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Estimation in Step-stress Partially Accelerated Life Test for Exponentiated Pareto Distribution under Progressive Censoring with Random Removal

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Authors' contributions

This work was carried out in collaboration between all authors. Author ASH designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors MAA and HGAEE managed the analyses of the study. Author HGAEE managed the literature searches. All authors read and approved the final manuscript.

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Abstract

Accelerated life testing or partially accelerated life testing is generally used in manufacturing industries since it affords significant minimization in the cost and test time. In this paper, a step-stress partially accelerated life test under progressive type-II censoring with random removals is considered. The lifetime of testing items under use condition follow the exponentiated Pareto distribution and the removals from the test are assumed to have binomial distribution. Maximum likelihood estimators for the model parameters and acceleration factor are obtained. Approximate confidence intervals for the parameters are formed via the normal approximation to the asymptotic distribution of maximum likelihood estimators. Simulation study is carried out to investigate the performance of estimators for different parameters values and sample size.

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ACRONYMS AND NOTATIONS

ALTs : Accelerated life tests
CP : Coverage probability
EP : Exponentiated Pareto
MLE : Maximum likelihood estimate/estimator
MSE : Mean square error
PALTs : Partially accelerated life tests
SS-PALT : Step stress-partially accelerated life test
RB : Relative bias
x_i *: Observed lifetime of unit i tested under SS-PALT*
τ *: Stress change time*
u_{1i}, u_{2i} *: Indicator functions:* $u_{1i} \equiv I(x_i < \tau), u_{2i} \equiv I(x_i > \tau)$
m_1, m_2 *: Number of units failed at normal use and accelerated conditions respectively*
m *: Total number of units failed under SS-PALT* $(m = m_1 + m_2)$
I(A) : State function which equals 1 if the A is true and 0 if not.

1 Introduction

Recent developments in technological areas lead to highly reliability products with very long lifetimes. When testing the life of these products under normal operating conditions, the result gives no or very little failure by the end of the test. So, the accelerated life tests (ALTs) or partially accelerated life tests (PALTs) are preferred to be applied to obtain information on the life of the products shortly and rapid. If all test items are subjected to higher than usual stress levels, then the test is called ALT. While, in PALTs items are tested at both accelerated and use conditions. The information data collected from the test executed in the ALTs or PALTs are used to estimate failure behavior of the items at normal use conditions. Nelson [1] indicated that the stress can be applied in various ways. The constant-stress PALTs and step-stress PALTs (SS-PALTs) are the commonly used methods. In SS-PALTs, a test item is first run at normal (use) condition and, if it does not fail for a specified time, then it is run at accelerated condition until the test terminates. In constant-stress PALTs each item is run at constant high stress until either failure occurs or the test is terminated.

In many life tests and reliability studies, the experimenter may not always obtain complete information on failure times for all experimental units. Data obtained from such experiments are called censored data. The most common censoring schemes are type-I censoring and type-II censoring. These two censoring schemes do not have the flexibility of allowing removal of units at points other than the terminal point of the experiment. Progressive censoring is a more general censoring scheme which allows the units to be removed from the test (Balakrishnan and Aggarwala [2]).

Progressive type-II censoring scheme can be described as follows. Consider an experiment in which n independent and identically units are placed on a life test and m failure are going to be observed. When the first failure occurs, say at time $t_{(1)}$, r_1 of units are randomly removed from the remaining $n-1$ surviving units. The test continues until the mth failure occurs at which time, all the remaining surviving units $r_m = n - m - r_1 - r_2 - \ldots - r_{m-1}$ are all removed from the test. Note that, in this scheme, $r_1, r_2, \ldots, r_m$ are all prefixed. However, in some practical situations, these numbers may occur at random. For example, Yuen and Tse [3] pointed out that the number of patients that withdraw from a clinical test at each stage is random and cannot be prefixed. Therefore, the statistical inference on lifetime distributions under progressive type II censoring with random removals has been studied by various authors, for instance; Wu et al. [4], Yan et al. [5], Tse et al. [6], and Dey and Dey [7].

Specifically, PALTs were studied under type-I and type-II censoring schemes by several authors; for example, see Goel [8] DeGroot and Goel [9], Bhattacharyya and Soejoeti [10], Bai and Chung [11], Abdel-Ghani [12], Abd-Elfattah et al. [13], Aly and Ismail [14], Hassan and Thobety [15]. Works for PALTs have been studied under progressive censoring, for instance; Ismail and Sarhan [16], Srivastava and Mittal [17], and Mohie EL-Din et al. [18].

The Pareto distribution is the most popular model for analyzing skewed data. The Pareto distribution was originally proposed to model the unequal distribution of wealth since it was observed the way that a larger portion of the wealth of any society is owned by a smaller percentage of the people. Ever since, it plays an important role in analyzing a wide range of real-world situations, not only in the field of economics. Examples of approximately Pareto distributed phenomena may be found in sizes of sand particles and clusters of Bose-Einstein condensate close to absolute zero. Exponentiated Pareto distribution has been received the greatest attention from theoretical and applied statisticians primarily due to its use in reliability and life testing studies since it has decreasing and upside-down bathtub shaped failure rates. It is considered to be useful for modeling and analyzing the life time data in medical and biological sciences, engineering, etc. A new two-parameter distribution, called the exponentiated Pareto (*EP*) distribution has been introduced by Gupta et al. [19]. The *EP* distribution can be defined by raising the distribution function of a Pareto distribution to a positive power. The probability density function of EP distribution, with two shape parameters α and θ, is given by:

$$f(t) = \alpha\theta\left(1+t\right)^{-(\theta+1)}\left[1-\left(1+t\right)^{-\theta}\right]^{\alpha-1}, \qquad \alpha,\theta,t>0 \tag{1}$$

The survival function of the *EP* distribution is as follows

$$S(t) = 1-\left[1-\left(1+t\right)^{-\theta}\right]^{\alpha}. \tag{2}$$

In this paper, the progressively type-II censored sampling with binomial removals data is used to obtain the point and approximate confidence interval estimates of the *EP* parameters in SS-PALTs. The paper can be arranged as follows. A description of the model, test procedure, and its assumptions are presented in Section 2. In Section 3, the maximum likelihood estimates of the model parameters are provided in Section 3. Approximate confidence interval estimates of the model parameters are derived in Section 4. Simulation study is performed to illustrate theoretical results in Section 5. Some concluding remarks are summarized at last section.

