Efficiency of Bayes Estimator for Rayleigh Distribution

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Abstract

Comparisons of estimates between Bayes and frequenters methods are interesting and challenging topics in statistics. In this paper, Bayes and maximum likelihood estimates are discussed for Rayleigh distribution based on complete and type-II censored sampling. The prior knowledge which is adequately represented by the natural conjugate distribution and the Jeffreys non-informative prior distribution of the parameter λ for Rayleigh distribution are used. Moreover, the Bayes risk functions under squared error loss function and maximum likelihood risk functions for complete and type-II censored sampling schemes are compared. Finally, numerical study is given to illustrate the results.

Keywords: Type -II censored sampling; Bayesian approach; Maximum likelihood procedure; Risk function.

1. Introduction

The essential element in lifetime data analysis is the presence of a nonnegative response, *X*, with appreciable dispersion and often with censoring. Due to sampling methods or the occurrence of some competing risk of removal from the study, several lifetimes of individuals may be censored. By censored data we mean that, in a potential sample of size *n*, a known number of observations is missing at either end (single censoring) or at both ends (double censoring). The type of censoring just described is often called Type II censoring. Single type-II censored data has been considered, among other authors, Epstein and Sobel (1953) presented the ratio of the expected experiment times of type-II censoring scheme and complete sampling when the lifetimes are exponential distributed. AL-Hussaini and Jaheen (1992, 1994) considered estimation of the parameters, reliability, and failure rate functions of the model based on Type-II censored samples from a Bayesian approach. Ali Mousa (1995) considered empirical Bayes estimation for one of the two shape parameters and the reliability function of the Burr type XII distribution based on Type-II censored data obtained from an accelerated life test. Tse and Tso (1996) studied the effects of type-II censored sampling schemes on the estimation of the unknown parameter assuming that the data being to exponential distribution.

The Rayleigh distribution is a suitable model for life testing studies. Polovko (1968), Dyer and Whisenand (1973), demonstrated the importance of this distribution in electro vacuumdevices and communication engineering. Ariyawansa and Templeton (1984) have also discussed some of its applications. Howlader and Hossian (1995) obtained Bayes estimators for the scale parameter and the reliability function (R (t)) in the case of type-II censored sampling. Lalitha and Anand (1996) used the modified maximum likelihood to estimate the scale parameter of the Rayleigh distribution. Mazloum (1997) concerned with the problem of estimating the scale parameter and the reliability under type-II censoring. Meintanis and Iliopoulos (2003) proposed a class of goodness of fit tests for the Rayleigh distribution. Abd Elfattah *et al* (2006) studied the efficiency of maximum likelihood estimate of the parameter of Rayleigh distribution under three cases, type-I, type-II and progressive type-II censored sampling schemes.

In this paper the risk functions under maximum likelihood estimation for complete and type-II censored samplings are obtained in Section (2). Also, the risk functions under Bayes estimation for complete and type-II censored samplings are obtained in Section (3). In Section (4), the comparisons between maximum likelihood and Bayes estimators based on the risk functions under complete and type-II censored samplings are performed. Furthermore, numerical computations are performed to illustrate these comparisons using Mathcad (2001). Tables and some graphs for these numerical results are displayed at the appendix.

2. Risk Function under Maximum Likelihood Estimation

Consider a life testing experiment in which n units put on test and successive failure times are recorded. Assume that the lifetimes are independent and identically distributed Rayleigh random variables with probability density function

$$f(x,\lambda)=\frac{2x}{\lambda}e^{\frac{-x^2}{\lambda}}, \qquad x>0, \lambda>0 \tag{1}$$

and the corresponding cumulative distribution function

$$F(x,\lambda)=1-e^{\frac{-x^2}{\lambda}}.$$

Suppose that $X_1, X_2, ..., X_n$ are the n complete observed in an experiment, then the likelihood function is

$$L(\lambda)=\left(\frac{2}{\lambda}\right)^n \prod_{i=1}^{n} x_i e^{\frac{-x_i^2}{\lambda}}. \tag{2}$$

And the natural logarithm of the likelihood function is

$$\ln L(\lambda)=n\ln 2-n\ln\lambda+\sum_{i=1}^{n}\ln x_i-\frac{\sum_{i=1}^{n}x_i^2}{\lambda}.$$

The MLE of the parameter λ is

$$\hat{\lambda}=\frac{\sum_{i=1}^{n}x_i^2}{n}.$$

The risk function of $\hat{\lambda}$ is given by

$$R\left(\hat{\lambda}\right) = \text{var}\left(\hat{\lambda}\right) + \left(bias\right)^2$$

$$= \text{var}\left(\hat{\lambda}\right) + \left[E\left(\hat{\lambda}\right) - \lambda\right]^2 = \frac{\lambda^2}{n}. \qquad (3)$$

where $E\left(\hat{\lambda}\right) = \lambda$, and $\text{var}\left(\hat{\lambda}\right) = \frac{\lambda^2}{n}$

