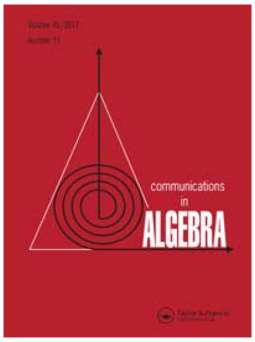

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Applications of two-sided α-generalized derivations to 3-prime near rings

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ABSTRACT
Let N be a zero symmetric 3-prime right near ring and $\alpha : N \to N$ be an endomorphism. In this paper, the notions of two-sided α-(generalized) derivations on N are studied. Some results characterize commutativity of 3-prime near rings are obtained. Examples proving the necessity of the 3-primeness hypothesis are given. When $\alpha = id_N$, one can easily obtain the main results of [5].

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1. Introduction

In this paper, N stands for a zero symmetric right near ring i.e. non empty set together with two binary operations "+" and "." such that (a) $(N, +, 0)$ is a group (not necessarily abelian) (b) $(N, .)$ is a semigroup (c) $\forall n_1, n_2, n_3 \in N$: $(n_1 + n_2)n_3 = n_1n_3 + n_2n_3$ ("right distributive law")and (d) $n0 = 0$ for all $n \in N$. $Z(N)$ is the multiplication center of N, that is, $Z(N) = \{x \in N : xy = yx \text{ for all } y \in N\}$. Note that $0 \in Z(N)$, so $Z(N) \neq \emptyset$. Usually N will be 3-prime near ring, that is, will have the property that $xNy = 0$ for $x, y \in N$ implies $x = 0$ or $y = 0$. Nonempty subset I of N is called a semigroup right ideal (res. semigroup left ideal) if $IN \subseteq I$(resp. $NI \subseteq I$); and I is said to be a semigroup ideal if its both a semigroup right and a semigroup left ideal. An additive mapping $d : N \to N$ is a derivation if $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$ or equivalently that $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$. As in [7], an additive mapping $F : N \to N$ is a right(resp. left)-generalized derivation with associated derivation d if $F(xy) = F(x)y + xd(y)$ $(resp. F(xy) = d(x)y + xF(y))$ for all $x, y \in N$. Recalling that N is 2-torsion free if $2x \neq 0$ for all $0 \neq x \in N$.

In (2004), Argac has introduced the notion of two-sided α-derivation of a near-ring N in the following way.

An additive mapping $d : N \to N$ is called two-sided α-derivation if there exists a function $\alpha : N \to N$ such that $d(xy) = d(x)\alpha(y) + xd(y)$ and $d(xy) = d(x)y + \alpha(x)d(y)$, for all $x, y \in N$. For $\alpha = id_N$ (the identity map on N), a two-sided α-derivation is of course a derivation.

Now we give an example of a two-sided α-derivation on a near-ring which is not a derivation.

Example 1. Let S be a zero-symmetric right near ring.

Let us define N and $d, \alpha : N \to N$ by:

$$N = \left\{ \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} : x, y \in S \right\},$$

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$$d\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} = \begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix},$$

and

$$\alpha\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & y \end{pmatrix}.$$

Clearly N is a zero symmetric right near ring, d is a two-sided α-derivation on N but not a derivation.

In 2014, Boua and Kamal [6] initiated the concepts of two-sided α-generalized derivation as follows:

Definition 1.1. Let N be a near-ring and d be a two-sided α-derivation of N. An additive mapping $F : N \to N$ is called two-sided α-generalized derivation associated with d if it satisfies $F(xy) = F(x)\alpha(y) + xd(y) = F(x)y + \alpha(x)d(y)$ for all $x, y \in N$.

Clearly every two-sided α-derivations is a two-sided α-generalized derivations.

Now we give an example of a two-sided α-generalized derivation F associated with a two-sided α-derivation d on a near-ring N such that F is not two-sided α-derivation of N.

Example 2. Let S be a zero-symmetric right near ring.

Let us define N and $d, F, \alpha : N \to N$ by:

$$N = \left\{\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} : x, y \in S\right\},$$

$$d\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} = \begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix},$$

$$F\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix},$$

and

$$\alpha\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & y \end{pmatrix}.$$

Clearly N is a zero symmetric right near ring, d is a two-sided α-derivation of N, and F is a two-sided α-generalized derivation associated with d, but F is not two-sided α-derivation of N.

For $\alpha = id_N$ (the identity map on N), a two-sided α-derivation is of course the usual derivation, there by F must be a generalized derivation associated with d. We will write for all $x, y \in N$, the symbol, $[x, y]$ stands for the commutator $xy - yx$ or sometimes the Lie product of x and y and the symbol xoy stands for the anticommutator $xy + yx$ or sometimes the Jordan product of x and y. Usually we denote for all $x, y \in N$, $[x, y]_\alpha = x\alpha(y) - yx$ and $(xoy)_\alpha = x\alpha(y) + yx$. In particular if $\alpha = id_N$, then $[x, y]_{id_N} = [x, y]$ and $(xoy)_{id_N} = xoy$, for all $x, y \in N$.

In the present paper, we generalized Theorems 2.9, 2.10, 3.1, 3.2, 3.3, and 3.5 of [5] and Corollary 4.1 of [2].

2. Preliminaries

Lemma 2.1 ([4, Lemmas 1.2(i), 1.2(iii) and 1.3(iii)]). *Let N be a 3-prime near ring.*

i *If $z \in Z(N)/\{0\}$, then z is not a zero divisor.*

ii *If $z \in Z(N)/\{0\}$ and $zx \in Z(N)$, then $x \in Z(N)$*

iii *If z centralizes a non zero semigroup left ideal, then $z \in Z(N)$.*

Lemma 2.2 ([4, Lemma 1.3(i)]). *Let N be a 3-prime near ring. If I is a nonzero semigroup left ideal and x is an element of N such that $xI = \{0\}$, then $x = 0$.*

Lemma 2.3 ([4, Lemma 1.4(i)]). *Let N be a 3-prime near ring and I is a nonzero semigroup ideal of N. If $x, y \in N$ and $xIy = \{0\}$, then $x = 0$ or $y = 0$.*

Lemma 2.4 ([4, Lemma 1.5]). *Let N be a 3-prime near ring. If $Z(N)$ contains a non-zero semigroup right ideal or a semigroup left ideal, then N is a commutative ring.*

Lemma 2.5 ([5, Lemma 2.8]). *Let N be a 3-prime near ring. If I is a nonzero semigroup left ideal or a semigroup right ideal, then I contains an element x such that $x^2 \neq 0$.*

Lemma 2.6. *Let N be a 3-prime near ring. If N admits an additive mapping F, then the following statements are equivalent:*

i $F(xy) = F(x)\alpha(y) + xd(y)$,

ii $F(xy) = xd(y) + F(x)\alpha(y)$ *for all* $x, y \in N$.

