical results are reported in Tables (3) to (6) and represented through some Figures from (3) to (6). From these tables, the following conclusions can be observed on the properties of estimated parameters from the OGE-IW distribution.

1- The biases of α in the percentiles method decrease as the value of β increases. Also, the biases of β increase as the value of β increases, for different set of parameters, in approximately all sets of parameters.

2- The biases and MSEs of MLEs, for β and α are smaller than the corresponding for λ.

3- For fixed values of λ, α and as the values of β increase, the biases and MSEs are decreasing, in approximately most of situations (see Table 4). As the values of α increase and for fixed values ofMSEs for all , the biases and β and λ estimates decrease in approximately, most sample sizes (see Table 5).

4- The biases and MSEs of ML estimates, for β and α are smaller than the corresponding for λ.

5- The MSEs of the MLEs, LS estimates and PEs decrease as the sample sizes increase for different selected set of parameters (see for example Figures 3 and 4).

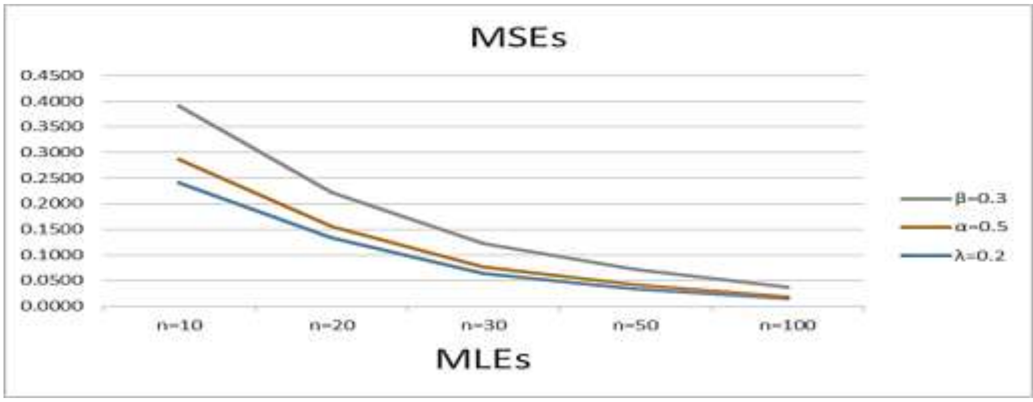

Figure 3: *MSE for MLE for the set 2*

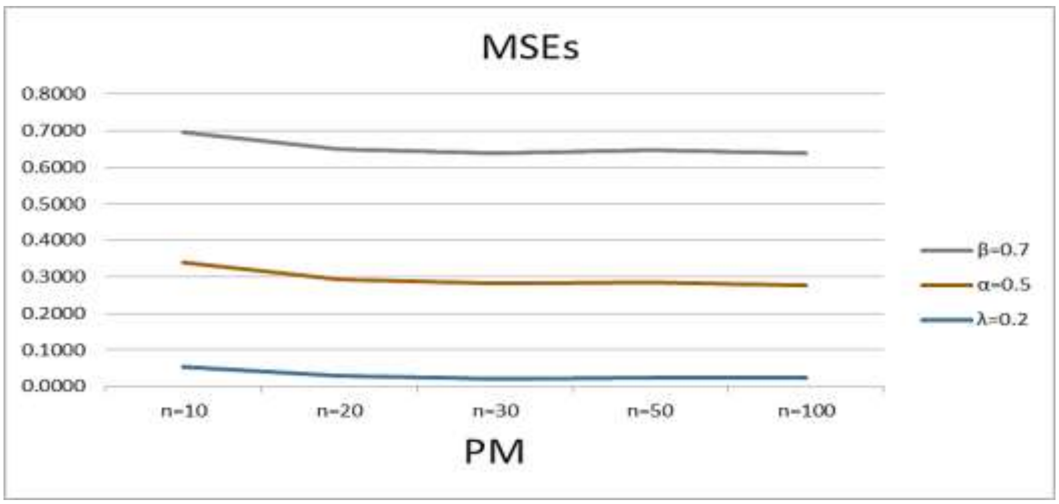

Figure 4: *MSE for PE for the set 4*

6- The MSEs for the LS estimates, $\tilde{\beta}$ and $\tilde{\lambda}$ take the smallest value among the corresponding MSEs for the other methods in almost all of the cases (see Tables (3) and (4)).

7- The biases of α in the PM decrease as the value of β increases. Also, the biases of β increase as the value of β increases, for different set of parameters, in approximately all sets of parameters.