2 Model and Test Procedure

The following assumptions of SS-PALT are considered:

1. n identical and independent units are put on the life under used condition and the lifetime of each testing unit has the *EP* distribution.
2. The test is terminated at the *mth* failure, where *m* is prefixed $m \le n$.
3. Each of the n units is first run under normal use condition. If it does not fail or remove from the test by a pre-specified time τ, it is put under accelerated condition.
4. At the *ith* failure a random number of the surviving units, r_i , $i=1,2,...,m-1$ are randomly selected and removed from the test. Finally, at the *mth* failure the remaining surviving units $R_m = n-m-\sum_{i=1}^{m-1} R_i$ are all removed from the test and the test is terminated.

5. Suppose that an individual unit being removed from the test is independent of the others but with the same removal probability p. Then, the number of units removed at each failure time follows a binomial distribution. That is, $R_1 \sim bino(n-m,p)$ and for $i=2,3,...,m-1$,

   $R_i \sim bino(n-m-\sum_{i=1}^{m} r_i, p)$ and $r_m = n-m-r_1-r_2-...-r_{m-1}$

6. The lifetime, say X, of a unit under SS-PALT can be rewritten as

$$X = \begin{cases} T & if \;\; T \le \tau \\ \tau + \dfrac{T-\tau}{\beta} & if \;\; T > \tau \end{cases} \tag{3}$$

where, T is the lifetime of an item at normal condition, and β is the acceleration factor. Thus, from the assumptions, the probability density function (3) of a total lifetime of test item takes the following form

$$f(x) = \begin{cases} f_1(x) = \alpha\theta(1+x)^{-(\theta+1)}\left[1-(1+x)^{-\theta}\right]^{\alpha-1}, & 0 < x \le \tau \\ f_2(x) = \alpha\theta\beta(1+\tau+\beta(x-\tau))^{-(\theta+1)}\left[1-(1+\tau+\beta(x-\tau))^{-\theta}\right]^{\alpha-1}, & x > \tau \end{cases} \tag{4}$$

3 Parameters Estimation

In this section, the maximum likelihood estimators of the model parameters; θ, α and β are obtained in step-stress PALT based on the progressively type II censored data with binomial removal. We assume that the number of units removed from the test at each failure time follows a binomial distribution.

Let $(x_i, r_i, u_{1i}, u_{2i})$, $i=1,2,...,m$, be the observed values of lifetime X obtained from a progressively type-II censored sample under a step-stress PALT. Hence, for the progressive censoring with the pre determined number of the removals $R=(R_1=r_1, R_2=r_2,...R_{m-1}=r_{m-1})$, the conditional likelihood function of the observations $x=\{(x_i, r_i, u_{1i}, u_{2i}), i=1,2,...,m\}$ can be defined as follows

$$L_1(x_i,\alpha,\beta,\theta,u_{1i},u_{2i} \mid R=r) = \prod_{i=1}^{m}\left\{f_1(x_i)(S_1(x_i))^{r_i}\right\}^{u_{1i}}\left\{f_2(x_i)(S_2(x_i))^{r_i}\right\}^{u_{2i}}, \tag{5}$$

where,

$$S_1(x) = 1-\left[1-(1+x)^{-\theta}\right]^{\alpha}, \; S_2(x) = 1-\left[1-(1+\tau+\beta(x-\tau))^{-\theta}\right]^{\alpha}.$$

Inserting the probability density functions $f_1(x), f_2(x)$ defined in (3) and their corresponding survival functions in (5), then we have

$$L_1(x_i,\alpha,\beta,\theta,u_{1i},u_{2i}\left|R=r\right.)=\prod_{i=1}^{m}\left(\alpha\theta\left(1+x\right)^{-(\theta+1)}\left[1-\left(1+x\right)^{-\theta}\right]^{\alpha-1}\left[1-\left[1-\left(1+x\right)^{-\theta}\right]^{\alpha}\right]^{r_i}\right)^{u_{1i}}$$
$$\theta\beta\left(1+\tau+\beta(x-\tau)\right)^{-(\theta+1)}\left[1-\left(1+\tau+\beta(x-\tau)\right)^{-\theta}\right]^{\alpha-1}\left(\left[1-\left[1-\left(1+\tau+\beta(x-\tau)\right)^{-\theta}\right]^{\alpha}\right]^{r_i}\right)^{u_{2i}}.\quad(6)$$

The number of units removed at each failure time assumed to follow a binomial distribution with the following probability mass function

$$P(R_1=r_1)=\binom{n-m}{r_1}p^{r_1}(1-p)^{n-m-r_1}.$$

While, for $i=2,3,...,m-1$

$$P(R_i=r_i\left|R_{i-1}=r_{i-1},...,R_1=r_1\right.)=\binom{n-m-\sum_{i=1}^{i-1}r_j}{r_i}p^{r_i}(1-p)^{n-m-\sum_{j=1}^{i}r_i}.$$

Moreover, suppose that R_i is independent of X_i for all i. Hence the likelihood function can be expressed as follows

$$L(x_i,\alpha,\beta,\theta,p)=L_1(x_i,\alpha,\beta,\theta,u_{1i},u_{2i}\left|R=r\right.)P(R=r),\quad(7)$$

Where

$$P(R=r)=P(R_1=r_1,R_2=r_2,...,R_{m-1}=r_{m-1})=P(R_{m-1}=r_{m-1}\left|R_{m-2}=r_{m-2},...,R_1=r_1\right.)\times$$
$$P(R_{m-2}=r_{m-2}\left|R_{m-3}=r_{m-3},...,R_1=r_1\right.)\times...P(R_2=r_2\left|R_1=r_1\right.)P(R_1=r_1).$$

That is,

$$P(R=r)=\frac{(n-m)!}{(n-m-\sum_{i=1}^{m-1}r_i)!\prod_{i=1}^{m-1}r_i}p^{\sum_{i=1}^{m-1}r_i}(1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i},$$

The natural logarithm of the conditional likelihood function, denoted by, $\ln L_1$, can be obtained from (6) as follows

$$\ln L_1=m\ln\alpha+m\ln\theta+m_2\ln\beta+(\alpha-1)\sum_{i=1}^{m_1}\ln(1-D_i^{-\theta})+\sum_{i=1}^{m_1}r_i\ln[1-(1-D_i^{-\theta})^{\alpha}]$$
$$-(\theta+1)\sum_{i=1}^{m_1}\ln D_i+(\alpha-1)\sum_{i=1}^{m_2}\ln(1-H_i^{-\theta})+\sum_{i=1}^{m_2}r_i\ln(1-(1-H_i^{-\theta})^{\alpha})-(\theta+1)\sum_{i=1}^{m_2}\ln H_i,$$

where, $m=\sum_{i=1}^{m_1}u_{1i}+\sum_{i=1}^{m_2}u_{2i}$ $\quad D_i=\left(1+x_i\right)$ and $\quad H_i=\left(1+\tau+\beta(x_i-\tau)\right)$