Proof.

$(i) \Rightarrow (ii)$: Assume that $F(xy) = F(x)\alpha(y) + xd(y)$, for all $x, y \in N$, so $F((x+x)y) = F(x+x)\alpha(y) + (x+x)d(y) = F(x)\alpha(y)+F(x)\alpha(y)++xd(y)+xd(y)$ for all $x, y \in N$, and $F((x+x)y) = F(xy)+F(xy) = F(x)\alpha(y) + xd(y) + F(x)\alpha(y) + xd(y)$ for all $x, y \in N$.

Comparing the two equations, then we get $F(x)\alpha(y) + xd(y) = xd(y) + F(x)\alpha(y)$ for all $x, y \in N$.

Similarly, we can prove the other implication. □

Lemma 2.7 ([3, Lemma 2.2]). *Let d be a two-sided α-derivation of a near ring N. Then N satisfies the following partial distributive laws:*

i $z(d(x)\alpha(y) + xd(y)) = zd(x)\alpha(y) + zxd(y)$ *for all* $x, y, z \in N$.

ii $z(d(x)y + \alpha(x)d(y)) = zd(x)y + z\alpha(x)d(y)$ *for all* $x, y, z \in N$.

Lemma 2.8 ([8, Lemma 4]). *Let N be a 3-prime near ring and d is a nonzero two-sided α-derivation of N. If I is a nonzero semigroup left ideal or a semigroup right ideal, then $d(I) \neq 0$.*

Lemma 2.9. *Let F be a two-sided α-generalized derivation associated with d of a near ring N. Then N satisfies the following partial distributive laws:*

i $z(F(x)\alpha(y) + xd(y)) = zF(x)\alpha(y) + zxd(y)$ *for all* $x, y, z \in N$.

ii $z(F(x)y + \alpha(x)d(y)) = zF(x)y + z\alpha(x)d(y)$ *for all* $x, y, z \in N$.

Proof. From the computation of $F(x(yz))$ and $F((xy)z)$, we obtain the required results. □

Lemma 2.10 ([8, Theorems 1,2]). *Let N be a 3-prime near ring and I is a nonzero semigroup left ideal or nonzero semigroup right ideal. If N admitting a non-trivial two-sided α-derivation d such that $d(I) \subseteq Z(N)$, then N is a commutative ring.*

We use the following results in the next sections

Lemma 2.11. *Let N be a 2-torsion, 3-prime near ring and I is a nonzero semigroup left ideal. If $\alpha : N \to N$, is a function satisfies $\alpha(xy) = \alpha(x)\alpha(y)$ for all $x, y \in I$. Then there exists $x, y \in I$ such that $(xoy)_\alpha \neq 0$.*

Proof. We argue by contradiction, we divide the proof of this lemma into two parts, in part 1, we show that N is a commutative ring, based in this property in part 2, we get the contradiction.

Assume on the contrary that

$$(xoy)_\alpha = 0 \qquad \text{for all } x, y \in I. \tag{1}$$

Then

$$x\alpha(y) = -yx \qquad \text{for all } x, y \in I. \tag{2}$$

Replacing x by zx, in Eq. (2) where $z \in N$ then $zx\alpha(y) = -yzx = z(-yx)$ for all $x, y \in I, z \in N$. So $((-y)z)x = (z(-y))x$ for all $x, y \in I, z \in N$. We have $((-y)z - z(-y))x = 0$ for all $x, y \in I, z \in N$. Then by Lemma (2.2), we get $-y \in Z(N)$ for all $y \in I$, and so $-I \subseteq Z(N)$. But $-I$ is a nonzero semigroup left ideal, hence N is a commutative ring by Lemma (2.4).

So from Eq. (1) we get $x\alpha(y) = \alpha(y)x = -yx$ for all $x, y \in I$. Then $(\alpha(y) + y)x = 0$ for all $x, y \in I$, by Lemma (2.2) we get $\alpha(y) = -y$ for all $y \in I$. But N is a commutative ring, so $-xy = \alpha(xy) = \alpha(x)\alpha(y) = (-x)(-y) = xy$ for all $x, y \in I$. So by torsionless we get $2xy = 0 = xy$ for all $x, y \in I$. Hence $x^2 = 0$ for all $x \in I$, contradicting Lemma (2.5) . □

Lemma 2.12. *Let N be a 3-prime near-ring with multiplicative center $Z(N)$.*
If $\alpha : N \to N$ is an automorphism and d is a two-sided α-derivation on N, then $d(Z(N)) \subseteq Z(N)$.

Proof. Let $z \in Z(N)$, we have $d(zx) = d(xz)$, for all $x \in N$. By Lemma 2.6 and defining property of d, we have

$$d(zx) = d(z)x + \alpha(z)d(x) = d(xz) = xd(z) + d(x)\alpha(z) \qquad \text{for all } x \in N. \tag{3}$$

But α is an automorphism, $z \in Z(N)$, so $\alpha(z) \in Z(N)$. Thus Eq. (3) implies that $xd(z) = d(z)x$ for all $x \in N$. It follows that $d(z) \in Z(N)$, for all $z \in Z(N)$. Hence $d(Z(N)) \subseteq Z(N)$. □

Lemma 2.13. *Let N be a 3-prime near ring, I is a nonzero semigroup left ideal and $\alpha : N \to N$, be a function. If $y \in N$ satisfies $[x, y]_\alpha = 0$ for all $x \in I$, Then $y \in Z(N)$.*

Proof. Let $y \in N$, such that $[x, y]_\alpha = 0$ for all $x \in I$; then

$$x\alpha(y) = yx \qquad \text{for all } x \in I. \tag{4}$$

Replacing x by tx where $t \in N$, in Eq. (40) and use it to get $tx\alpha(y) = tyx = ytx$, for all $x \in I; t \in N$. So $(ty - yt)x = 0$ for all $x \in I$ and $t \in N$ and hence $y \in Z(N)$ by Lemma 2.2. □

3. Commutativity conditions involving two-sided α-derivations

In this section, N is assumed to be a zero-symmetric right near rings and $\alpha : N \to N$ be an automorphism.