8- As it seems from Figure (5)**,** the MSEs of the MLEs of α take the smallest values corresponding to the other estimates $\tilde{\alpha}$ and $\bar{\alpha}$ for the same sample size and for all set of parameters. Also, from Figure (5) the MSEs of MLEs of α for all set of parameters have the smallest values for the same sample size. Generally, the set 3 of parameters has the smallest MSEs corresponding to other set of parameters.

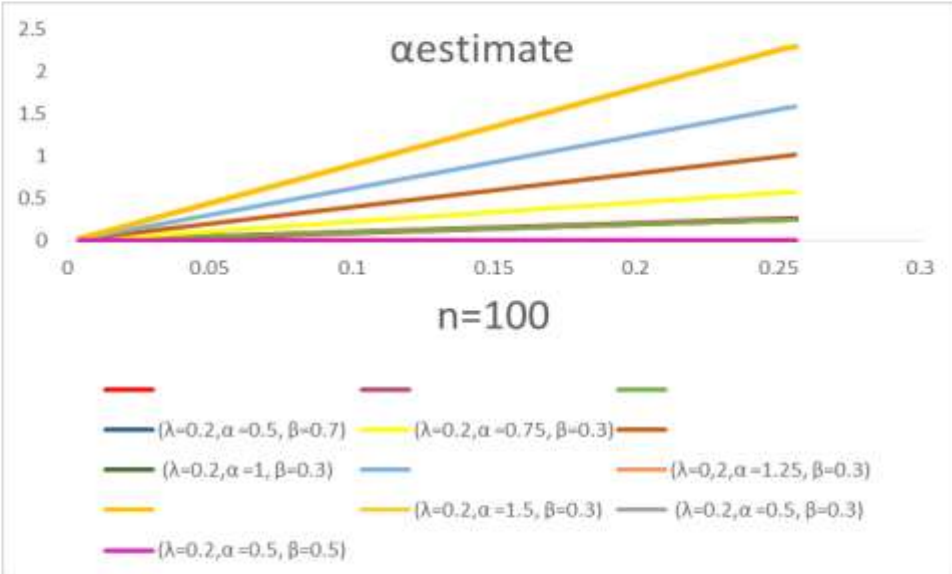

Figure 5: *MSEs of* $\hat{\alpha}$, $\tilde{\alpha}$*and* $\bar{\alpha}$ *for all set of parameters*

9- For fixed values of λ , α and as the values of β increase, the biases and MSEs are decreasing, in approximately most of situations (see Table 4). As the values of α increase and for fixed values ofMSEs for all , the biases and β and λ estimates decrease in approximately, most sample sizes (see Table 5).

10- As it seems from Figure (6), the MSEs of the MLEs of β take the smallest values corresponding to the other estimates $\tilde{\beta}$and $\bar{\beta}$ for the same sample size. Also the MSEs of β for all sets of parameters have the smallest values for the same sample size. The set 6 of parameters gives the smallest MSEs for different β estimates corresponding to other set of parameters.

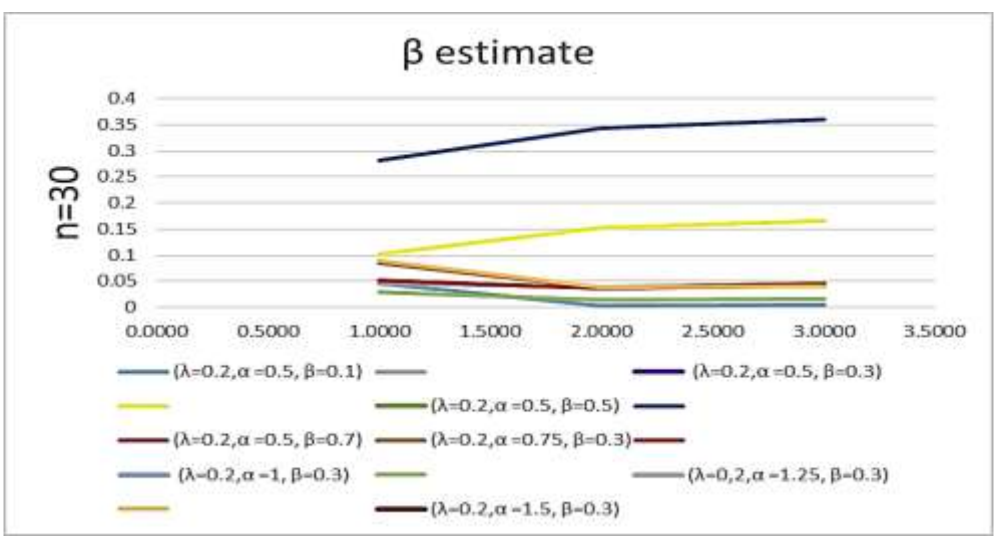

***Figure 6**: MSEs of* $\hat{\beta}$, $\tilde{\beta}$ *and* $\bar{\beta}$ *for all set of parameters*

Table 3: Biases and MSEs of estimates for set1and set2, for the Odds Generalized Exponential Inverse Weibull distribution

