Quiz 5

- A- When is the point estimate at the center of the CI (confidence interval)?
- (a) It is never at the center of the CI.
- (b) It is at the center of the CI when you estimate μ but not at the center of the
- CI when you estimate σ.
- (c) It is at the center of the CI when you estimate σ but not at the center of the
- CI when you estimate μ.
- (d) It is at the center of the CI when you estimate μ and when you estimate σ.

- B- Select the correct interpretation of the term 99% confidence interval.
- (a) It is an interval containing 99% of the data.
- (b) It means that the population parameter will be in our interval with probability 99% and outside of that interval with probability 1%.
- (c) It means that if we repeat the procedure many times, approximately 99% of the intervals so constructed will contain the true population parameter.
- (d) It is an interval over which the area under the density curve is 0.99.

- **Q2:** A researcher wants to estimate the mean great point average of all college students in the USA. How many great point averages does she need to obtain so that the sample mean is within 0.07 from the population mean? Assume that a 95% confidence is desired and the population standard deviation is 0.84.

$$n = (Z_{\alpha/2}\,\sigma / E)\text{^}2 = [1.9684 / .07]\text{^}2 = 553.2$$

$$n = 554$$

- A random sample of the weights of 18 green M&Ms has a mean of 0.86 g and a standard deviation of 0.04 g. Assume that the population has normal distribution.

(a) Give an appropriate point estimate for the standard deviation of weights of all M&Ms.

s= .04

(b) Construct an 80% confidence interval estimate of the standard deviation of weights of all M&Ms. (Round off to three decimal places.) Name the distribution that you use. Give the number of degrees of freedom.

n-1 = 17 df $\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$

Alpha = 1- .08=.2 → $\chi_L^2 = \chi_{90}^2$ *=10.086 and* $\chi_R^2 = \chi_{10}^2$

sqroot(17*.04*.04/24.769)<sigma<sqroot(17*.04*.04/10.086)

.033<sigma <.o52,

area	.995	.990	.975	.950	.900	.750	.500	.250	.100	.050	.025	.010	.005
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.10153	0.45494	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	0.01003	0.02010	0.05064	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146	7.37776	9.21034	10.59663
3	0.07172	0.11483	0.21580	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473	9.34840	11.34487	12.83816
4	0.20699	0.29711	0.48442	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773	11.14329	13.27670	14.86026
5	0.41174	0.55430	0.83121	1.14548	1.61031	2.67460	4.35146	6.62568	9.23636	11.07050	12.83250	15.08627	16.74960
6	0.67573	0.87209	1.23734	1.63538	2.20413	3.45460	5.34812	7.84080	10.64464	12.59159	14.44938	16.81189	18.54758
7	0.98926	1.23904	1.68987	2.16735	2.83311	4.25485	6.34581	9.03715	12.01704	14.06714	16.01276	18.47531	20.27774
8	1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	10.21885	13.36157	15.50731	17.53455	20.09024	21.95495
9	1.73493	2.08790	2.70039	3.32511	4.16816	5.89883	8.34283	11.38875	14.68366	16.91898	19.02277	21.66599	23.58935
10	2.15586	2.55821	3.24697	3.94030	4.86518	6.73720	9.34182	12.54886	15.98718	18.30704	20.48318	23.20925	25.18818
11	2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.34100	13.70069	17.27501	19.67514	21.92005	24.72497	26.75685
12	3.07382	3.57057	4.40379	5.22603	6.30380	8.43842	11.34032	14.84540	18.54935	21.02607	23.33666	26.21697	28.29952
13	3.56503	4.10692	5.00875	5.89186	7.04150	9.29907	12.33976	15.98391	19.81193	22.36203	24.73560	27.68825	29.81947
14	4.07467	4.66043	5.62873	6.57063	7.78953	10.16531	13.33927	17.11693	21.06414	23.68479	26.11895	29.14124	31.31935
15	4.60092	5.22935	6.26214	7.26094	8.54676	11.03654	14.33886	18.24509	22.30713	24.99579	27.48839	30.57791	32.80132
16	5.14221	5.81221	6.90766	7.96165	9.31224	11.91222	15.33850	19.36886	23.54183	26.29623	28.84535	31.99993	34.26719
17	5.69722	6.40776	7.56419	8.67176	10.08519	12.79193	16.33818	20.48868	24.76904	27.58711	30.19101	33.40866	35.71847

Components of a Formal Hypothesis Test

Null Hypothesis: H_0

- The null hypothesis (denoted by H_0) is a statement that the value of a population parameter (such as *proportion, mean*, or *standard deviation*) is equal to some claimed value.
- We test the null hypothesis directly.
- Either reject H_0 or fail to reject H_0 (in other words, accept H_0).

Alternative Hypothesis: H_1

- The alternative hypothesis (denoted by H_1) is the statement that the parameter has a value that somehow differs from the null hypothesis.
- The symbolic form of the alternative hypothesis must use one of these symbols: $\neq$, $<$, $>$.
  (not equal, less than, greater than)

General rules:

- If the null hypothesis is <u>rejected</u>, the alternative hypothesis is <u>accepted</u>.
- If the null hypothesis is <u>accepted</u>, the alternative hypothesis is <u>rejected</u>.
- Acceptance or rejection of the null hypothesis is an **initial conclusion**.
- Always state the **final conclusion** expressed in terms of the **original claim**, not in terms of the null hypothesis or the alternative hypothesis.

Type I and Type II Errors

		True State of Nature	
		The null hypothesis is true	**The null hypothesis is false**
Decision	We decide to reject the null hypothesis	**Type I error** (rejecting a true null hypothesis) $P(\text{type I error}) = \alpha$	Correct decision
	We fail to reject the null hypothesis	Correct decision	**Type II error** (failing to reject a false null hypothesis) $P(\text{type II error}) = \beta$

Requirements for Testing Claims About a Population Proportion p

1) The sample observations are a simple random sample.