Theorem 3.1. *Let N be a 3-prime near-ring. If I is a nonzero semigroup ideal and d is a nonzero two-sided α- derivation on N, then the following assertions are equivalent:*

- i $[x, y]_\alpha \in Z(N)$ *for all* $x, y \in I$;
- ii $[x, d(y)]_\alpha \in Z(N)$ *for all* $x, y \in I$;
- iii $d([x, y]) = 0$ *for all* $x, y \in I$;
- iv $d([x, y]_\alpha) = 0$ *for all* $x, y \in I$;
- v *N is a commutative ring.*

Proof. $(v) \Rightarrow (i)$, $(v) \Rightarrow (ii)$, $(v) \Rightarrow (iii)$, and $(v) \Rightarrow (iv)$ are obvious.

$(i) \Rightarrow (v)$: Assume that

$$[x, y]_\alpha \in Z(N) \qquad \text{for all } x, y \in I. \tag{5}$$

Replace x by $x\alpha(y)$ in Eq. (5) and noting that $[x\alpha(y), y]_\alpha = [x, y]_\alpha \alpha(y)$, we get

$$[x, y]_\alpha \alpha(y) \in Z(N) \qquad \text{for all } x, y \in I. \tag{6}$$

Now by Lemma 2.1(ii), we conclude that for each $y \in I$ either

$$[x, y]_\alpha = 0 \qquad \text{for all } x \in I, \qquad \text{or } \alpha(y) \in Z(N). \tag{7}$$

But α is an automorphism so Eq. (7) implies for each $y \in I$ either

$$[x, y]_\alpha = 0 \qquad \text{for all } x \in I, \qquad \text{or } y \in Z(N). \tag{8}$$

It follows by Lemma 2.13 $y \in Z(N)$ for all $y \in I$ i.e. $I \subseteq Z(N)$. Hence N is a commutative ring by Lemma 2.4.

$(ii) \Rightarrow (v)$: Assume that

$$[x, d(y)]_\alpha \in Z(N) \qquad \text{for all } x, y \in I. \tag{9}$$

Replace x by $x\alpha(d(y))$ in Eq. (9), we get

$$[x, d(y)]_\alpha \alpha(d(y)) \in Z(N) \qquad \text{for all } x, y \in I. \tag{10}$$

Now as above by Lemma 2.1(ii), we conclude that for each $y \in I$ either

$$[x, d(y)]_\alpha = 0 \qquad \text{for all } x \in I, \qquad \text{or } \alpha(d(y)) \in Z(N). \tag{11}$$

As above Eq. (11) implies that for each $y \in I$ either

$$[x, d(y)]_\alpha = 0 \qquad \text{for all } x \in I, \qquad \text{or } d(y) \in Z(N). \tag{12}$$

It follows by Lemma 2.13, $d(I) \subseteq Z(N)$. Hence N is a commutative ring by Lemma 2.10.

$(iii) \Rightarrow (v)$: In view of our hypothesis, we have

$$d([x, y]) = 0 \qquad \text{for all } x, y \in I. \tag{13}$$

Then, replacing y by yx in Eq. (13), we obtain

$$0 = d([x, yx]) = d([x, y]x) = \alpha([x, y])d(x) \qquad \text{for all } x, y \in I. \tag{14}$$

Since α is an automorphism, then

$$[x, y]\alpha^{-1}(d(x)) = 0 \qquad \text{for all } x, y \in I. \tag{15}$$

Again replace y by zy in Eq. (15) where $z \in N$ and use it to get

$$xzy\alpha^{-1}(d(x))) = zyx\alpha^{-1}(d(x)) = zxy\alpha^{-1}(d(x)) \qquad \text{for all } x, y \in I, z \in N. \tag{16}$$

This implies that

$$[x, z]I\alpha^{-1}(d(x))) = 0 \qquad \text{for all } x \in I, z \in N. \tag{17}$$

Then by Lemma 2.3, for each fixed $x \in I$ either $d(x) = 0$ or $[x, z] = 0$ for all $z \in N$. This means that for each fixed $x \in I$, we have either $d(x) = 0$ or $x \in Z(N)$. Clearly if $d(x) = 0$, then $d(x) \in Z(N)$. But by Lemma 2.12, we have $d(x) \in Z(N)$, for all $x \in Z(N)$. Therefore, in both cases we find that $d(x) \in Z(N)$, for all $x \in I$ and hence $d(I) \subseteq Z(N)$. It follows from Lemma 2.10, that N is a commutative ring.

$(iv) \Rightarrow (v)$: Assume that $d([x, y]_\alpha) = 0$ for all $x, y \in I$; since $[x\alpha(y), y]_\alpha = [x, y]_\alpha \alpha(y)$,for all $x, y \in I$. So $0 = d([x\alpha(y), y]_\alpha) = d([x, y]_\alpha \alpha(y)) = [x, y]d(\alpha(y))$ for all $x, y \in I$. Then as above $d(\alpha(I)) \subseteq Z(N)$ and we get N is a commutative ring by Lemma 2.10 . □

Corollary 3.2. *Let N be a 3-prime near-ring. If I is a nonzero semigroup ideal and d is a nonzero derivation on N, then the following assertions are equivalent:*
i $[x, d(y)] \in Z(N)$ *for all* $x, y \in I$
ii $[d(x), y] \in Z(N)$ *for all* $x, y \in I$

Proof.
$(i) \Rightarrow (ii)$: Assume that

$$[x, d(y)] \in Z(N) \qquad \text{for all } x, y \in I. \tag{18}$$

Then if we replace x and y by y and x, respectively, in Eq. (18), we get $[y, d(x)] \in Z(N)$ for all $x, y \in I$. So $-[d(x), y] = [y, d(x)] \in Z(N)$ for all $x, y \in I$. But Eq. (18) and Theorem 3.1 yield N is a commutative ring. Thus $[d(x), y] \in Z(N)$ for all $x, y \in I$.
Similarly, we can prove the other implication. □

So by Corollary 3.2 and if we put $\alpha = id_N$ in Theorem 3.1, we obtain the following corollary.

Corollary 3.3 ([5, Theorem 2.9]). *Let N be a 3-prime near-ring. If I is a nonzero semigroup ideal and d is a nonzero derivation on N, then the following assertions are equivalent:*
i $[x, y] \in Z(N)$ *for all* $x, y \in I$;
ii $[d(x), y] \in Z(N)$ *for all* $x, y \in I$;
iii *N is a commutative ring.*

if we put $\alpha = id_N$ in Theorem 3.1, we obtain the following corollary which is a generalization of ([2], Corollary 4.1).