			Set1≡(λ=0.2,α =0.5, β=0.1)			Set2≡(λ=0.2,α =0.5, β=0.3)		
Sample Size	Method	Properties	λ	α	β	λ	α	β
10	ML	MSE	0.8860	0.0370	0.0770	0.2400	0.0460	0.1050
		Bias	0.4290	0.0610	0.1210	0.1120	0.0990	0.0350
	LS	MSE	0.0230	0.2850	0.0065	0.0160	0.2690	0.0390
		Bias	-0.1170	-0.5060	0.0070	-0.1140	-0.4950	-0.1850
	PM	MSE	0.0530	0.3050	0.0150	0.0310	0.2960	0.0470
		Bias	-0.1060	-0.5110	0.0150	-0.1250	-0.5030	-0.1880
20	ML	MSE	0.7060	0.0200	0.0630	0.1340	0.0210	0.0670
		Bias	0.3580	0.0190	0.1110	0.0530	0.0650	-0.0017
	LS	MSE	0.0170	0.2560	0.0041	0.0160	0.2560	0.0390
		Bias	-0.1200	-0.4970	0.0046	-0.1140	-0.4960	-0.1850
	PM	MSE	0.0290	0.2670	0.0082	0.0270	0.2650	0.0480
		Bias	-0.1330	-0.5000	0.0046	-0.1360	-0.4980	-0.1920
30	ML	MSE	0.5010	0.0140	0.0470	0.0640	0.0130	0.0460
		Bias	0.2660	0.0110	0.0850	0.0018	0.0540	-0.0380
	LS	MSE	0.0180	0.2530	0.0039	0.0150	0.2560	0.0380
		Bias	-0.1220	-0.4960	0.0023	-0.1130	-0.4960	-0.1840
	PM	MSE	0.0250	0.2610	0.0056	0.0240	0.2600	0.0460
		Bias	-0.1440	-0.4990	-0.0021	-0.1460	-0.4980	-0.2000
50	ML	MSE	0.4320	0.0086	0.0320	0.0340	0.0074	0.0310
		Bias	0.1850	0.0047	0.0600	-0.0300	0.0430	-0.0620
	LS	MSE	0.0180	0.2570	0.0042	0.0150	0.2560	0.0370
		Bias	-0.1210	-0.5030	0.0050	-0.1100	-0.5030	-0.1800
	PM	MSE	0.0240	0.2620	0.0042	0.0240	0.2620	0.0450
		Bias	-0.1490	-0.5060	-0.0045	-0.1500	-0.5050	-0.2030
100	ML	MSE	0.1420	0.0044	0.0160	0.0150	0.0036	0.0190
		Bias	0.1040	-0.0012	0.0370	-0.0540	0.0350	-0.0780
	LS	MSE	0.0170	0.2530	0.0043	0.0130	0.2530	0.0330
		Bias	-0.1200	-0.5020	0.0079	-0.1010	-0.5010	-0.1690
	PM	MSE	0.0240	0.2560	0.0039	0.0230	0.2560	0.0430
		Bias	-0.1500	-0.5020	-0.0015	-0.1490	-0.5020	-0.2000

Table 4: Biases and MSEs of estimates for set3and set4, for the Odds Generalized Exponential Inverse Weibull distribution