Now, suppose that only the first k ordered observations $X_{(1)} < X_{(2)} < ... < X_{(k)}$ in a random sample of size n from one-parameter Rayleigh distribution are available, where k is fixed before the experiment is conducted. The likelihood function of the sample in this case is given by

$$L\left(\lambda\right) = \frac{n!}{(n-k)!} \prod_{i=1}^{k} \frac{2x_{(i)}}{\lambda} e^{\frac{-x_{(i)}^2}{\lambda}} \left[e^{\frac{-x_{(k)}^2}{\lambda}}\right]^{(n-k)} .. \qquad (4)$$

The MLE of the parameter λ under type-II censoring can be shown to be of the form

$$\hat{\lambda}_{II} = \frac{1}{k}\left[\sum_{i=1}^{k} x_{(i)}^2 + (n-k)x_{(k)}^2\right].$$

To obtain the expected value and mean squared error of $\hat{\lambda}_{II}$, let us now perform the transformation, $U_{(i)} = X_{(i)}^2, \quad i = 1, 2, ..., k$. This random sample is distributed as exponential distribution from type-II censoring sampling. Also, consider the following transformation:

Let, $W_1 = nU_{(1)}$ and $W_i = (n-i+1)\left(U_{(i)} - U_{(i-1)}\right), \qquad i = 2, 3, ..., k$.

Hence, $\sum_{i=1}^{k} U_{(i)} + (n-k)U_{(k)} = \sum_{i=1}^{k} W_i$.

It can be shown that $W_1, W_2, ..., W_k$ are independent exponential random variables with parameter λ. Then, the expected value of $\hat{\lambda}_{II}$ is $E\left(\hat{\lambda}_{II}\right) = \lambda$.

Thus $\hat{\lambda}_{II}$ is unbiased estimator for λ, and the variance of $\hat{\lambda}_{II}$ is

$$\text{var}\left(\hat{\lambda}_{II}\right) = \text{var}\left[\frac{1}{k}\sum_{i=1}^{k} W_i\right] = \frac{\lambda^2}{k}.$$

So the mean square error of the estimated parameter under type-II censoring, $\hat{\lambda}_{II}$, is given by

$$R\left(\hat{\lambda}_{II}\right) = \frac{\lambda^2}{k}. \qquad (5)$$

3. Risk Function under Bayes Estimation

In this Section, the risk functions of the unknown parameter λ are obtained for complete and type-II censored sampling in two cases, conjugate prior and Non-Informative Prior distribution.

Case (a): Conjugate Prior Distribution for λ

Under the assumption that the parameter λ is unknown, we can use the conjugate inverted gamma prior with probability density function

$$p(\lambda)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}\lambda^{-(\alpha+1)}e^{-\beta/\lambda}, \qquad \lambda>0 \tag{6}$$

where, to be a proper density, we must have $\alpha>0$ and $\beta>0$. This prior distribution has advantages over many other distributions because of its analytical tractability, richness, and easy interpretability. The posterior density function of λ given the data, denoted by $\pi(\lambda|\underline{x})$, can be obtained by combining the likelihood function under complete sampling which given by equation (2) with the prior density,

$$\pi(\lambda|\underline{x})=\frac{(\beta+d)^{n+\alpha}}{\Gamma(n+\alpha)}\lambda^{-(n+\alpha+1)}e^{-(\beta+d)/\lambda}, \qquad \lambda>0,$$

where $d=\sum_{i=1}^{n}x_i^2$ and $\Gamma(.)$ is the gamma function. It follows that λ has an inverted gamma distribution with parameters $(n+\alpha,\beta+d)$, which can be symbolic by $InGa(n+\alpha,\beta+d)$.

From a decision-theoretic view point, in order to select a single value as representing our estimator of λ, one must first specify a loss function, $L(\lambda,\tilde{\lambda})$. This loss function represents the cost involved in using the estimate $\tilde{\lambda}$ when the true value is λ. A commonly used loss function for estimating λ is the squared error loss, $L(\lambda,\tilde{\lambda})=(\tilde{\lambda}-\lambda)^2$. Under this loss function, the Bayes estimator of λ, denoted by $\tilde{\lambda}$, is the mean of the posterior density function. The posterior mean for inverted gamma distribution with parameters $(n+\alpha,\beta+d)$ has the form

$$\tilde{\lambda}=E(\lambda|\underline{x})=\frac{\beta+d}{n+\alpha-1}.$$

The risk function of Bayes estimator $\tilde{\lambda}$ is given by

$$R(\tilde{\lambda})=\mathrm{var}(\tilde{\lambda})+\left[E(\tilde{\lambda})-\lambda\right]^2.$$

$$=\frac{\beta^2+\lambda^2\left(n+(\alpha-1)^2\right)-2\beta\lambda(\alpha-1)}{[n+\alpha-1]^2} \tag{7}$$

Under type-II censored sampling, Combining the likelihood function which given by equation (4) and the conjugate prior distribution of λ, the posterior density function of λ, given $\underline{x}$, is then

$$\pi(\lambda|\underline{x})=\frac{(\beta+s^2)^{k+\alpha}}{\Gamma(k+\alpha)}\lambda^{-(k+\alpha+1)}e^{-(\beta+s^2)/\lambda}$$