Corollary 3.4. *Let N be a 3-prime near-ring. If I is a nonzero semigroup ideal and d is a nonzero derivation on N, then the following assertions are equivalent:*
i $d([x, y]) = 0$ *for all* $x, y \in I$;
ii *N is a commutative ring.*

If N is 2-torsion free the conclusion of Theorem 3.1 is true if we replace the product $[x, y]_\alpha$ with $(xoy)_\alpha$. In fact, we obtain the following results.

Theorem 3.5. *Let N be a 2-torsion free 3-prime near-ring. If I is a nonzero semigroup ideal and d is a nonzero two-sided α- derivation on N, then the following assertions are equivalent:*
i $(xoy)_\alpha \in Z(N)$ *for all* $x, y \in I$;
ii $(xod(y))_\alpha \in Z(N)$ *for all* $x, y \in I$;
iii $d(xoy) = 0$ *for all* $x, y \in I$;
iv $d((xoy)_\alpha) = 0$ *for all* $x, y \in I$;
v *N is a commutative ring.*

Proof. $(v) \Rightarrow (i)$, $(v) \Rightarrow (ii)$, $(v) \Rightarrow (iii)$ and $(v) \Rightarrow (iv)$ are obvious.

$(i) \Rightarrow (v)$: Assume that

$$(xoy)_\alpha \in Z(N) \qquad \text{for all } x, y \in I. \tag{19}$$

Replace x by $x\alpha(y)$ in Eq. (19) and noting that $(x\alpha(y)oy)_\alpha = (xoy)_\alpha\alpha(y)$, we get

$$(xoy)_\alpha\alpha(y) \in Z(N) \qquad \text{for all } x, y \in I. \tag{20}$$

Now by Lemma 2.1(ii) we conclude that either

$$(xoy)_\alpha = 0 \qquad \text{for all } x, y \in I, \qquad \text{or } \alpha(y) \in Z(N) \qquad \text{for all } y \in I. \tag{21}$$

But by Lemma 2.11 Eq. (21) implies

$$\alpha(y) \in Z(N) \qquad \text{for all } y \in I. \tag{22}$$

Hence it follows N is a commutative ring by Lemma 2.4.

$(ii) \Rightarrow (v)$:

$$(xod(y))_\alpha \in Z(N) \qquad \text{for all } x, y \in I. \tag{23}$$

Replace x by $x\alpha(d(y))$ in Eq. (23) we get

$$(xod(y))_\alpha \alpha(d(y)) \in Z(N) \qquad \text{for all } x, y \in I. \tag{24}$$

Now by Lemma 2.1(ii) we conclude that either

$$(xod(y))_\alpha = 0 \qquad \text{for all } x, y \in I, \qquad \text{or } \alpha(d(y)) \in Z(N) \qquad \text{for all } y \in I. \tag{25}$$

As above we get either

$$(xod(y))_\alpha = 0 \qquad \text{for all } x, y \in I, \qquad \text{or } d(y) \in Z(N) \qquad \text{for all } y \in I. \tag{26}$$

Clearly if $d(y) \in Z(N)$ for all $y \in I$, then N is a commutative ring by Lemma 2.10. But if $(xod(y))_\alpha = 0$ for all $x, y \in Z(N)$, then

$$x\alpha(d(y)) = -d(y)x \qquad \text{for all } x, y \in I. \tag{27}$$

Replacing x by zx, where $z \in N$ in Eq. (27) then $zx\alpha(d(y)) = -d(y)zx = z(-d(y)x)$ for all $x, y \in I, z \in N$. So $(d(-y)z)x = (zd(-y))x$ for all $x, y \in I, z \in N$. We have $(d(-y)z - zd(-y))x = 0$ for all $x, y \in I, z \in N$. Then by Lemma 2.2 we get $d(-y) \in Z(N)$ for all $y \in I$, and so $d(-I) \subseteq N$. Hence N is a commutative ring by Lemma 2.10.

$(iii) \Rightarrow (v)$: Assume that $d(xoy) = 0$ for all $x, y \in I$; since $xoyx = (xoy)x$,for all $x, y \in I$; so $0 = d(xoyx) = d((xoy)x) = \alpha(xoy)d(x)$ for all $x, y \in I$. So $(xoy)\alpha^{-1}(d(x)) = 0$ for all $x, y \in I$. Then

$$xy\alpha^{-1}(d(x)) = -yx\alpha^{-1}(d(x)) \qquad \text{for all } x, y \in I. \tag{28}$$

Replace y by zy where $z \in N$ in Eq. (28) and use it to obtain

$$xzy\alpha^{-1}(d(x)) = -zyx\alpha^{-1}(d(x)) = (-z)((-x)y)\alpha^{-1}(d(x)), \qquad \text{for all } x, y \in I, z \in N. \tag{29}$$

That is,

$$(xz - (-z)(-x))y\alpha^{-1}(d(x)) = 0 \qquad \text{for all } x, y \in I, z \in N. \tag{30}$$

Which leads to

$$(-(-x)z + z(-x))y\alpha^{-1}(d(x)) = 0 \qquad \text{for all } x, y \in I, z \in N. \tag{31}$$

And thus

$$(-(-x)z + z(-x))I\alpha^{-1}(d(x)) = 0 \qquad \text{for all } x \in I, z \in N. \tag{32}$$

Now, we get for each $x \in I$ either

$$-x \in Z(N) \qquad \text{or } d(x) = 0. \tag{33}$$

Thus by Lemma 2.12 we get $d(-I) \subseteq Z(N)$, and hence N is a commutative ring by Lemma 2.10.

$(iv) \Rightarrow (v)$: Assume that $d((xoy)_\alpha) = 0$ for all $x, y \in I$; since $(x\alpha(y)oy)_\alpha = (xoy)_\alpha \alpha(y)$, for all $x, y \in I$. So $0 = d((x\alpha(y)oy)_\alpha) = \alpha((xoy)_\alpha)d(\alpha(y))$ for all $x, y \in I$. But α is an automorphism, so $(xoy)\alpha^{-1}(d(\alpha(y))) = 0$ for all $x, y \in I$.. Then as above we can get $(-y)tx\alpha^{-1}(d(\alpha(y))) = t(-y)x\alpha^{-1}(d(\alpha(y)))$, for all $x, y \in I, t \in Z$. It follows that $[-y, t]I\alpha^{-1}d(\alpha(y)) = 0$, for all $y \in I, t \in N$.