			Set3≡(λ=0.2,α =0.5, β=0.5)			Set4≡(λ=0.2,α =0.5, β=0.7)		
Sample Size	Method	Properties	λ	α	β	λ	α	β
10	ML	MSE	0.0650	0.0540	0.1300	0.0460	0.0560	0.2800
		Bias	-0.0780	0.1370	-0.2410	-0.1370	0.1480	-0.4850
	LS	MSE	0.0200	0.2710	0.1500	0.0170	0.2660	0.3420
		Bias	-0.1050	-0.4950	-0.3760	-0.1060	-0.4930	-0.5800
	PM	MSE	0.0750	0.2890	0.1600	0.0550	0.2840	0.3570
		Bias	-0.1180	-0.4990	-0.3870	-0.1110	-0.4950	-0.5830
20	ML	MSE	0.0360	0.0250	0.1080	0.0290	0.0260	0.2730
		Bias	-0.1000	0.0980	-0.2570	-0.1510	0.1090	-0.5010
	LS	MSE	0.0170	0.2550	0.1500	0.0170	0.2550	0.3420
		Bias	-0.1080	-0.4950	-0.3790	-0.1090	-0.4950	-0.5800
	PM	MSE	0.0250	0.2640	0.1630	0.0300	0.2620	0.3570
		Bias	-0.1380	-0.4960	-0.3940	-0.1290	-0.4950	-0.5850
30	ML	MSE	0.0260	0.0160	0.1020	0.0270	0.0170	0.2810
		Bias	-0.1110	0.0820	-0.2660	-0.1590	0.0940	-0.5200
	LS	MSE	0.0500	0.2520	0.1530	0.0220	0.2520	0.3430
		Bias	-0.1040	-0.4950	-0.3780	-0.1070	-0.4950	-0.5790
	PM	MSE	0.0230	0.2590	0.1650	0.0220	0.2590	0.3590
		Bias	-0.1450	-0.4970	-0.3990	-0.1420	-0.4970	-0.5950
50	ML	MSE	0.0200	0.0096	0.0940	0.0270	0.0110	0.2830
		Bias	-0.1200	0.0690	-0.2760	-0.1610	0.0800	-0.5220
	LS	MSE	0.0150	0.2560	0.1480	0.0160	0.2560	0.3400
		Bias	-0.1080	-0.5030	-0.3780	-0.1070	-0.5030	-0.5780
	PM	MSE	0.0230	0.2610	0.1660	0.0230	0.2610	0.3640
		Bias	-0.1500	-0.5050	-0.4040	-0.1470	-0.5040	-0.6000
100	ML	MSE	0.0190	0.0056	0.0920	0.0270	0.0069	0.2910
		Bias	-0.1250	0.0590	-0.2810	-0.1640	0.0700	-0.5350
	LS	MSE	0.0170	0.2530	0.1470	0.0150	0.2530	0.3370
		Bias	-0.1040	-0.5010	-0.3750	-0.1050	-0.5010	-0.5750
	PM	MSE	0.0230	0.2550	0.1650	0.0230	0.2550	0.3620
		Bias	-0.1500	-0.5020	-0.4020	-0.1480	-0.5020	-0.5990

Table 5: Biases and MSEs of estimates for set5and set6, for the Odds Generalized Exponential Inverse Weibull distribution

			Set5≡(λ=0.2,α =0.75, β=0.3)			Set6≡(λ=0.2,α =1, β=0.3)		
Sample Size	Method	Properties	λ	α	β	λ	α	β
10	ML	MSE	0.3540	0.1070	0.1370	0.3190	0.1860	0.1040
		Bias	0.1810	0.1390	0.0740	0.1050	0.2060	0.0110
	LS	MSE	0.0150	0.6060	0.0370	0.0150	1.0770	0.0370
		Bias	-0.1100	-0.7430	-0.1800	-0.1090	-0.9900	-0.1780
	PM	MSE	0.0390	0.6570	0.0480	0.0320	1.1690	0.0450
		Bias	-0.1130	-0.7530	-0.1750	-0.1140	-1.0020	-0.1720
20	ML	MSE	0.2520	0.0480	0.1110	0.1710	0.0830	0.0670
		Bias	0.1370	0.0750	0.0570	0.0420	0.1300	-0.0230
	LS	MSE	0.0150	0.5780	0.0380	0.0150	1.0270	0.0380
		Bias	-0.1120	-0.7450	-0.1830	-0.1110	-0.9930	-0.1820
	PM	MSE	0.0260	0.2700	0.0470	0.0290	1.0640	0.0460
		Bias	-0.1340	0.6010	0.0470	-0.1330	-0.9960	-0.1890
30	ML	MSE	0.1690	0.0300	0.0860	0.0890	0.0500	0.0510
		Bias	0.0990	0.0520	0.0400	0.0073	0.0990	-0.0370
	LS	MSE	0.0150	0.5790	0.0370	0.0150	1.0290	0.0370
		Bias	-0.1090	-0.7520	-0.1790	-0.1090	-1.0030	-0.1780
	PM	MSE	0.0270	0.5970	0.0460	0.0230	1.0620	0.0430
		Bias	-0.1390	-0.7550	-0.1920	-0.1380	-1.0070	-0.1870
50	ML	MSE	0.1020	0.0200	0.0590	0.0330	0.0310	0.0280
		Bias	0.0470	0.0430	0.0031	-0.0410	0.0880	-0.0720
	LS	MSE	0.0150	0.5770	0.0380	0.0150	1.0260	0.0370
		Bias	-0.1130	-0.7550	-0.1840	-0.1110	-1.0060	-0.1810
	PM	MSE	0.0230	0.5870	0.0430	0.0220	1.0440	0.0420
		Bias	-0.1480	-0.7570	-0.2000	-0.1450	-1.0090	-0.1950
100	ML	MSE	0.0400	0.0091	0.0320	0.0150	0.0150	0.0180
		Bias	0.0052	0.0320	-0.0200	0.0410	0.0700	-0.0820
	LS	MSE	0.0150	0.5680	0.0370	0.0140	1.0100	0.0360
		Bias	-0.1110	-0.7510	-0.1810	-0.0820	-0.0820	-0.1770
	PM	MSE	0.0230	0.5720	0.0430	0.0220	1.0170	0.0410
		Bias	-0.1500	-0.7510	-0.2000	-0.1450	-1.0020	-0.1920