So the posterior density function of λ has the inverted gamma distribution with parameters $(k+\alpha, \beta+s^2)$ and $s^2 = \left[\sum_{i=1}^{k} x_{(i)}^2 + (n-k)x_{(k)}^2\right]$. The Bayes estimator ($\tilde{\lambda}_{II}$) for λ under type-II censored sampling which is the posterior mean for In Ga $\left(k+\alpha, \beta+s^2\right)$ is given by

$$\tilde{\lambda}_{II} = E\left(\lambda|\underline{x}\right) = \frac{\beta+s^2}{k+\alpha-1}.$$

Therefore, the risk function of $\tilde{\lambda}_{II}$ is given by

$$R\left(\tilde{\lambda}_{II}\right) = \frac{\beta^2 + \lambda^2\left(k+(\alpha-1)^2\right) - 2\beta\lambda(\alpha-1)}{(k+\alpha-1)^2} \tag{8}$$

Case (b): Non-Informative Prior for λ

If prior information about λ is scanty, it may be appropriate to resort to the use of a diffuse prior distribution. Improper prior density for λ, which can reasonably be accepted, is the Jeffreys' (1961) prior distribution. The non-informative prior distribution of the parameter λ under complete sampling is given by

$$g(\lambda) \propto \sqrt{I(\lambda)}.$$

Therefore,

$$g(\lambda) \propto \frac{1}{\lambda}, \qquad \lambda > 0$$

where, $I(\lambda) = \frac{n}{\lambda^2}$ is the Fisher information for complete sampling. Using the likelihood function and the prior distribution of λ then the posterior distribution of λ is

$$\pi\left(\lambda|\underline{x}\right) = \frac{d^n}{\Gamma(n)}\, \lambda^{-(n+1)}\, e^{-d/\lambda}$$

we can noted that the posterior distribution of λ for complete sampling is inverted gamma distribution with parameters (n, d). Assuming a squared error loss function, the Bayes estimate of a parameter λ under complete sampling, λ', is given by

$$\lambda'' = E\left(\lambda|\underline{x}\right) = \frac{d}{n-1}.$$

So, the risk function of the estimator λ'' is given by

$$R(\lambda'') = \frac{n\lambda^2}{(n-1)^2} + \left(\frac{\lambda}{(n-1)}\right)^2 = \frac{\lambda^2(n+1)}{(n-1)^2}. \tag{9}$$

While in the second case, the Jeffreys non-informative prior density function of the parameter λ under type-II censored sampling is given by

$$g(\lambda) \propto \sqrt{I_{II}(\lambda)}.$$

Therefore,

$$g(\lambda) \propto \frac{1}{\lambda}, \qquad \lambda > 0$$

where, $I_{II}(\lambda) = \frac{k}{\lambda^2}$ is the Fisher information for type-II censored sampling. The posterior density function of λ given $\underline{x}$ is obtained by combining the likelihood function under type-II censored sampling and the prior distribution of λ as follow

$$\pi(\lambda|\underline{x}) = \frac{(s^2)^k}{\Gamma(k)} \lambda^{-(k+1)} e^{-s^2/\lambda}.$$

Therefore λ given the data is distributed as inverted gamma distribution with parameters (k, s^2). Under the assumption that the loss function is the squared error Bayes estimator is the mean of the inverted gamma distribution with parameters (k, s^2) and has the form

$$\lambda''_{II} = E(\lambda|\underline{x}) = \frac{s^2}{k-1}.$$

The risk function of Bayes estimator under a squared error loss function based on type-II censored sampling is

$$R(\lambda''_{II}) = E(\lambda''_{II} - \lambda)^2 = \mathrm{var}(\lambda''_{II}) + \left[E(\lambda''_{II}) - \lambda\right]^2 = \frac{\lambda^2 (k+1)}{(k-1)^2}. \qquad (10)$$

4. Efficiency of Estimators under Type-II Censored Sampling

This **Section** deals with the comparison between complete and type-II censored sampling. This comparison is performed theoretically and numerically based on the risk functions. This problem will be discussed in the two cases, i.e., conjugate and non-informative prior for λ. Moreover, the comparison between the risk functions for the ML and Bayes estimators under type-II censored sampling are made.

It is obvious from the theoretical results that appear in equations (3), (5), (7), (8), (9) and (10) that the risk functions for complete sampling are smaller than the risk functions for type-II censored sampling in the two cases. Furthermore, it should be mentioned here that, when α and β tend to be zero, the risk function in the case of the conjugate prior of λ tend to be the risk function in the case of the non-informative prior of λ. Furthermore, it should be mentioned here that, when α and β tend to be zero, the risk function in the case of the conjugate prior of λ tend to be the risk function in the case of the non-informative prior of λ. It is clear that the analytically comparison between ML and Bayes estimators under type-II censored sampling is difficult. An alternative is to calculate the risk functions for them numerically for various n, k, α and β using Mathcad (2001). The procedures are performed as:

Step (1): Five sets of generated random samples of sizes $n = 10, 15, 20, 25$ and 30 from the Rayleigh distribution are used to obtain the risk functions.

Step (2): For each n the ratio of the risk function of maximum likelihood estimator under type-II censored sampling to the risk function under complete sampling (R_{ML}) is computed for all k such that $n/2 \le k \le n$.