So by Lemma 2.3, we get either $-y \in Z(N)$ or $d(\alpha(y)) = 0$ for all $y \in I$. Therefore by Lemma 2.12, we get $d(\alpha(-y)) \in Z(N)$ for all $y \in I$. Thus N is a commutative ring by Lemma 2.10 . □

Corollary 3.6. *Let N be a 3-prime near-ring. If I is a nonzero semigroup ideal and d is a nonzero derivation on N, then the following assertions are equivalent:*
i $xod(y) \in Z(N)$ *for all* $x, y \in I$
ii $d(x)oy \in Z(N)$ *for all* $x, y \in I$.

Proof.
$(i) \Rightarrow (ii)$: Assume that

$$(xod(y)) \in Z(N) \qquad \text{for all } x, y \in I. \tag{34}$$

Then if we replace x and y by y and x, respectively, in Eq. (34), we get $(yod(x)) \in Z(N)$ for all $x, y \in I$. So $-(d(x)oy) = (yod(x)) \in Z(N)$ for all $x, y \in I$. But Eq. (34) and Theorem 3.5 yield N is a commutative ring. Thus $(d(x)oy) \in Z(N)$ for all $x, y \in I$.
Similarly, we can prove the other implication. □

So by Corollary 3.6 and if we put $\alpha = id_N$ in Theorem 3.5 we obtain the following corollary.

Corollary 3.7 ([5, Theorem 2.10]). *Let N be a 3-prime near-ring. If I is a nonzero semigroup ideal and d is a nonzero derivation on N, then the following assertions are equivalent:*
i $xoy \in Z(N)$ *for all* $x, y \in I$;
ii $d(x)oy \in Z(N)$ *for all* $x, y \in I$;
iii *N is a commutative ring.*

If we put $\alpha = id_N$ in Theorem 3.5, we obtain the following corollary, which is a generalization of Corollary 4.1 of [2].

Corollary 3.8. *Let N be a 3-prime near-ring. If I is a nonzero semigroup ideal and d is a nonzero derivation on N, then the following assertions are equivalent:*
i $d(xoy) = 0$ *for all* $x, y \in I$;
ii *N is a commutative ring.*

The following example shows that the 3-primeness hypothesis in Theorems 3.1 and 3.5 cannot be omitted.

Example 3. Let S be a 2-torsion free zero-symmetric right near ring.
Let us define N, I and $d, \alpha, F : N \to N$ by:

$$N = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} : x, y \in S \right\},$$

$$I = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : x \in S \right\},$$

$$d\begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \alpha\begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ y & 0 & x \\ 0 & 0 & 0 \end{pmatrix}, \quad \text{and}$$

$$F\begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y \\ 0 & 0 & 0 \end{pmatrix}.$$

It is clear that N is a 2-torsion free non-3-prime right near-ring and I is a nonzero semigroup ideal . Moreover, d is a nonzero two-sided α-derivation of N and F is a nonzero two-sided α-generalized derivation of N satisfying the conditions

1. $[A,B]_\alpha \in Z(N)$,
2. $[d(A),B]_\alpha \in Z(N)$
3. $d[A,B] = 0$
4. $d[A,B]_\alpha = 0$
5. $(AoB)_\alpha \in Z(N)$
6. $(Aod(B))_\alpha \in Z(N)$
7. $d(AoB) = 0$
8. $d(AoB)_\alpha = 0$

for all $A, B \in I$, but N is not a commutative ring.

4. Commutativity conditions involving two-sided α-generalized derivations

In this section, N is assumed to be a zero symmetric right near ring and $\alpha : N \to N$ be an automorphism.

Theorem 4.1. *Let N be a 3-prime near-ring and I is a nonzero semigroup ideal. If N admits a two-sided α-generalized derivation F associated with a nonzero α- derivation d, such that $F([x,y]) = [d(x),y]$ for all $x,y \in I$; or $F([x,y]) = [d(x),\alpha(y)]$ for all $x,y \in I$; then N is a commutative ring.*

Proof. Assume that

$$F([x,y]) = [d(x),y] \qquad \text{for all } x,y \in I. \tag{35}$$

Replacing y by yx in Eq. (35), we get

$$[d(x),yx] = F[x,yx] = F([x,y]x) \qquad \text{for all } x,y \in I. \tag{36}$$

Moreover, since $[d(x),x] = F([x,x]) = 0$ for all $x \in I$. so $xd(x) = d(x)x$ for all $x \in I$. Now by $xd(x) = d(x)x$ for all $x \in I$, we can prove $[d(x),yx] = [d(x),y]x$ for all $x,y \in I$

$$\begin{aligned} [d(x),yx] &= d(x)yx - yxd(x) \\ &= d(x)yx - yd(x)x, \text{ since } \ xd(x) = d(x)x \\ &= (d(x)y - yd(x))x \\ &= [d(x),y]x \end{aligned}$$

for all $x,y \in I$. From Eqs. (35), (36) and $[d(x),yx] = [d(x),y]x$ for all $x,y \in I$, we get $F([x,y])x = F([x,y]x) = F([x,y])x + \alpha([x,y])d(x)$, for all $x,y \in I$. So that

$$[x,y]\alpha^{-1}(d(x)) = 0 \qquad \text{for all } x,y \in I. \tag{37}$$

Substituting zy for y in Eq. (37), where $z \in N$, and use it to get $zyx\alpha^{-1}(d(x)) = zxy\alpha^{-1}(d(x)) = xzy\alpha^{-1}(d(x))$, for all $x,y \in I, z \in N$. So $[x,z]I\alpha^{-1}(d(x)) = 0$ for all $x \in I, z \in N$. It follows that for each $x \in I$ either

$$x \in Z(N) \qquad \text{or } d(x) = 0. \tag{38}$$

Clearly if $d(x) = 0$, then $d(x) \in Z(N)$. But if $x \in I \cap Z(N)$, then Eq. (35) implies $d(x)$ and centralizes I. So by Lemma 2.1(iii), $d(x) \in Z(N)$, for all $x \in I \cap Z(N)$. Thus in both cases Eq. (38) yields $d(I) \subseteq Z(N)$,and hence N is a commutative ring by Lemma 2.10. Similarly, we can prove the result for the case $F([x,y]) = [d(x),\alpha(y)]$ for all $x,y \in I$. This proves the theorem. □

Take $F = d$ in Theorem 4.1, we obtain the following corollary

Corollary 4.2. *Let N be a 3-prime near-ring and I is a nonzero semigroup ideal. If N admits a two-sided α-derivation d, such that $d([x,y]) = [d(x),y]$ for all $x,y \in I$, or $d([x,y]) = [d(x),\alpha(y)]$ for all $x,y \in I$; then N is a commutative ring.*

If we put $\alpha = id_N$ in Theorem 4.1, we obtain the following corollary.