Table 6: Biases and MSEs of estimates for set7and set8, for the Odds Generalized Exponential Inverse Weibull distribution

			Set7≡(λ=0.2,α =1.25, β=0.3)			Set8≡(λ=0.2,α =1.5, β=0.3)		
Sample Size	Method	Properties	λ	α	β	λ	α	β
10	ML	MSE	0.1240	0.2790	0.2790	0.5260	0.4040	0.1700
		Bias	-0.0089	0.2890	-0.0500	0.2100	0.2730	0.0930
	LS	MSE	0.9730	1.4260	1.4260	0.0160	2.4260	0.0380
		Bias	-0.0500	-1.1360	-0.1360	-0.1100	-1.4860	-0.1800
	PM	MSE	1.4260	1.3760	0.0230	0.0370	2.5950	0.0460
		Bias	0.5750	-1.0910	-0.1350	-0.1080	-1.5000	-0.1650
20	ML	MSE	0.0250	0.0250	0.0290	0.3260	0.1870	0.1150
		Bias	-0.0550	0.1910	-0.0790	0.1280	0.1640	0.0420
	LS	MSE	0.3690	1.5120	0.0170	0.0160	2.3140	0.0390
		Bias	0.1010	-1.2010	-0.1230	-0.1130	-1.4910	-0.1840
	PM	MSE	0.7390	1.4770	0.0200	0.0290	2.3920	0.0460
		Bias	0.2380	-1.1670	-0.1230	-0.1270	-1.4940	-0.1810
30	ML	MSE	0.0330	0.0730	0.0300	0.2020	0.1160	0.0900
		Bias	-0.0490	0.1440	-0.0740	0.0790	0.1230	0.0200
	LS	MSE	0.1360	1.5800	0.0150	0.0160	2.3180	0.0390
		Bias	-0.0001	-1.2410	-0.1180	-0.1130	-1.5050	-0.1840
	PM	MSE	0.3430	1.5750	0.0160	0.0240	2.3850	0.0410
		Bias	0.0530	-1.2230	-0.1200	-0.1340	-1.5090	-0.1820
50	ML	MSE	0.0180	0.0460	0.0210	0.0720	0.0710	0.0410
		Bias	-0.0620	0.1190	-0.0870	0.0000	0.1120	-0.0360
	LS	MSE	0.0140	1.5970	0.0140	0.0160	2.3090	0.0400
		Bias	-0.0550	-1.2560	-0.1160	-0.1150	-1.5100	-0.1870
	PM	MSE	0.0800	1.6100	0.0150	0.0220	2.3490	0.0410
		Bias	-0.0720	-1.2530	-0.1180	-0.1420	-1.5130	-0.1900
100	ML	MSE	0.0078	0.0210	0.0120	0.0280	0.0310	0.0210
		Bias	-0.0650	0.0890	-0.0860	-0.0320	0.0860	-0.0540
	LS	MSE	0.0035	1.5760	0.0130	0.0170	2.2740	0.0420
		Bias	-0.0580	-1.2520	-0.1130	-0.1200	-1.5030	-0.1930
	PM	MSE	0.0110	1.5890	0.0140	0.0220	2.2890	0.0400
		Bias	-0.1060	-1.2530	-0.1200	-0.1430	-1.5030	-0.1890