Step (3): The ratio of the risk function of Bayes estimator under type-II censored sampling to the risk function under complete sampling $\left(R_B\right)$ is obtained.

Step (4): Steps from (1) to (3) are repeated 1000 times and the risk function is computed by averaging the risk functions over the 1000 repetitions [see program (4) at appendix (c)].

Table (1) shows the numerical values of the ratios of the risk function for type-II censored sampling to the risk function for complete sampling based on ML and Bayes estimators for different values of α and β. It is clear from the numerical results that:

(i) For ML and Bayes estimators the expected loss under a squared error of λ for type-II censored sampling with k failures is greater than the expected loss under a squared error of λ for complete sampling.

(ii) For large values of α and β together, the risk function under a squared error loss for Bayes estimator is smaller than the risk function for MLE.

(iii) For fixed n and k, when α large and β small, the risk function under a square error loss for Bayes estimator is more than the risk function for MLE except the small sample.

(iv) For fixed n and k, when α small and β large, the risk function under a square error loss for Bayes estimator is more than the risk function for MLE

(v) For moderate values of α and β together, the risk function under a squared error loss for Bayes estimator is smaller than the risk function for MLE.

(vi) For small values of α and β together, the risk function under a squared error loss for Bayes estimator is greater than the risk function for MLE except when $\alpha=1$ and $\beta=0$, the risk function under a squared error loss for Bayes estimator is equal the risk function for MLE.

(vii) For ML and Bayes estimators when k increase the ratio of risk function under type-II censored sampling to the risk functions under complete sampling decrease for fixed α, β and n, we take n = 10 as special case (see graph (1)).

(viii) The ratio of the risk function for Bayes estimator under type-II censored sampling to complete sampling has no trend with respect to α or β (we take n = 10 and k = 5 as special case). Graph (2) and (3) appear this notice.

Therefore, the results of this comparison indicate that Bayes estimator for some cases is more efficient than MLE, for some appropriate choice of α and β.

Appendix

Table (1)

Ratio of Risk Function (R) under Type-II Censored Sampling for the ML and Bayes Estimators of λ for Different Values of n and k to Complete Sampling

n	k	R_{ML}	R_B				
			$\alpha = 0, \beta = 0$	$\alpha = 0, \beta = 1$	$\alpha = 1, \beta = 0$	$\alpha = 1, \beta = 1$	$\alpha = 2, \beta = 3$
10	5	2.000	2.761	2.991	2.000	2.051	1.728
	6	1.667	2.062	2.179	1.667	1.695	1.510
	7	1.429	1.636	1.698	1.429	1.444	1.340
	8	1.250	1.353	1.382	1.250	1.258	1.204
	9	1.111	1.151	1.162	1.111	1.114	1.092
	10	1.000	1.000	1.000	1.000	1.000	1.000
15	7	2.143	2.722	2.919	2.143	2.185	1.904
	8	1.875	2.250	2.376	1.875	1.903	1.712
	9	1.667	1.914	1.997	1.667	1.686	1.554
	10	1.500	1.664	1.718	1.500	1.513	1.423
	11	1.364	1.470	1.505	1.364	1.372	1.312
	12	1.250	1.316	1.338	1.250	1.255	1.217
	13	1.154	1.191	1.203	1.154	1.157	1.135
	14	1.071	1.087	1.093	1.071	1.073	1.063
	15	1.000	1.000	1.000	1.000	1.000	1.000
20	10	2.000	2.335	2.456	2.000	2.027	1.857
	11	1.818	2.063	2.151	1.818	1.838	1.711
	12	1.667	1.847	1.912	1.667	1.681	1.586
	13	1.538	1.671	1.719	1.538	1.549	1.477
	14	1.429	1.526	1.561	1.429	1.437	1.383
	15	1.333	1.403	1.428	1.333	1.339	1.300
	16	1.250	1.299	1.316	1.250	1.254	1.227
	17	1.176	1.209	1.220	1.176	1.179	1.161
	18	1.111	1.130	1.137	1.111	1.113	1.102
	19	1.053	1.061	1.064	1.053	1.053	1.048
	20	1.000	1.000	1.000	1.000	1.000	1.000
25	12	2.083	2.380	2.490	2.083	2.107	1.948
	13	1.923	2.154	2.239	1.923	1.941	1.816
	14	1.786	1.966	2.033	1.786	1.800	1.700
	15	1.667	1.808	1.861	1.667	1.678	1.599
	16	1.563	1.674	1.715	1.563	1.572	1.508
	17	1.471	1.558	1.590	1.471	1.478	1.428
	18	1.389	1.456	1.481	1.389	1.394	1.355
	19	1.316	1.368	1.386	1.316	1.320	1.290
	20	1.250	1.289	1.303	1.250	1.253	1.230
	21	1.190	1.218	1.229	1.190	1.193	1.176
	22	1.136	1.155	1.162	1.136	1.138	1.127
	23	1.087	1.099	1.103	1.087	1.088	1.081
	24	1.042	1.047	1.049	1.042	1.042	1.039
	25	1.000	1.000	1.000	1.000	1.000	1.000
30	15	2.000	2.215	2.296	2.000	2.017	1.893
	16	1.875	2.050	2.116	1.875	1.889	1.787
	17	1.765	1.908	1.961	1.765	1.776	1.692
	18	1.667	1.784	1.828	1.667	1.676	1.606
	19	1.579	1.675	1.711	1.579	1.587	1.529
	20	1.500	1.578	1.608	1.500	1.506	1.459
	21	1.429	1.492	1.516	1.429	1.434	1.395
	22	1.364	1.415	1.434	1.364	1.368	1.336
	23	1.304	1.345	1.361	1.304	1.308	1.282
	24	1.250	1.282	1.294	1.250	1.253	1.233
	25	1.200	1.225	1.234	1.200	1.202	1.187
	26	1.154	1.172	1.179	1.154	1.155	1.144
	27	1.111	1.124	1.128	1.111	1.112	1.104
	28	1.071	1.079	1.082	1.071	1.072	1.067
	29	1.034	1.038	1.039	1.034	1.035	1.032
	30	1.000	1.000	1.000	1.000	1.000	1.000