Since $P(R=r)$ does not involve the parameters; θ, α and β, so the maximum likelihood estimators, $\hat{\alpha},\hat{\theta}$, and $\hat{\beta}$, can be obtained by maximizing $\ln L_1$ directly. Hence, the maximum likelihood estimates (MLEs) of θ,α and β can be obtained by solving the following non-linear equations

$$\frac{\partial \ln L_1}{\partial \alpha}=\frac{m}{\alpha}+\sum_{i=1}^{m_1}\ln(1-D_i^{-\theta})+\sum_{i=1}^{m_2}\ln(1-H_i^{-\theta})-\sum_{i=1}^{m_1}\frac{r_i\ln(1-D_i^{-\theta})}{[(1-D_i^{-\theta})^{-\alpha}-1]}-\sum_{i=1}^{m_2}\frac{r_i\ln(1-H_i^{-\theta})}{((1-H_i^{-\theta})^{-\alpha}-1)},$$

$$\begin{aligned}\frac{\partial \ln L_1}{\partial \theta}=&\frac{m}{\theta}+(\alpha-1)\sum_{i=1}^{m_1}\frac{\ln D_i}{(D_i^{\theta}-1)}-\sum_{i=1}^{m_1}\frac{r_i\alpha(1-D_i^{-\theta})^{\alpha-1}D_i^{-\theta}\ln D_i}{[1-(1-D_i^{-\theta})^{\alpha}]}-\sum_{i=1}^{m_1}\ln D_i\\&+(\alpha-1)\sum_{i=1}^{m_2}\frac{\ln H_i}{(H_i^{\theta}-1)}-\sum_{i=1}^{m_2}\frac{\alpha r_i(1-H_i^{-\theta})^{\alpha-1}H_i^{-\theta}\ln H_i}{(1-(1-H_i^{-\theta})^{\alpha})}-\sum_{i=1}^{m_2}\ln H_i,\end{aligned}$$

$$\begin{aligned}\frac{\partial \ln L_1}{\partial \beta}=&\frac{m_2}{\beta}+(\alpha-1)\sum_{i=1}^{m_2}\frac{\theta H_i^{-\theta-1}(x_i-\tau)}{(1-H_i^{-\theta})}-\sum_{i=1}^{m_2}\frac{\alpha\theta r_i(1-H_i^{-\theta})^{\alpha-1}H_i^{-\theta-1}(x_i-\tau)}{1-(1-H_i^{-\theta})^{\alpha}}\\&-(\theta+1)\sum_{i=1}^{m_2}\frac{(x_i-\tau)}{H_i}.\end{aligned}$$

The maximum likelihood estimators of the model parameters are determined by setting $\frac{\partial \ln L_1}{\partial \alpha},\frac{\partial \ln L_1}{\partial \theta},\frac{\partial \ln L_1}{\partial \beta}$ to be zero. These equations cannot be solved analytically and statistical software can be used to solve them numerically via iterative technique.

Similarly, since $L_1(x_i,\alpha,\theta,\beta,u_{1i},u_{2i}\mid R=r)$ does not involve the binomial parameter p, the MLE of p can be derived by maximizing (6) directly. Hence the MLE of p is obtained by solving the following equation

$$\frac{\partial \ln L}{\partial p}=\frac{\sum_{i=1}^{m-1}r_i}{p}-\frac{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i}{1-p}.$$

Hence,

$$\hat{p}=\frac{\sum_{i=1}^{m-1}r_i}{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i}.$$

4 Approximate Confidence Interval

The most common method to set confidence bounds for the parameters is to use the asymptotic normal distribution of the MLEs (see VanderWiel and Meeker [20]). The asymptotic variances and covariance matrix of the MLE of the parameters can be approximated by numerically inverting the asymptotic Fisher-information matrix F. It is composed of the negative second and mixed derivatives of the natural logarithm of the likelihood function evaluated at the MLE. The asymptotic Fisher information matrix F can be written as follows

$$F=\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}=\begin{bmatrix} \frac{-\partial^2 \ln L_1}{\partial\alpha^2} & \frac{-\partial^2 \ln L_1}{\partial\alpha\partial\theta} & \frac{-\partial^2 \ln L_1}{\partial\alpha\partial\beta} \\ \frac{-\partial^2 \ln L_1}{\partial\theta\partial\alpha} & \frac{-\partial^2 \ln L_1}{\partial\theta^2} & \frac{-\partial^2 \ln L_1}{\partial\theta\partial\beta} \\ \frac{-\partial^2 \ln L_1}{\partial\beta\partial\alpha} & \frac{-\partial^2 \ln L_1}{\partial\beta\partial\theta} & \frac{-\partial^2 \ln L_1}{\partial\beta^2} \end{bmatrix}\downarrow(\hat{\alpha},\hat{\beta},\hat{\theta})$$

The second and mixed partial derivatives of the natural logarithm of likelihood function with respect to α,θ and β are obtained as follows

$$\frac{\partial^2 \ln L_1}{\partial\alpha^2}=\frac{-m}{\alpha^2}-\sum_{i=1}^{m_1}\frac{r_i\left(\ln(1-D_i^{-\theta})\right)^2(1-D_i^{-\theta})^{-\alpha}}{[(1-D_i^{-\theta})^{-\alpha}-1]^2}-\sum_{i=1}^{m_2}\frac{r_i\left(\ln(1-H_i^{-\theta})\right)^2(1-H_i^{-\theta})^{-\alpha}}{[(1-H_i^{-\theta})^{-\alpha}-1]^2},$$