Corollary 4.3 ([5, Theorem 3.1])**.** *Let N be a 3-prime near-ring and I is a nonzero semigroup ideal. If N admits a generalized derivation F associated with a nonzero derivation d, such that $F([x,y]) = [d(x),y]$ for all $x,y \in I$; then N is a commutative ring.*

Put $\alpha = id_N$, and $F = d$ in Theorem 4.1 we obtain the following corollary

Corollary 4.4. *Let N be a 3-prime near-ring and I is a nonzero semigroup ideal. If N admits a non zero derivation d, such that $d([x,y]) = [d(x),y]$ for all $x,y \in I$; then N is a commutative ring.*

Theorem 4.5. *Let N be a 3-prime near-ring and I is a nonzero semigroup ideal. If N admits a two-sided α-generalized derivation F associated with a nonzero two-sided α- derivation d, such that $d([x,y]) = [F(x),y]$ for all $x,y \in I$; or $d([x,y]) = [F(x),\alpha(y)]$ for all $x,y \in I$; then N is a commutative ring.*

Proof. Assume that

$$F([x,y]) = [d(x),y] \qquad \text{for all } x,y \in I. \tag{39}$$

Replacing y by yx in Eq. (39) , we get

$$[d(x),yx] = F([x,yx]) = F([x,y]x) \qquad \text{for all } x,y \in I. \tag{40}$$

Moreover, since $[d(x),x] = F([x,x]) = 0$ for all $x \in I$. so $d(x)x = xd(x)$ for all $x \in I$. From Eqs. (36) and (39), we get $F([x,y])x = F([x,y]x) = F([x,y])x + \alpha([x,y])d(x)$, for all $x,y \in I$. So that

$$[x,y]\alpha^{-1}(d(x)) = 0 \qquad \text{for all } x,y \in I. \tag{41}$$

Substituting zy for y in Eq. (41), where $z \in N$, and use it to get $zyx\alpha^{-1}(d(x)) = zxy\alpha^{-1}(d(x)) = xzy\alpha^{-1}(d(x))$, for all $x,y \in I$, $z \in N$. So $[x,z]I\alpha^{-1}(d(x)) = 0$ for all $x \in I, z \in N$. So again by Lemma 2.3 we obtain

$$x \in Z(N) \qquad \text{or } d(x) = 0, \qquad \text{for all } x \in Z(N). \tag{42}$$

Clearly if $d(x) = 0$, then $d(x) \in Z(N)$. As above if $x \in I \cap Z(N)$, then $F(x) \in Z(N)$, and $F(x^2) \in Z(N)$. So $F(x^2) = F(x)\alpha(x) + xd(x) \in Z(N)$ for all $x \in I \cap Z(N)$. But $\alpha(x)$, $F(x)$ and $F(x^2)$ are in $Z(N)$ for all $x \in I \cap Z(N)$. Thus by Lemma 2.9 $xd(x) \in Z(N)$ for all $x \in I \cap Z(N)$, and so if $I \cap Z(N) \neq \{0\}$, then $d(x) \in Z(N)$ for all $x \in I \cap Z(N)$, by Lemma 2.1(ii). If $I \cap Z(N) = \{0\}$, then Eq. (42) gives $d(x) = 0$, for all $x \in I$. Hence in both cases we have $d(I) \subseteq Z(N)$ and N is a commutative ring by Lemma 2.10.

Similarly, we can prove the result for the case $d([x,y]) = [F(x),\alpha(y)]$ for all $x,y \in I$. This proves the theorem. □

Put $\alpha = id_N$ in Theorem 4.5 we obtain the following corollary.

Corollary 4.6 ([5, Theorem 3.2])**.** *Let N be a 3-prime near-ring and I is a nonzero semigroup ideal. If N admits a generalized derivation F associated with a nonzero derivation d, such that $d([x,y]) = [F(x),y]$ for all $x,y \in I$; then N is a commutative ring.*

We now study analogous conditions involving anticommutators xoy.

Theorem 4.7. *Let N be a 2-torsion free 3-prime near-ring and I a nonzero semigroup ideal. Then N admits no two-sided α-generalized derivation F with associated two-sided α- derivation d, such that* $d(Z(N)) \neq \{0\}$ *and* $d(xoy) = F(x)oy$.

Proof. Assume that

$$d(xoy) = F(x)oy \qquad \text{for all } x, y \in I. \tag{43}$$

Let $z \in Z(N)$ such that $d(z) \neq 0$. Now we prove $F(x)oyz = (F(x)oy)z$ for all $x, y \in I$, and $z \in Z(N)$

$$\begin{aligned} F(x)oyz &= F(x)yz + yzF(x) \\ &= F(x)yz + yF(x)z, \text{ since } z \in Z(N) \\ &= (F(x)oy)z \end{aligned}$$

for all $x, y \in I$ and $z \in Z(N)$. Also we prove $d(xoyz) = d((xoy)z)$ for all $x, y \in I$, and $z \in Z(N)$

$$\begin{aligned} d(xoyz) &= d(xyz + yzx) \\ &= d(xyz + yxz), \text{ since } z \in Z(N) \\ &= d((xy + yx)z) \\ &= d((xoy)z) \end{aligned}$$

for all $x, y \in I$ and $z \in Z(N)$. Replace y by yz in Eq. (43), so we obtain

$$(F(x)oy)z = d((xoy)z) \qquad \text{for all } x, y \in I. \tag{44}$$

So we get $d((xoy)z) = d((xoy)z) = d((xoy)z + \alpha(xoy)d(z)$, for all $x, y \in I$.

So that $\alpha(xoy)d(z) = 0$, for all $x, y \in I$ but $d(z) \in Z(N)/\{0\}$, then $xoy = 0$ for all $x, y \in I$. So by torsionless this contradicts Lemma 2.5. □

Put $\alpha = id_N$ in Theorem 4.5 we obtain the following corollary

Corollary 4.8 ([5, Theorem 3.3])**.** *Let N be a 2-torsion free 3-prime near-ring and I nonzero semigroup ideal. Then N admits no generalized derivation F with associated derivation d, such that either* $d(Z(N)) \neq \{0\}$ *and* $d(xoy) = F(x)oy$.

Theorem 4.9. *Let N be a 2-torsion free 3-prime near-ring and I a nonzero semigroup ideal. Then N admits no two-sided α-generalized derivation F with associated two-sided α- derivation d, such that* $d(Z(N)) \neq \{0\}$ *and* $d(xoy) = F(x)o\alpha(y)$.