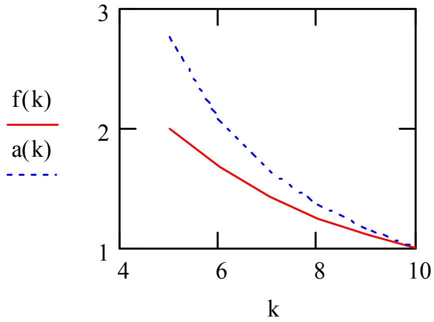
Continued Table (1)

Ratio of Risk Function (R) under Type-II Censored Sampling for the ML and Bayes Estimators of λ for Different Values of n and k to Complete Sampling

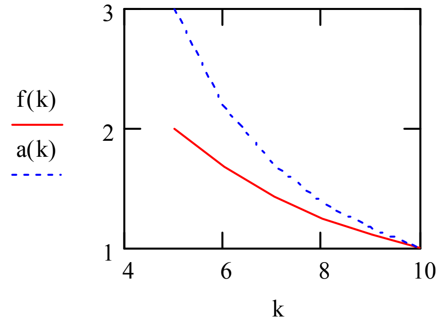
n	k	R_{ML}	R_B				
			α =3, β = 2	α = 5, β = 7	α = 7, β = 5	α = 1, β = 10	α = 10, β = 1
10	5	2.000	1.621	1.364	1.637	3.360	1.730
	6	1.667	1.443	1.276	1.456	2.422	1.527
	7	1.429	1.299	1.196	1.309	1.845	1.359
	8	1.250	1.182	1.124	1.108	1.462	1.219
	9	1.111	1.083	1.059	1.086	1.195	1.101
	10	1.000	1.000	1.000	1.000	1.000	1.000
15	7	2.143	1.715	1.990	1.638	3.613	2.044
	8	1.875	1.577	1.776	1.517	2.860	1.833
	9	1.667	1.458	1.602	1.413	2.334	1.656
	10	1.500	1.356	1.458	1.322	1.950	1.504
	11	1.364	1.266	1.337	1.242	1.661	1.374
	12	1.250	1.187	1.234	1.171	1.438	1.261
	13	1.154	1.118	1.145	1.108	1.260	1.163
	14	1.071	1.056	1.068	1.051	1.117	1.076
	15	1.000	1.000	1.000	1.000	1.000	1.000
20	10	2.000	1.763	1.532	1.813	3.090	2.077
	11	1.818	1.638	1.457	1.680	2.629	1.897
	12	1.667	1.530	1.388	1.564	2.272	1.741
	13	1.538	1.435	1.325	1.462	1.990	1.606
	14	1.429	1.351	1.267	1.372	1.762	1.486
	15	1.333	1.277	1.214	1.293	1.576	1.381
	16	1.250	1.210	1.164	1.222	1.420	1.287
	17	1.176	1.149	1.119	1.158	1.290	1.204
	18	1.111	1.095	1.076	1.100	1.178	1.129
	19	1.053	1.045	1.037	1.048	1.083	1.061
	20	1.000	1.000	1.000	1.000	1.000	1.000
25	12	2.083	1.811	1.626	1.739	3.190	2.271
	13	1.923	1.706	1.553	1.645	2.793	2.094
	14	1.786	1.611	1.486	1.561	2.373	1.801
	15	1.667	1.527	1.424	1.485	2.211	1.678
	16	1.563	1.451	1.367	1.417	1.993	1.675
	17	1.471	1.382	1.314	1.354	1.810	1.569
	18	1.389	1.319	1.267	1.297	1.654	1.472
	19	1.316	1.262	1.219	1.244	1.519	1.383
	20	1.250	1.209	1.176	1.195	1.403	1.304
	21	1.190	1.161	1.136	1.150	1.302	1.232
	22	1.136	1.116	1.099	1.109	1.212	1.166
	23	1.087	1.074	1.064	1.070	1.133	1.106
	24	1.042	1.036	1.031	1.034	1.063	1.051
	25	1.000	1.000	1.000	1.000	1.000	1.000
30	15	2.000	1.790	1.634	1.726	2.899	2.253
	16	1.875	1.701	1.569	1.647	2.613	2.101
	17	1.765	1.620	1.509	1.574	2.372	1.964
	18	1.667	1.547	1.453	1.507	2.166	1.841
	19	1.579	1.480	1.401	1.446	1.990	1.731
	20	1.500	1.418	1.352	1.390	1.837	1.632
	21	1.429	1.361	1.306	1.338	1.704	1.541
	22	1.364	1.309	1.264	1.289	1.587	1.459
	23	1.304	1.260	1.223	1.244	1.483	1.384
	24	1.250	1.215	1.186	1.202	1.391	1.315
	25	1.200	1.173	1.150	1.163	1.308	1.251
	26	1.154	1.134	1.117	1.126	1.234	1.193
	27	1.111	1.097	1.085	1.092	1.167	1.139
	28	1.071	1.063	1.055	1.059	1.106	1.089
	29	1.034	1.030	1.027	1.029	1.051	1.043
	30	1.000	1.000	1.000	1.000	1.000	1.000