$$\begin{aligned}\frac{\partial \ln L_1}{\partial\theta}=&\frac{m}{\theta}+(\alpha-1)\sum_{i=1}^{m_1}\frac{\ln D_i}{(D_i^{\theta}-1)}-\sum_{i=1}^{m_1}\frac{r_i\alpha(1-D_i^{-\theta})^{\alpha-1}D_i^{-\theta}\ln D_i}{[1-(1-D_i^{-\theta})^{\alpha}]}-\sum_{i=1}^{m_1}\ln D_i\\&+(\alpha-1)\sum_{i=1}^{m_2}\frac{\ln H_i}{(H_i^{\theta}-1)}-\sum_{i=1}^{m_2}\frac{\alpha r_i(1-H_i^{-\theta})^{\alpha-1}H_i^{-\theta}\ln H_i}{(1-(1-H_i^{-\theta})^{\alpha})}-\sum_{i=1}^{m_2}\ln H_i,\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 \ln L_1}{\partial\alpha\partial\theta}=&\sum_{i=1}^{m_1}\frac{\ln D_i}{(D_i^{\theta}-1)}+\sum_{i=1}^{m_2}\frac{\ln H_i}{(H_i^{\theta}-1)}-\sum_{i=1}^{m_1}\frac{r_i\ln D_i}{(D_i^{\theta}-1)[(1-D_i^{-\theta})^{-\alpha}-1]}-\sum_{i=1}^{m_1}\frac{\alpha r_i\ln(1-D_i^{-\theta})(1-D_i^{-\theta})^{-\alpha-1}D_i^{-\theta}\ln D_i}{[(1-D_i^{-\theta})^{-\alpha}-1]^2}\\&-\sum_{i=1}^{m_2}\frac{r_i\ln H_i}{(H_i^{\theta}-1)[(1-H_i^{-\theta})^{-\alpha}-1]}-\sum_{i=1}^{m_2}\frac{\alpha r_i\ln(1-H_i^{-\theta})(1-H_i^{-\theta})^{-\alpha-1}H_i^{-\theta}\ln H_i}{[(1-H_i^{-\theta})^{-\alpha}-1]^2},\end{aligned}$$

$$\frac{\partial^2 \ln L_1}{\partial\alpha\partial\beta}=\sum_{i=1}^{m_2}\frac{\theta H_i^{-\theta-1}(x_i-\tau)}{(1-H_i^{-\theta})}-\sum_{i=1}^{m_2}\frac{r_i\theta H_i^{-\theta-1}(x_i-\tau)}{((1-H_i^{-\theta})^{-\alpha}-1)(1-H_i^{-\theta})}-\sum_{i=1}^{m_2}\frac{r_i\alpha\theta(1-H_i^{-\theta})^{-\alpha-1}\ln(1-H_i^{-\theta})H_i^{-\theta}}{((1-H_i^{-\theta})^{-\alpha}-1)^2},$$

$$\begin{aligned}\frac{\partial^2 \ln L_1}{\partial\theta\partial\beta}=&(\alpha-1)\sum_{i=1}^{m_2}\left[\frac{(x_i-\tau)}{(H_i^{\theta}-1)H_i}-\frac{\theta H_i^{\theta-1}\ln H_i(x_i-\tau)}{(H_i^{\theta}-1)^2}\right]-\sum_{i=1}^{m_2}\frac{(x_i-\tau)}{H_i}\\&-\alpha\sum_{i=1}^{m_2}\frac{r_i(x_i-\tau)[(\alpha-1)\theta(1-H_i^{-\theta})^{\alpha-2}H_i^{-2\theta-1}\ln H_i-\theta(1-H_i^{-\theta})^{\alpha-1}H_i^{-\theta-1}\ln H_i+(1-H_i^{-\theta})^{\alpha-1}H_i^{-\theta-1}]}{(1-(1-H_i^{-\theta})^{\alpha})}\\&-\alpha\theta\sum_{i=1}^{m_2}\frac{r_i(1-H_i^{-\theta})^{2(\alpha-1)}H_i^{-2\theta-1}\ln H_i(x_i-\tau)}{(1-(1-H_i^{-\theta})^{\alpha})^2}\end{aligned}$$

$$\frac{\partial^2 \ln L_1}{\partial \beta^2} = \frac{-m_2}{\beta^2} + (\alpha-1)\sum_{i=1}^{m_2} \frac{(1-H_i^{-\theta})[-\theta(\theta+1)H_i^{-\theta-2}(x_i-\tau)^2] - \theta^2 H_i^{-2(\theta+1)}(x_i-\tau)^2}{(1-H_i^{-\theta})^2}$$
$$-\sum_{i=1}^{m_2} \frac{\alpha\theta r_i (x_i-\tau)^2[(\alpha-1)\theta(1-H_i^{-\theta})^{\alpha-2} H_i^{-2(\theta+1)} - (\theta+1)(1-H_i^{-\theta})^{\alpha-1} H_i^{-\theta-2}]}{[1-(1-H_i^{-\theta})^{\alpha}]}$$
$$-\sum_{i=1}^{m_2} \frac{\alpha^2\theta^2 r_i (1-H_i^{-\theta})^{2(\alpha-1)} H_i^{-2(\theta+1)}(x_i-\tau)^2}{[1-(1-H_i^{-\theta})^{\alpha}]^2} + (\theta+1)\sum_{i=1}^{m_2} \frac{(x_i-\tau)^2}{(H_i)^2}.$$

$$\frac{\partial^2 \ln L_1}{\partial \theta^2} = \frac{-m}{\theta^2} - (\alpha-1)\sum_{i=1}^{m_1} \frac{(\ln D_i)^2 D_i^{\theta}}{(D_i^{\theta}-1)^2} - \sum_{i=1}^{m_1} \frac{r_i\alpha(\ln D_i)^2[(\alpha-1)(1-D_i^{-\theta})^{\alpha-2}.D_i^{-2\theta} - (1-D_i^{-\theta})^{\alpha-1}.D_i^{-\theta}]}{[1-(1-D_i^{-\theta})^{\alpha}]}$$
$$-\sum_{i=1}^{m_1} \frac{r_i\alpha^2(\ln D_i)^2 D_i^{-2\theta}(1-D_i^{-\theta})^{2(\alpha-1)}}{[1-(1-D_i^{-\theta})^{\alpha}]^2} - \sum_{i=1}^{m_1} \frac{r_i\alpha(\ln H_i)^2[(\alpha-1)(1-H_i^{-\theta})^{\alpha-2}.H_i^{-2\theta} - (1-H_i^{-\theta})^{\alpha-1}.H_i^{-\theta}]}{[1-(1-H_i^{-\theta})^{\alpha}]}$$
$$-\sum_{i=1}^{m_1} \frac{r_i\alpha^2(\ln H_i)^2 H_i^{-2\theta}(1-H_i^{-\theta})^{2(\alpha-1)}}{[1-(1-H_i^{-\theta})^{\alpha}]^2} - (\alpha-1)\sum_{i=1}^{m_2} \frac{\left(\ln H_i\right)^2 H_i^{\theta}}{(H_i^{\theta}-1)^2},$$

In relation to the asymptotic variance-covariance matrix of the maximum likelihood estimators of the parameters, it can be approximated by numerically inverting the above Fisher's information matrix F. Thus, the approximate $100(1-\gamma)\%$ and two sided confidence intervals for α,θ and β can be, respectively, easily obtained by

$$\hat{\alpha} \pm z_{\gamma/2}\sigma_{\hat{\alpha}},\ \hat{\theta} \pm z_{\gamma/2}\sigma_{\hat{\theta}},\ \hat{\beta} \pm z_{\gamma/2}\sigma_{\hat{\beta}} \quad , \quad ,$$

where, z is the $\left[\dfrac{100(1-\gamma)}{2}\right]^{th}$ standard normal percentile and $\sigma(.)$ is the standard deviation for the maximum likelihood estimates.

