Outline of the course

- Introduction to Simulation.
- Hand Simulation.
- Review of basic Probability Theory.
- Random Number Generation
- Generation of Random Varieties.
- Analysis of Output.
- Elementary Queuing Models

Modeling and Simulation

Model

- It is a simplification of the reality
- A (usually miniature) representation of an actual system; an example for imitation or emulation
- A description of observed behavior, simplified by ignoring certain details.
- Models allow complex systems to be understood and their behavior predicted within the scope of the model
- A model can be **Analytical** (Queuing Theory) or by Simulation.

Queuing Theory

- Queuing is the study of waiting lines, or queues.
- The objective of queuing analysis is to design systems that enable organizations to perform optimally according to some criterion.
- That requires a clear understanding of the appropriate service measurement.
- Possible service measurements
  - Average time a customer spends in line.
  - Average length of the waiting line.
  - The probability that an arriving customer must wait for service.

Applications of Queuing Theory

- Telecommunications
- Computer Networks
- Predicting computer performance
- Health services (eg. control of hospital bed assignments)
- Airport traffic, airline ticket sales
- Layout of manufacturing systems.

Discrete-Event Simulation

- Discrete Event System
  - a system whose *state* changes at discrete points in time due to the occurrence of asynchronous *events*
- Example: M/M/1 queuing system
  - State
    - number of customers in system
  - Events
    - customer arrival
    - customer departure

Queuing System

- A Queuing system is determined by three basic components
  - Arrival characteristics: Customers arrive according to some arrival pattern.
  - Service facility characteristics: Customers receive service and leave the system.
  - Queue characteristics: Arriving customers may have to wait in one or more queues for service.

Arrival Characteristics

- Size of the arrival population – either infinite or limited
- Arrival distribution:
  - Either fixed or random, The random process is more common in businesses.
  - measured by time between consecutive arrivals, or arrival rate
  - The **Poisson distribution** is often used for random arrivals

Arrival Characteristics

- Under three conditions the arrivals can be modeled as a Poisson process
  - ***Orderliness*** : one customer, at most, will arrive during a predefined time interval.
  - ***Stationary*** : for a given time frame, the probability of arrivals within a certain time interval is the same for all time intervals of equal length.
  - ***Independence*** : the arrival of one customer has no influence on the arrival of another.

Service facility characteristics

1. Configuration of service facility
   - Number of servers
2. Service distribution
   - The time it takes to serve 1 arrival
   - Can be fixed or random
   - Exponential distribution is often used

Queue Configuration

- A single service queue.
- Multiple service queue with single waiting line.
- Multiple service queue with multiple waiting lines.
- Multistage service system.

Queue Characteristics

- Queue length (max possible queue length) – either limited or unlimited
- There are several commonly used rules:
  - First come first served (FCFS - FIFO).
  - Last come first served (LCFS - LIFO).
  - Estimated service time.
  - Random selection of customers for service.

The Waiting Line Characteristics

- Factors that influence the modeling of queues
  - Line configuration
  - Balking
  - Reneging
  - Jockeying

Balking, Reneging, Jockeying

- Balking occurs if customers avoid joining the line when they perceive the line to be too long
- Reneging occurs when customers abandon the waiting line before getting served
- Jockeying (switching) occurs when customers switch lines once they perceived that another line is moving faster

M/M/1 System

- M stands for "Memoryless" (a property of the exponential distribution)
  - M/M/1 stands for Poisson arrival process (which is memoryless)
  - M/M/1 stands for exponentially distributed transmission times
  - M/M/1 stands for one server

- "M/M/1" is a special case of more general (Kendall notation)
- Example of packet :
  - Arrival process is Poisson with rate l
  - Processing times are exponentially distributed with mean 1/m
  - One server
  - Independent interarrival times and processing times

The Exponential Distribution - Characteristics

- The memoryless property (Markov)
  - No additional information about the time left for the completion of a service, is gained by recording the time elapsed since the service started.
  - $P(T \geq s+t \,/\, T \geq s) = P(T \geq t)$
- The Exponential and the Poisson distributions are related to one another.
  - If customer arrivals follow a **Poisson distribution with mean rate λ,** their interarrival times are **exponentially distributed with mean time $1/\lambda$.**

Kendall Notation

A/B/m/K/N/X

Arrival Process
- **M**: Markovian
- **D**: Deterministic
- **Er**: Erlang
- **G**: General

Service Process
- **M**: Markovian
- **D**: Deterministic
- **Er**: Erlang
- **G**: General

Number of servers
m=1,2,…

Service Discipline
FIFO, LIFO, Round Robin, …

Storage Capacity
K= 1,2,…
(if ∞ then it is omitted)

Number of customers
N= 1,2,…
(for **closed networks**, otherwise it is omitted)

Kendall Notation

- M/M/1 Queue Poisson arrivals (*exponential inter-arrival*), and exponential service, 1 server, infinite capacity and population, FCFS (FIFO). The most basic and important queuing model -Was the subject of our course
- M/M/m Queue *Same, but m servers*
- M/M/m/k Queue system Same as M/M/m, but there is buffer space for at most k packets. Packets arriving at a full buffer are dropped, k is omitted when k = ∞
- M/D/1 Queue Poisson arrivals and <u>CONSTANT</u> service times, 1 server, infinite capacity and population, FIFO.

Name (Kendall Notation)	Example
Simple system (M / M / 1)	Customer service desk in a store
Multiple server (M / M / s)	Airline ticket counter
Constant service (M / D / 1)	Automated car wash
General service (M / G / 1)	Auto repair shop
General service G/G/3/20/1500/SPF	3 servers, 17 queues (20-3), 1500 total jobs, Shortest Packet First