Graph (1)

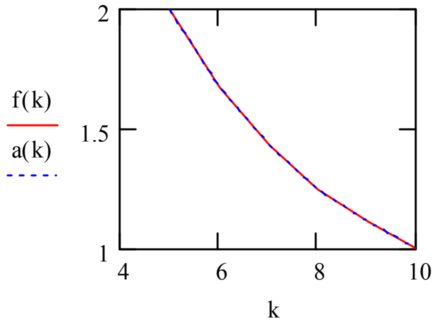
α = 0 and β = 0



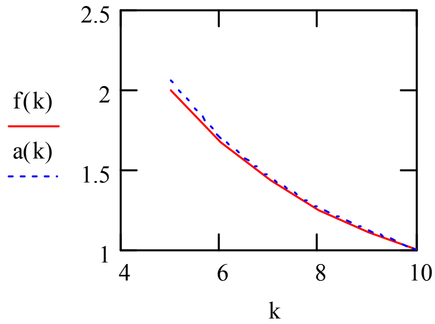
α = 0 and β = 1



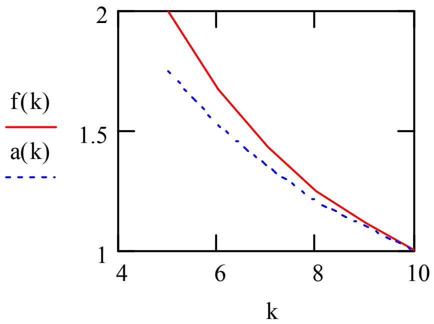
α = 1 and β = 0



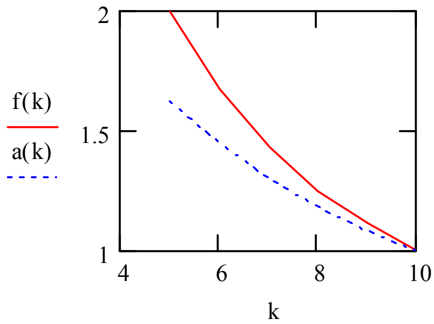
α = 1 and β = 1



α = 2 and β = 3

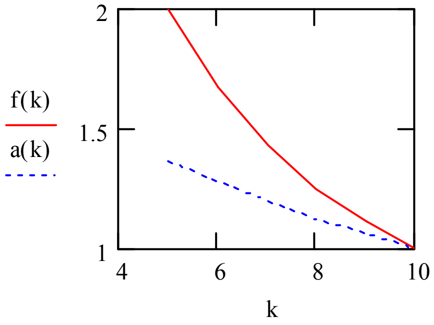


α = 3 and β = 2

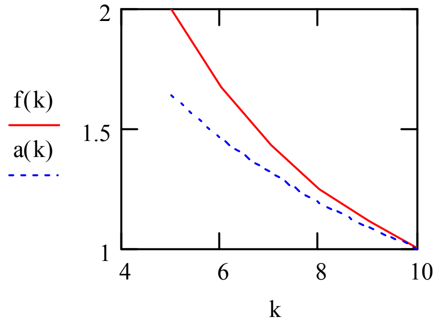


Continued Graph (3)