5 Simulation Study

In this section, a simulation study is performed to examine the performance of the parameter estimates and the influence of the available information from the progressively censored data on these estimates. The performance of estimators has been considered in terms of their relative bias (RB), mean square error (MSE) and coverage probability (CP). For different choices of n, m, p and R_i, i = 1, 2, ..., m; the results are concluded in Tables 1, 2 and 3. A simulation study is performed according to the following steps:

1. The value of n and m is specified.
2. Three selected set of parameters are considered as; set $I \equiv (\alpha = 1.5, \beta = 1.2, \theta = 0.5)$ set $II \equiv (\alpha = 1.5, \beta = 3, \theta = 1.5)$ and set $III \equiv (\alpha = 1.5, \beta = 3, \theta = 0.5)$. Three levels of p are selected as 0.4, 0.6 and 0.8. Assuming that the value of τ equals 7 throughout all experiments.

3. Generate a random sample with size n and censoring size m with random removals R_i , i = 1, 2, ..., m, from the random variable X given by (4).
4. Generate a group value random number $R_i \sim bino(n-m-\sum_{j=1}^{m} r_j , p)$ and
   $r_m = n-m-r_1-r_2-...-r_{m-1}$.
5. The MLEs of the model parameters are computed for sample sizes n= 50, 75, 100, and150.
6. Compute the values of relative biases, mean square errors and the coverage rate of the *95%* confidence interval of MLE of the parameters numerically for each sample size.
7. The above steps are repeated 1000 times for different values of n, m, R_i i = 1, 2, ..., m, and p.
8. The simulation results are listed in Tables 1 to 3 and represented through some figures.

i. From the results shown in Tables 1 - 3, one can observe that the relative biases and the MSEs of the parameter estimates decrease as the sample size increases.
ii. For set I, the MSEs of $\hat{\alpha}$ have the smallest values at p=0.8 for different m and n, while the MSEs of $\hat{\theta}$ and $\hat{\beta}$ have the smallest values at p=0.6 for different m and n (see for example Figs. 1 and 2).
iii. For set III, the MSEs of $\hat{\alpha}$ and $\hat{\theta}$ have the smallest values at p=0.6 for different m and n, while the MSE of $\hat{\beta}$ has the smallest values at p=0.4 for different m and n (see for example Figs. 3 and 4).
iv. For set II, the MSEs of $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\theta}$ have the smallest values at p=0.4 for different m and n, (see for example Fig. 5).
v. As seen from the Tables 1-3, the coverage probabilities are close to the nominal confidence level 0.95% for different n, m and selected set of parameters.

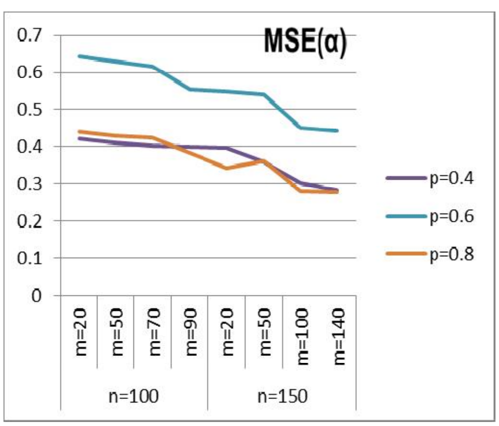

Fig. 1. MSE of $\hat{\alpha}$ for set I for different values of p at sample sizes n=100 and 150

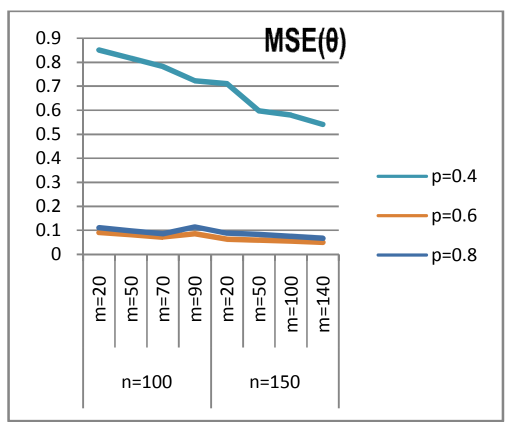

Fig. 2. MSE of $\hat{\theta}$ for set I for different values of p at sample sizes n=100 and 150

Table 1. Relative bias, mean square error and coverage probability for set $I \equiv (\alpha = 1.5, \beta = 1.2, \theta = 0.5)$