6. Data Analysis

In this section, we provide a data analysis in order to assess the goodness-of-fit of the OGE-IW model comparing with some known distributions such as the exponential (E) generalized exponential (GE) generalized inverse Weibull (GIW), Kumaraswamy inverse Weibull (KIW), Marshpall–Olkin extended inverse Weibull (MOEIW) and IW. The data set refers to Lee and Wang (2003) which represent remission times (in months) of a random sample of 128 bladder cancer patients. The data are as follows:

0.08 2.09 3.48 4.87 6.94 8.66 13.11 23.63 0.2 2.23 0.52 4.98 6.97 9.02 13.29 0.4 2.26 3.57 5.06 7.09 0.22 13.8 25.74 0.5 2.46 3.46 5.09 7.26 9.47 14.24 0.82 0.51 2.54 3.7 5.17 7.28 9.74 14.76 26.31 0.81 0.62 3.28 5.32 7.32 10.06 14.77 32.15 2.64 3.88 5.32 0.39 10.34 14.38 34.26 0.9 2.69 4.18 5.34 7.59 10.66 0.96 36.66 1.05 2.69 4.23 5.41 7.62 10.75 16.62 43.01 0.19 2.75 4.26 5.41 7.63 17.12 46.12 1.26 2.83 4.33 0.66 11.25 17.14 79.05 1.35 2.87 5.62 7.87 11.64 17.36 0.4 3.02 4.34 5.71 7.93 11.79 18.1 1.46 4.4 5.85 0.26 11.98 19.13 1.76 3.25 4.5 6.25 8.37 12.02 2.02 0.31 4.51 6.54 8.53 12.03 20.28 2.02 3.36 6.76 12.07 0.73 2.07 3.36 6.39 8.65 12.63 22.69 5.49 .

Measures of fit statistic using the maximized log-likelihood $\left(-2\log L\right)$, Akaike information criterion $\left(AIC\right)$, the corrected Akaike information criterion $\left(CAIC\right)$, and Hannan-Quinn information criterion $\left(HQIC\right)$, are provided in Table 7. The model with minimum values for $-2\log L$ or AIC or BIC or $CAIC$ or $HQIC$ can be chosen as the best model to fit the data. The ML estimates and their standard errors (SE) for OGE-IW, GE, E, GIW, KIW, MOEIW and IW models are given in Table 8.

Table 7: The statistics $-2\log L$, AIC, $CAIC$, BIC, and $HQIC$, for the 128 bladder cancer patients data

Distribution	-2logL	AIC	BIC	CAIC	HQIC
OGE-IW	801.263	807.263	807.585	807.457	810.740
GE	805.022	809.022	809.236	809.118	811.339
E	827.296	829.296	829.403	829.328	830.455
GIW	874.450	880.450	863.673	880.644	883.926
KIW	971.574	979.574	980.003	979.899	984.209
MOEIW	810.707	816.707	817.029	816.901	820.183
IW	857.352	861.352	861.566	861.448	863.669

Table 8: ML estimates of the model parameters and the corresponding SEs for the 128 bladder cancer patient's data

Distribution	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{a}$	$\hat{b}$	$\hat{\theta}$
OGE-IW	0.057 (0.05)	0.87 (0.076)	0.336 (0.244)	- -	- -	- -
GE	0.111 (0.013)	0.922 (0.107)	- -	- -	- -	- -
E	0.075 (0.008)	- -	- -	- -	- -	- -
GIW	0.75 (0.25)	0.53 (0.038)	1.797 (0.324)	- -	- -	- -
KIW	- -	3.796 (0.238)	2.239 (2.846)	0.0077 (0.006)	0.093 (0.012)	- -
MOEIW	- -	0.047 (0.049)	1.39 (0.104)	- -	- -	198.304 (221.399)
IW	16.142 (0.125)	0.464 (0.042)	- -	- -	- -	- -

The results show that the OGE-IW distribution provides a significantly better fit than the other models.

7. Conclusion

In this article, we propose a new model, called the odds generalized exponential-inverse Weibull distribution based on T-X family presented by Alzaatreh *et al*. (2013). Some statistical properties of current distribution are derived and discussed. The estimation of the model parameters is approached by maximum likelihood, least squares and percentiles methods. Simulation study is carried out to compare the performance of different estimates. Simulation study revealed that the PEs perform well than the MLEs and LS estimates, in approximately, most of situations. An application to a real data set indicates that the new model is superior to the fits than the other well-known distributions.

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