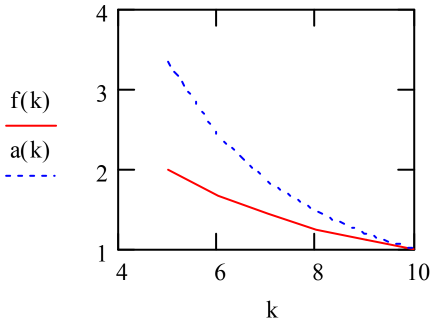
α = 5 and β = 7



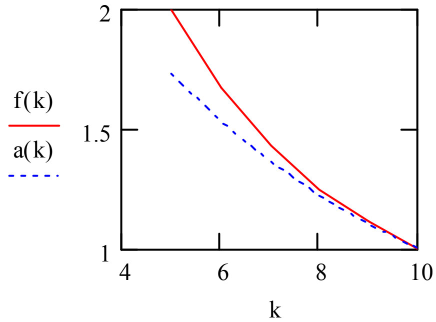
α = 7 and β = 5



α = 1 and β = 10



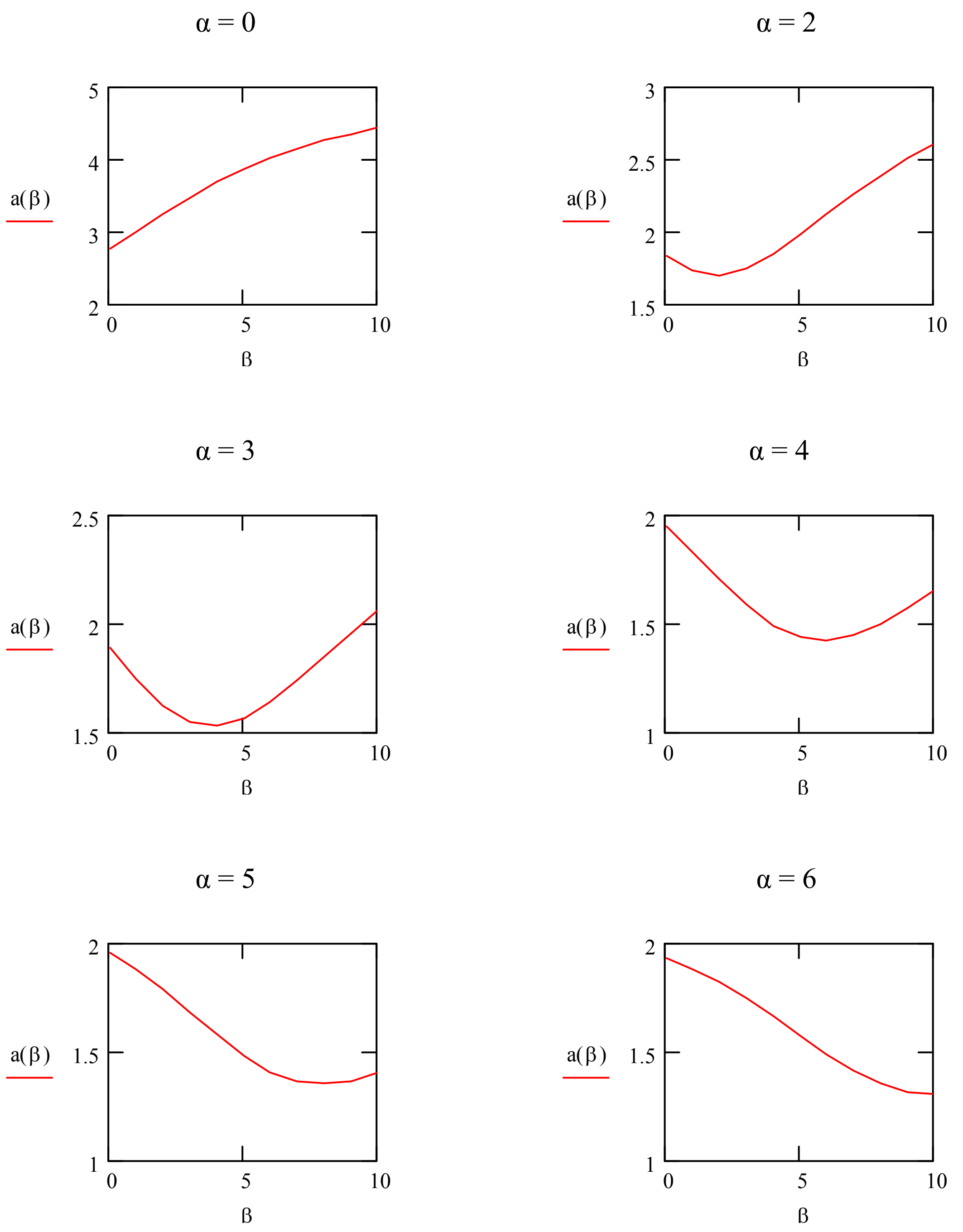
α = 10 and β = 1



a(k) the ratio of the risk function for Bayes estimator under type-II censoring to the risk function under complete sampling when n = 10