			P=0.4			P=0.6			P=0.8		
			RB	**MSE**	**CP**	**RB**	**MSE**	**CP**	**RB**	**MSE**	**CP**
n=50	m=20	α	-0.36584	0.470873	1	0.257643	0.622268	0.995	0.315152	0.462891	0.9925
		β	2.058367	8.96585	0.9975	2.589204	7.82268	0.99	1.80752	6.334238	0.9925
		θ	-0.65677	1.11526	1	0.81565	0.253836	0.9975	0.703428	0.424852	0.9975
n=50	m=30	α	-0.34593	0.443213	0.995	0.248641	0.591796	0.99	0.304162	0.43105	0.995
		β	1.812186	8.5526	0.9975	2.317242	7.392273	0.995	1.711599	5.905246	0.9875
		θ	-0.61194	1.026	0.995	0.787201	0.251647	0.99	0.697659	0.421494	0.99
n=50	m=40	α	-0.29986	0.404714	0.9975	0.230459	0.596698	1	0.285065	0.433268	0.99
		β	1.582549	8.127966	0.9875	1.934896	6.905664	0.9925	1.706332	5.636779	0.9875
		θ	-0.53155	0.99599	0.995	0.874282	0.251056	0.9875	0.935538	0.42028	0.9975
n=75	m=20	α	-0.14493	0.441182	1	0.225889	0.544343	1	0.291654	0.39424	0.995
		β	1.830523	8.014031	0.9975	2.0502	6.362951	0.995	1.593127	5.423162	0.9975
		θ	-0.6572	0.99139	0.9975	0.876078	0.227749	1	0.863305	0.37745	1
n=75	m=50	α	-0.36411	0.43256	0.9975	0.255503	0.54188	0.99	0.344289	0.396933	0.9875
		β	2.398017	6.011121	0.9875	2.170691	4.991654	1	1.850099	4.753422	0.995
		θ	-0.71927	0.937114	0.995	0.823148	0.098317	0.9975	0.646523	0.120882	0.995
n=75	m=70	α	-0.34776	0.431113	0.985	0.244038	0.644473	0.99	0.325547	0.439897	0.995
		β	2.325195	5.644994	1	2.053912	4.897775	0.995	1.719185	4.650491	0.995
		θ	-0.6665	0.867901	0.985	0.750672	0.090373	0.9925	0.55696	0.111293	1
n=100	m=20	α	-0.01985	0.422563	0.9975	0.231416	0.643355	1	0.31757	0.439181	0.995
		β	1.793146	6.26963	0.9825	1.746095	4.499268	0.9875	1.59607	4.574203	0.985
		θ	-0.46352	0.851032	0.9975	0.890247	0.089337	0.9975	0.550567	0.108948	0.9975
n=100	m=50	α	-0.01697	0.41156	1	0.203816	0.63013	0.9925	0.277345	0.428551	0.9975
		β	1.660347	5.935514	0.98	1.681772	4.051209	0.99	1.576908	3.98056	1
		θ	-0.39353	0.816596	1	0.808374	0.080527	0.995	0.487173	0.096466	0.995
n=100	m=70	α	0.006452	0.40239	0.9875	0.137432	0.615122	0.99	0.221015	0.424595	0.995
		β	1.674137	5.939133	1	1.649779	4.02792	0.99	1.561328	4.231271	0.9875
		θ	-0.35136	0.781813	0.995	0.823111	0.070576	0.995	0.518569	0.084215	0.985
n=100	m=90	α	-0.13906	0.40025	1	0.126759	0.554054	1	0.200533	0.383337	0.9875
		β	2.010123	5.65996	0.9875	1.616122	4.388505	0.995	1.542001	4.63472	1
		θ	-1.16627	0.722694	0.98	0.817767	0.084936	0.9925	0.489233	0.11312	1

			P=0.4			P=0.6			P=0.8		
			RB	**MSE**	**CP**	**RB**	**MSE**	**CP**	**RB**	**MSE**	**CP**
n=150	m=20	α	-0.38613	0.395632	1	0.162184	0.54729	1	0.262008	0.343217	1
		β	1.61644	4.596727	0.9925	1.762585	3.982524	0.9975	1.45078	4.305654	0.9975
		θ	-1.14427	0.709742	0.9975	0.940723	0.062229	1	0.69313	0.086997	0.9975
n=150	m=50	α	-0.29493	0.357698	0.9925	0.153731	0.540267	0.9975	0.248161	0.363045	0.9925
		β	1.43557	4.282914	0.9975	1.584315	3.84409	0.9975	1.338698	4.248263	0.9875
		θ	-0.99092	0.596428	1	0.726195	0.059019	0.995	0.731186	0.081842	0.9875
n=150	m=100	α	-0.33186	0.302878	0.9925	0.142419	0.451148	0.9975	0.264684	0.28105	0.9975
		β	1.475529	3.747086	1	1.832497	3.25632	0.9975	1.353053	4.02589	0.995
		θ	-0.97294	0.57978	0.9975	0.668818	0.055027	0.9925	0.661044	0.07536	1
n=150	m=140	α	-0.33612	0.283574	1	0.088666	0.444267	0.9925	0.143085	0.276758	1
		β	1.353553	3.599838	0.99	1.644942	2.9998	0.995	1.429233	3.99856	1
		θ	-0.96232	0.540338	1	0.66027	0.049757	0.99	0.65007	0.066142	1

Table 2. Relative bias, mean square error and coverage probability for set $II \equiv (\alpha = 1.5, \beta = 3, \theta = 1.5)$

			P=0.4			P=0.6			P=0.8		
			RB	**MSE**	**CP**	**RB**	**MSE**	**CP**	**RB**	**MSE**	**CP**
n=50	m=20	α	0.287456	0.456963	1	0.191188	0.51968	0.995	0.338223	0.525631	0.9925
		β	0.660062	4.96523	1	0.930987	4.36523	0.9925	1.03535	5.3652	0.99
		θ	0.181381	0.221326	1	0.229919	0.278596	0.995	0.362677	0.302156	0.9925
n=50	m=30	α	0.279113	0.429653	0.9925	0.184524	0.501632	0.9975	0.32785	0.522365	0.995
		β	0.584587	2.918183	0.985	0.880664	2.946317	0.9975	0.928046	5.0125	0.9925
		θ	0.177829	0.199652	1	0.227872	0.26896	0.9925	0.356269	0.298563	0.99
n=50	m=40	α	0.272844	0.422565	0.9875	0.173109	0.474194	0.99	0.317653	0.515632	0.9925
		β	0.522486	2.427945	0.985	0.900225	2.535726	0.995	0.713118	4.241114	0.99
		θ	0.17196	0.17659	0.99	0.333098	0.239431	1	0.389068	0.256355	0.99
n=75	m=20	α	0.270547	0.41256	0.9975	0.178151	0.47025	0.9975	0.318584	0.495331	0.9975
		β	0.496628	2.271742	0.9925	0.840278	2.355989	0.995	0.672595	3.998728	0.99
		θ	0.162541	0.142653	1	0.309462	0.224777	0.9925	0.366895	0.23569	0.9975
n=75	m=50	α	0.274962	0.37956	0.9975	0.214579	0.44653	0.9975	0.335681	0.47856	0.99
		β	0.487214	2.320027	0.9975	0.972986	2.199652	0.995	0.683912	3.7565	0.9925
		θ	0.128803	0.123792	0.9925	0.245352	0.194324	0.9975	0.296036	0.191999	0.99