Proof. Assume that

$$d(xoy) = F(x)o\alpha(y) \qquad \text{for all } x, y \in I. \tag{45}$$

Let $z \in Z(N)$ such that $d(z) \neq, 0$. Hence

$$\begin{aligned} d((xoy)z) &= d((xy + yx)z) \\ &= d(xyz + yxz) \\ &= d(xyz + yzx) \\ &= d(xoyz), \end{aligned}$$

for all $x, y \in, I$ and $z \in Z(N)$. But $z \in Z(N)$, so $\alpha(z) \in Z(N)$, since α is an automorphism, hence by using Eq. (45), we get

$$\begin{aligned} d((xoy)z) &= F(x)o\alpha(yz) \\ &= F(x)o(\alpha(y)\alpha(z)) \end{aligned}$$

$$= F(x)\alpha(y)\alpha(z) + \alpha(y)\alpha(z)F(x)$$
$$= (F(x)\alpha(y) + \alpha(y)F(x))\alpha(z)$$
$$= F(x)\alpha(y)\alpha(z) + \alpha(y)F(x)\alpha(z)$$
$$= (F(x)o\alpha(y))\alpha(z))$$
$$= (d(x)oy)\alpha(z)),$$

for all $x, y \in I$ and $z \in Z(N)$. So $d(xoyz) = d((xoy)z) = d((xoy)\alpha(z) + (xoy)d(z)$, for all $x, y \in I$. for all $x, y \in I$ but $d(z) \in Z(N)/\{0\}$, then $xoy = 0$ for all $x, y \in I$. So by tensionless this contradicts Lemma 2.5 . □

Theorem 4.10. *Let N be a 2-torsion free 3-prime near-ring and I a nonzero semigroup ideal. Then there exists no nonzero two-sided α-generalized derivation F with associated α-derivation d,such that $[d(x), x] = 0$ for all $x \in I$ and $d(x)oy = F(xoy)$ for all $x, y \in I$.*

Proof. Assume that

$$d(x)oy = F(xoy) \qquad \text{for all } x, y \in I, \tag{46}$$

and

$$[d(x), x] = 0, \qquad \text{for all } x \in I \tag{47}$$

Replacing y by yx in Eq. (46) and use Eq. (47) we get,

$$\begin{aligned} d(x)oyx &= F(xoyx) \\ d(x)yx + yxd(x) &= F(xyx + yxx) \\ d(x)yx + yd(x)x &= F((xy + yx)x) \\ (d(x)y + yd(x))x &= F((xoy)x) \\ (d(x)oy)x &= F((xoy)x) \\ F((xoy))x &= F((xoy))x + \alpha(xoy)d(x), \end{aligned}$$

for all $x, y \in I$. Hence $\alpha(xoy)d(x) = 0$, for all $x, y \in, I$. So

$$yx\alpha^{-1}(d(x)) = -xy\alpha^{-1}d(x) \qquad \text{for all } x, y \in I. \tag{48}$$

Substituting zy for y, in Eq. (48) and use it to get

$$zyx\alpha^{-1}(d(x)) = z(-xy\alpha^{-1}(d(x))) = (-x)zy\alpha^{-1}(d(x))) \qquad \text{for all } x, y \in I, z \in N. \tag{49}$$

This equation can be written as $((-x)z - z(-x))I\alpha^{-1}(d(x)) = 0$, for all $x \in I$, and by Lemma 2.3, we get

$$-x \in Z(N) \qquad \text{or } d(x) = 0 \qquad \text{for all } x \in I. \tag{50}$$

Now if $d(Z(N)) \neq 0$, so, as in the proof of Theorem 4.7, we can show that $xoy = 0$ for all $x, y \in I$, by tensionless contradicting Lemma 2.5. Thus if $d(Z(N)) = 0$, $d(-x) = 0 = d(x)$ for all $x \in I$, gives $d(I) = 0$, which contradicts Lemma 2.8. □

Corollary 4.11. *Let N be a 2-torsion free 3-prime near-ring and I a nonzero semigroup ideal. Then N admits no two-sided α- derivation d, such that $[d(x), x] = 0$ for all $x \in I$, and $d(xoy) = d(x)oy$ for all $x, y \in I$.*

Notice that the condition $[d(x), x] = 0$ for all $x \in I$, that appears in Theorem 4.10 will obtain from the condition $d(x)oy = F(xoy)$, for all $x, y \in I$, that appears in the same theorem when $\alpha = id_N$. This can be proved in the following Lemma

Lemma 4.12. *Let N be a 2-torsion free 3-prime near-ring and I a nonzero semigroup ideal. If N admits a generalized derivation F with associated derivation d, such that $d(x)oy = F(xoy)$ for all $x, y \in I$. then $[d(x), x] = 0$ for all $x \in I$, and*

Proof. Assume that

$$d(x)oy = F(xoy), \qquad \text{for all } x, y \in I. \tag{51}$$

If we replace y by yx in Eq. (51), we obtain

$$d(x)oyx = F(xoyx) \qquad \text{for all } x, y \in I. \tag{52}$$

So

$$\begin{aligned}
d(x)oyx &= F(xoyx)\\
d(x)yx + yxd(x) &= F(xyx + yxx)\\
d(x)yx + yxd(x) &= F((xy + yx)x)\\
d(x)yx + yxd(x) &= F((xoy)x)\\
d(x)yx + yxd(x) &= F(xoy)x + (xoy)d(x)\\
d(x)yx + yxd(x) &= (d(x)oy)x + (xoy)d(x)\\
d(x)yx + yxd(x) &= (d(x)y + yd(x))x + (xy + yx)d(x)\\
d(x)yx + yxd(x) &= (d(x)yx + yd(x)x + xyd(x) + yxd(x),
\end{aligned}$$

for all $x, y \in I$. Hence we get

$$yd(x)x = -xyd(x) \qquad \text{for all } x, y \in I. \tag{53}$$

So replace y by ty in Eq. (53) where $t \in N$ and use it to get

$$tyd(x)x = t(yd(x)x) = t(-xyd(x)) = t(-x)yd(x) = -x(ty)d(x) = (-x)tyd(x)$$
$$\text{for all } x, y \in I, t \in N. \tag{54}$$

So we have $t(-x)yd(x) = (-x)tyd(x)$, for all $x, y \in I$ and $t \in N$. Hence

$$[-x, t]yd(x) = 0, \qquad \text{for all } x, y \in I, t \in N. \tag{55}$$

But N is 3-prime near ring, so we get

$$-x \in Z(N) \qquad \text{or } d(x) = 0 \qquad \text{for all } x \in I. \tag{56}$$

Now if $d(Z(N)) \neq 0$, so, as in the proof of Theorem 4.7, we can show that $xoy = 0$ for all $x, y \in I$, by tensionless contradicting Lemma 2.5 . So $d(Z(N)) = 0$, and in both cases Eq. (56) yields $d(x) = 0$, for all $x \in I$. Hence $[d(x), x] = 0$ for all $x \in I$. □

So if we put $\alpha = id_N$, in Theorem (4.10), we get the following corollary.