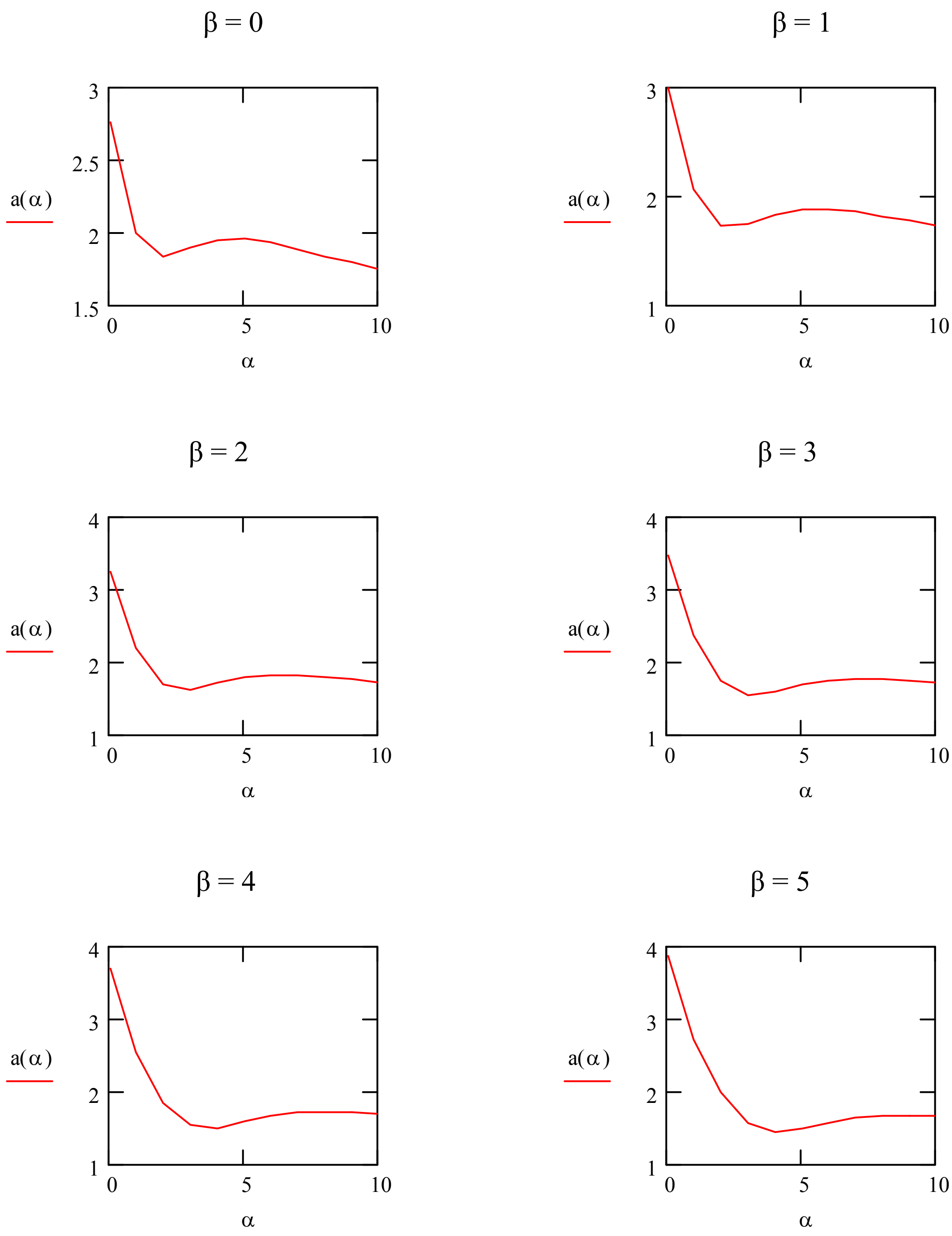
f(k) the ratio of the risk function for MLE under type-II censoring to the risk function under complete sampling when n = 10

Graph (4)

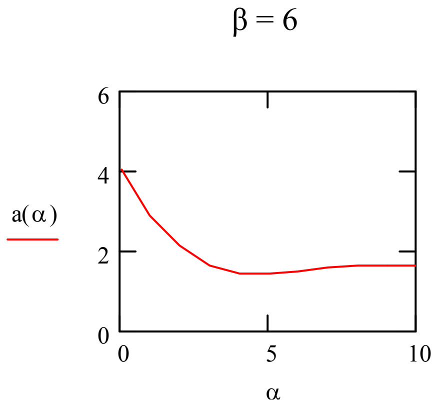


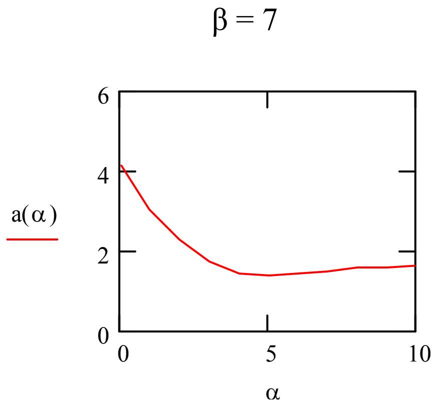
a(β) the ratio of the risk function for Bayes estimator under type-II censoring to the risk function under complete sampling when n = 10 and k = 5

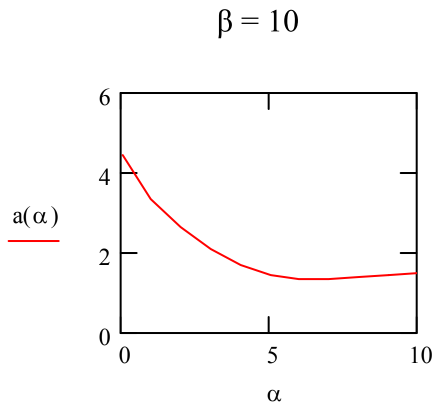
Graph (5)



Continued Graph (5)







a(α) the ratio of the risk function for Bayes estimator under type-II censoring to the risk function under complete sampling when n = 10 and k = 5

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