			P=0.4			P=0.6			P=0.8		
			RB	**MSE**	**CP**	**RB**	**MSE**	**CP**	**RB**	**MSE**	**CP**
n=75	m=70	α	0.291551	0.344659	0.985	0.200759	0.4026	0.99	0.345532	0.46529	0.995
		β	0.418437	2.02635	0.9825	0.849179	2.126261	0.99	0.585288	3.612796	0.9975
		θ	0.12856	0.122232	1	0.220058	0.178092	0.99	0.284947	0.170263	0.9975
n=100	m=20	α	0.277746	0.316589	0.995	0.196207	0.389856	0.9925	0.330249	0.433569	0.995
		β	0.453414	1.918718	0.975	0.801555	2.126385	0.9925	0.62323	3.55962	0.9975
		θ	0.133727	0.11963	1	0.213986	0.179488	0.9925	0.29445	0.165359	0.9975
n=100	m=50	α	0.19481	0.281742	0.995	0.162402	0.359856	0.9925	0.239706	0.436311	0.995
		β	0.436853	1.9228	0.985	0.810003	2.178826	0.9925	0.610566	3.46566	0.99
		θ	0.108382	0.113265	1	0.154132	0.175263	0.9925	0.163939	0.156903	0.995
n=100	m=70	α	0.175323	0.245219	1	0.130854	0.317774	0.995	0.209773	0.411563	0.9925
		β	0.447201	1.94134	1	0.802002	2.160874	0.995	0.619092	3.333263	0.9875
		θ	0.10395	0.10326	0.9975	0.164041	0.15698	0.9925	0.16155	0.157098	0.995
n=100	m=90	α	0.176289	0.243081	0.995	0.119829	0.304061	0.9975	0.208156	0.409965	0.9975
		β	0.45461	1.930303	0.9975	0.778	2.09629	0.99	0.620817	3.1326	0.9975
		θ	0.16459	0.095848	0.9975	0.137289	0.135569	1	0.228583	0.15563	0.995
n=150	m=20	α	0.361161	0.223652	0.9875	0.1349	0.300965	0.995	0.395972	0.401563	1
		β	0.469598	1.89563	0.985	0.802073	2.06596	0.985	0.63683	3.00963	1
		θ	0.148327	0.095553	0.9875	0.202261	0.131347	0.99	0.127029	0.14523	1
n=150	m=50	α	0.343505	0.22123	0.99	0.129485	0.256323	0.9825	0.377411	0.38785	0.9975
		β	0.432965	1.854228	0.995	0.746069	2.054241	0.99	0.591549	2.96523	0.995
		θ	0.134775	0.09312	0.995	0.186369	0.12122	0.9975	0.116128	0.122326	0.995
n=150	m=100	α	0.300788	0.20365	1	0.11321	0.233656	1	0.269165	0.34956	1
		β	0.448176	1.78563	0.9975	0.807924	2.0085	0.9975	0.667582	2.785	0.9975
		θ	0.127189	0.089886	0.9975	0.175936	0.115347	0.995	0.111462	0.10236	0.995
n=150	m=140	α	0.292634	0.196323	1	0.110182	0.201236	0.995	0.261437	0.28965	0.9975
		β	0.415215	1.799953	1	0.748147	1.963256	0.995	0.615555	2.66532	0.9975
		θ	0.112078	0.081091	1	0.15798	0.105546	1	0.101834	0.0996	0.995

Table 3. Relative bias, mean square error and coverage probability for set $III \equiv (\alpha = 1.5, \beta = 3, \theta = 0.5)$

			P=0.4			P=0.6			P=0.8		
			RB	**MSE**	**CP**	**RB**	**MSE**	**CP**	**RB**	**MSE**	**CP**
n=50	m=20	α	0.906507	1.9652	0.9925	1.026663	1.212353	0.995	1.107203	1.28804	0.995
		β	0.943828	2.447085	0.99	0.810852	3.02	0.99	0.888061	2.268926	0.9925
		θ	2.25084	1.20065	1	1.447395	0.688709	0.9975	1.878515	1.3569	0.995
n=50	m=30	α	0.878308	1.623782	0.9975	0.991061	1.170955	0.99	1.068763	1.24381	0.9975
		β	0.852658	2.224574	1	0.755192	3.011434	0.995	0.824736	2.093338	0.9975
		θ	2.214567	1.206739	1	1.430451	0.679904	0.99	1.854767	1.12563	0.9925
n=50	m=40	α	0.847302	1.560821	0.995	0.934263	1.108447	1	1.007716	1.176307	0.99
		β	0.618451	1.575762	0.995	0.719332	2.808705	0.9925	0.788854	1.973897	0.995
		θ	2.536376	1.1985	0.9925	1.901741	0.70659	0.9875	2.36806	1.11236	1
n=75	m=20	α	0.85188	1.573756	0.995	0.958053	1.133868	1	1.032344	1.202387	0.9975
		β	0.583186	1.485656	0.9825	0.66838	2.612546	0.995	0.731948	1.832355	0.995
		θ	2.384657	1.102563	0.9925	1.778973	0.699953	1	2.220014	1.096533	0.9925
n=75	m=50	α	0.916308	1.563562	0.99	1.115182	1.301435	0.99	1.194216	1.37441	0.9975
		β	0.604022	1.561555	0.9925	0.748663	2.42156	1	0.815617	1.65963	0.995
		θ	1.893311	1.053932	0.9875	1.490169	0.691507	0.9975	1.894987	1.049231	0.9975
n=75	m=70	α	0.933022	1.556963	0.9925	1.041146	1.233496	0.99	1.109594	1.289015	0.99
		β	0.517004	1.350518	0.9975	0.609597	2.367516	0.995	0.641057	1.55802	0.99
		θ	1.805633	0.993829	0.9975	1.40549	0.654429	0.9925	1.828146	1.017993	0.99
n=100	m=20	α	0.895172	1.32563	0.995	1.006242	1.190123	1	1.069626	1.24012	0.9925
		β	0.541312	1.393307	0.995	0.607709	2.383244	0.9875	0.64558	1.502563	0.9925
		θ	1.844461	0.8212	1	1.385076	0.650685	0.9975	1.807091	1.015655	0.9925
n=100	m=50	α	0.661687	1.282169	0.9975	0.725575	0.867575	0.9925	0.731711	0.96563	0.9925
		β	0.532717	1.371419	0.99	0.623517	2.336563	0.99	0.66879	1.49652	0.9925
		θ	0.94008	0.76543	0.9975	1.017023	0.464423	0.995	1.366499	0.936563	0.9925
n=100	m=70	α	0.573932	1.099959	0.99	0.601322	0.723268	0.99	0.610889	0.902563	0.995
		β	0.538883	1.38462	0.99	0.618251	2.0256	0.99	0.662971	1.4526	0.995
		θ	0.93689	0.69532	0.995	1.061907	0.481244	0.995	1.416323	0.769845	0.9925
n=100	m=90	α	0.562927	1.065036	0.9975	0.574977	0.694834	1	0.590074	0.996523	0.9975
		β	0.538524	1.380419	0.995	0.599598	1.56326	0.995	0.641916	1.33652	0.99
		θ	1.257445	0.495633	0.995	0.966495	0.467438	0.9925	1.317057	0.765148	1