Corollary 4.13 ([5, Theorem 4.5]). *Let N be a 2-torsion free 3-prime near-ring and I a nonzero semigroup ideal. Then there exists no nonzero generalized derivation F associated with non zero derivation d,such that $d(x)oy = F(xoy)$ for all $x, y \in I$.*

Put $\alpha = id_N$ in Corollary 4.11 we obtain the following corollary

Corollary 4.14. *Let N be a 2-torsion free 3-prime near-ring and I a nonzero semigroup ideal. Then N admits no derivation d, such that $d(xoy) = d(x)oy$.*

Theorem 4.15. *Let N be a 2-torsion free 3-prime near-ring and I a nonzero semigroup ideal. Then there exists no nonzero two-sided α-generalized derivation F with associated α -derivation d,such that $[d(x), \alpha(x)] = 0$ for all $x \in I$ and $d(x)o\alpha(y) = F(xoy)$ for all $x, y \in I$.*

Proof. Assume that

$$d(x)o\alpha(y) = F(xoy) \qquad \text{for all } x, y \in I. \tag{57}$$

and

$$[d(x), \alpha(x)] = 0, \qquad \text{for all } x \in I. \tag{58}$$

Replacing y by yx in Eq. (57), we get

$$d(x)o\alpha(yx) = F(xoyx) = F(xyx + yxx) = F((xy + yx)x) = F((xoy)x) \qquad \text{for all } x, y \in I. \tag{59}$$

Since $d(x)o\alpha(yx) = F((xoy)x) = F((xoy))\alpha(x) + (xoy)d(x)$, for all $x, y \in I$. So by (57), we get $d(x)o\alpha(yx) = (d(x)o\alpha(y))\alpha(x) + (xoy)d(x)$, for all $x, y \in I$. So $d(x)\alpha(yx) + \alpha(yx)d(x) = (d(x)\alpha(y) + \alpha(y)d(x))\alpha(x) + (xoy)d(x)$, for all $x, y \in, I$. So $d(x)\alpha(yx) + \alpha(yx)d(x) = (d(x)\alpha(y)\alpha(x) + \alpha(y)d(x)\alpha(x) + (xoy)d(x)$, for all $x, y \in, I$. But from Eq. (58), we get $d(x)\alpha(yx) + \alpha(yx)d(x) = (d(x)\alpha(y)\alpha(x) + \alpha(y)\alpha(x)d(x) + (xoy)d(x)$, for all $x, y \in, I$. So $(xoy)d(x) = 0$, for all $x, y \in I$. Then using similar approach as we have used in the last paragraph of the proof of Theorem 4.10, we get the required result. □

If we put $F = d$ in Theorem 4.15, we obtain the following Corollary

Corollary 4.16. *Let N be a 2-torsion free 3-prime near-ring and I a nonzero semigroup ideal. Then N admits no two-sided α- derivation d, such that $[d(x), \alpha(x)] = 0$, for all $x \in I$ and $d(xoy) = d(x)o\alpha(y)$ for all $x, y \in I$.*

The following example shows that the 3-primeness hypothesis in Theorems 4.1, 4.5, 4.7, 4.9,4.10 and 4.15 cannot be omitted.

Example 4. Let S be a 2-torsion free zero-symmetric right near ring.

Let us define N, I and $d, \alpha, F : N \to N$ by:

$$N = \left\{ \begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & y & 0 \end{pmatrix} : x, y \in S \right\},$$

$$I = \left\{ \begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : x \in S \right\},$$

$$d\begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & y & 0 \end{pmatrix} = \begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \alpha\begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & y & 0 \end{pmatrix} = \begin{pmatrix} 0 & y & 0 \\ 0 & 0 & 0 \\ 0 & x & 0 \end{pmatrix}, \quad \text{and}$$

$$F\begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & y & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & y & 0 \end{pmatrix}.$$

It is clear that N is a 2-torsion non-3-prime right near-ring and I is a nonzero semigroup ideal. Moreover, d is a nonzero two-sided α derivation of N and F is a nonzero two-sided α-generalized derivation of N satisfying the conditions

1. $F[A, B] = [d(A), B]$

2. $d[A, B] = [F(A), B]$
3. $F[A, B] = [d(A), \alpha(B)]$
4. $d[A, B] = [F(A), \alpha(B)]$
5. $d(Z(N)) \neq \{0\}$
6. $F(AoB) = d(A)oB$
7. $d(AoB) = F(A)oB$
8. $F(AoB) = d(A)o\alpha(B)$
9. $d(AoB) = F(A)o\alpha(B)$
10. [d(A), A]=0

for all $A, B \in I$, but N is not a commutative ring.

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References

[1] Argac, N. (2004). On near-rings with two-sided α-derivations. *Turk. J. Math.* 28:195–204.
[2] Ashraf, M., Ali, S. (2008). On (σ, τ)-derivation of prime near-rings-ii. *Sarajevo J. Math.* 4(16):23–30.
[3] Ashraf, M., Ali, A., Ali, S. (2004). (σ, τ)-derivations on prime near rings. *Arch. Math.* 40(3):281–286.
[4] Bell, H. E. (1997). On derivations in near-rings. Vol. II. In: Saad, G., Thomsen, M. J., eds. *Nearrings, Nearfields and K-loops* (Hamburg, 1995). Math. Appl., Vol. 426. Dordrecht: Kluwer Academic Publications, pp. 191–197.
[5] Bell, H. E., Boua, A., Oukhtite, L. (2015). Semigroup ideals and commutativity in 3-prime near rings. *Commun. Algebra* 43:1757–1770.
[6] Boua, A., Kamal, A. A. M. (2014). Structure of near-rings satisfying certain polynomial identities. *Int. J. Pure Appl. Math.* 95(4):597–610.
[7] Golbasi, O. (2006). Notes on prime near-rings with generalized derivation. *Southeast Asian Bull. Math.* 30:49–54.
[8] Golbasi, O., Aydin, N. (2007). On near-ring ideals with (σ, τ)-derivation. *Arch. Math.* 43:87–92.