			P=0.4			P=0.6			P=0.8		
			RB	**MSE**	**CP**	**RB**	**MSE**	**CP**	**RB**	**MSE**	**CP**
n=150	m=20	α	1.004306	1.02653	0.9975	0.860034	0.605963	1	0.93416	0.85632	0.995
		β	0.547742	1.392861	1	0.622665	1.42563	0.9975	0.668826	1.302156	0.985
		θ	1.024771	0.475632	1	0.628025	0.239536	1	2.991843	0.705963	0.99
n=150	m=50	α	0.958751	0.99652	1	0.827519	0.596532	0.9975	0.899805	0.832563	0.9825
		β	0.512578	1.31111	0.9975	0.587297	1.425223	0.9975	0.634574	1.298563	0.99
		θ	0.937822	0.464282	0.995	0.579491	0.220866	0.995	2.764986	0.465329	0.9975
n=150	m=100	α	0.634439	0.85632	1	0.578435	0.60123	0.9975	0.521361	0.801562	1
		β	0.603897	1.29632	0.9975	0.705539	1.2356	0.9975	0.792789	1.12563	0.9975
		θ	0.904999	0.450492	0.995	0.553103	0.21088	0.9925	2.682004	0.365962	0.995
n=150	m=140	α	0.615309	0.778563	0.9975	0.561686	0.5896	0.9925	0.505751	0.579183	0.995
		β	0.557785	1.1235	0.9975	0.651596	1.12563	0.995	0.73392	1.021566	0.995
		θ	0.830681	0.416783	0.995	0.503626	0.192633	0.99	2.478344	0.325634	1

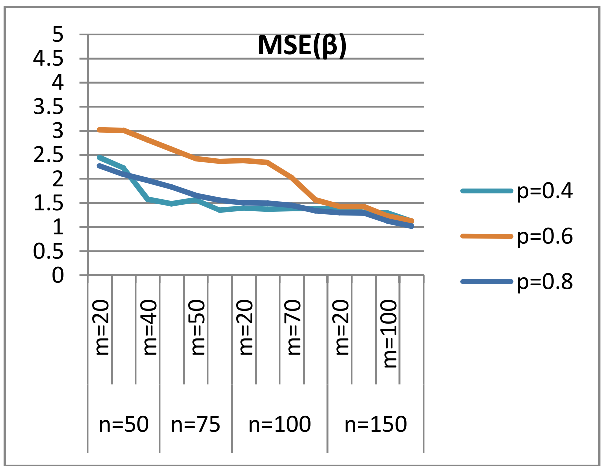

Fig. 3. MSE of $\hat{\theta}$ for set III for different values of p for different sample sizes

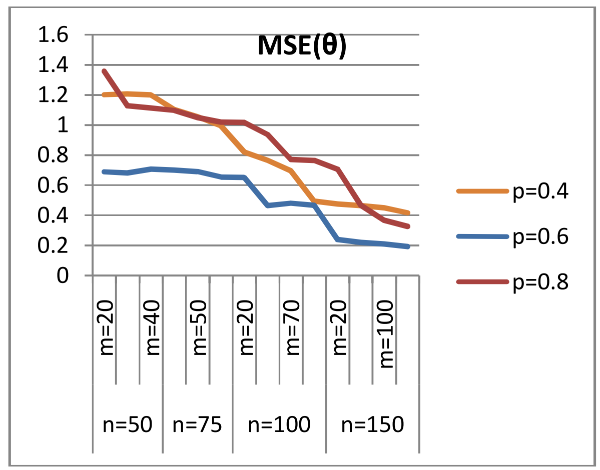

Fig. 4. MSE of $\hat{\theta}$ for set III for different values of p for different sample sizes

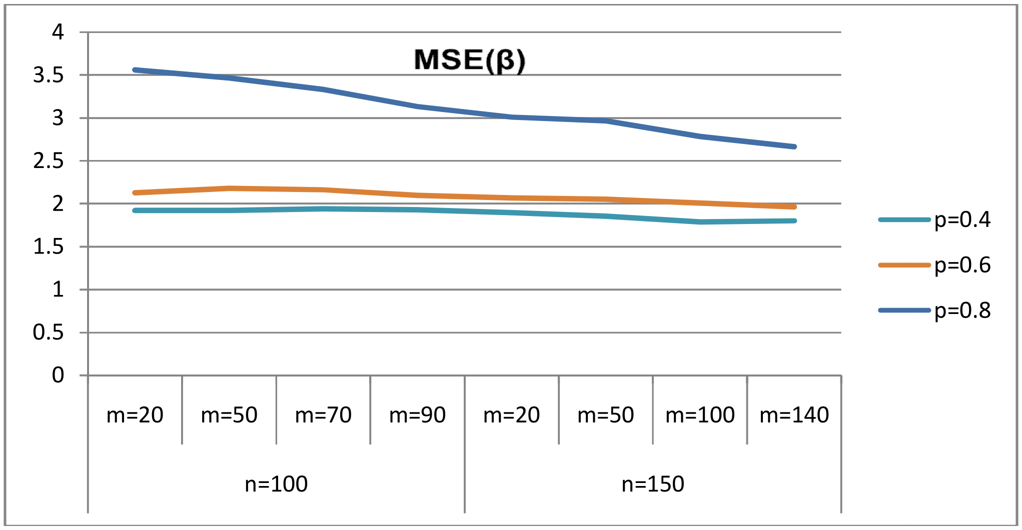

Fig. 5. MSE of $\hat{\beta}$ for set II for different values of p at sample sizes n=100 and 150

6 Conclusion

In this paper, a SS-PALT is presented under the progressive type II censored data with binomial removals. The life times of the testing items are assumed to follow exponentiated Pareto distribution. The MLEs of the considered parameters are obtained and studied their performance through their relative biases and MSEs. Also, approximate confidence intervals for the parameters are constructed. Generally, based on the above analysis, one can say that a large sample size with a large censoring sample gives a better estimate in the sense of having smaller relative bias and MSE. Also the values of binomial parameter p have influence on the accuracy of the parameter estimates. Furthermore, the coverage probabilities are close to the nominal confidence level 0.95% for different censoring schemes, samples sizes and set of parameters.

Competing Interests

Authors have declared that no competing interests exist.

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