2) The conditions for a binomial distribution are satisfied.

3) The conditions $np \geq 5$ and $nq \geq 5$ are both satisfied, so the binomial distribution of sample proportions can be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$. Note: p is the assumed proportion not the sample proportion.

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Test Statistic for Testing a Claim About a Proportion

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Note: p is the value specified in the null hypothesis; $q = 1 - p$

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Critical Region

The critical region (or rejection region) is the set of all values of the test statistic that cause us to reject the null hypothesis.

For example, see the red-shaded region in the previous figure.

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Critical Value

A critical value is a value that separates the critical region (where we reject the null hypothesis) from the values of the test statistic that do not lead to rejection of the null hypothesis.

See the previous figure where the critical value is $z = 1.645$. It corresponds to a significance level of $\alpha = 0.05$.

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Significance Level

The significance level (denoted by α) is the probability that the test statistic will fall in the critical region (when the null hypothesis is actually true).

Compute the test statistic:

$$\hat{p} = 13/14 = 0.929$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.929 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{14}}} = 3.21$$

Conclusion of the test

Since the test statistic ($z=3.21$) falls in the critical region ($z>1.645$), we **reject the null hypothesis**.

Final conclusion: the original claim is accepted, the XSORT method of gender selection indeed increases the likelihood of having a baby girl.

P-Value method:

If *P*-value $\leq \alpha$, **reject H_0**.

If *P*-value $> \alpha$, **fail to reject H_0**.

If the P is low, the null must go.
If the P is high, the null will fly.

Traditional method:

If the test statistic falls within the critical region, reject H_0.

If the test statistic does not fall within the critical region, fail to reject H_0.

P-Value method:

If *P*-value is small ($\le \alpha$), reject H_0.

If *P*-value is not small ($> \alpha$), fail to reject H_0.

If the P is low, the null must go.
If the P is high, the null will fly.

Do we prove a claim?

- **A statistical test cannot prove a hypothesis or a claim.**
- **Our conclusion can be only stated like this: the available evidence is not strong enough to warrant rejection of a hypothesis or a claim**

Section 8-4

Testing a Claim About a Mean: σ Known

Notation

n = sample size

$\bar{x}$ = sample mean

μ = claimed population mean (from H_0)

σ = known value of the population standard deviation

Requirements for Testing Claims About a Population Mean (with σ Known)

1) The sample is a simple random sample.

2) The value of the population standard deviation σ is known.

3) Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$.

Test Statistic for Testing a Claim About a Mean (with σ Known)

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Example:

People have died in boat accidents because an obsolete estimate of the mean weight of men (166.3 lb) was used.

A random sample of $n = 40$ men yielded the mean $\bar{x}$ = 172.55 lb. Research from other sources suggests that the population of weights of men has a standard deviation given by $\sigma = 26$ lb.

Test the claim that men have a mean weight greater than 166.3 lb.

Example:

Requirements are satisfied: σ is known (26 lb), sample size is 40 ($n > 30$)

We can express claim as $\mu > 166.3$ lb

It does not contain equality, so it is the alternative hypothesis.

H_0: $\mu = 166.3$ lb null hypothesis

H_1: $\mu > 166.3$ lb alternative hypothesis (and original claim)

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Example:

For a significance level to $\alpha = 0.05$

Next we calculate z

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}} = \frac{172.55 - 166.3}{\frac{26}{\sqrt{40}}} = 1.52$$

It is a right-tailed test, so P-value is the area is to the right of $z = 1.52$;

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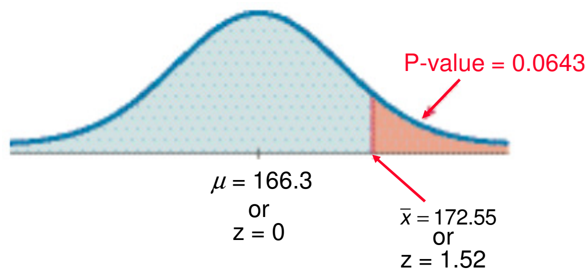
Example:

Table A-2: area to the left of $z = 1.52$ is 0.9357, so the area to the right is $1 - 0.9357 = 0.0643$.
The *P*-value is 0.0643

The *P*-value of 0.0643 is greater than the significance level of $\alpha = 0.05$, we fail to reject the null hypothesis.



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Example:

The traditional method:

Use critical value $z = 1.645$ instead of finding the *P*-value. Since $z = 1.52$ does not fall in the critical region, again fail to reject the null hypothesis.
i.e. we reject H_1; alternative hypothesis (and original claim)

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Section 8-6

Testing a Claim About a Standard Deviation or Variance

Notation

n = sample size

s = *sample* standard deviation

s^2 = *sample* variance

σ = claimed value of the *population* standard deviation (from H_0)

σ^2 = claimed value of the *population* variance (from H_0)

Requirements for Testing Claims About σ or σ^2

1. The sample is a simple random sample.

2. The population has a normal distribution. (This is a much stricter requirement than the requirement of a normal distribution when testing claims about means.)

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Chi-Square Distribution

Test Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

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Critical Values for Chi-Square Distribution

- Use Table A-4.
- The degrees of freedom df = $n-1$.

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Table A-4

Table A-4 is based on cumulative areas from the right.

Critical values are found in Table A-4 by first locating the row corresponding to the appropriate number of degrees of freedom (where df = $n-1$).

Next, the significance level α is used to determine the correct column.

The following examples are based on a significance level of $\alpha = 0.05$.

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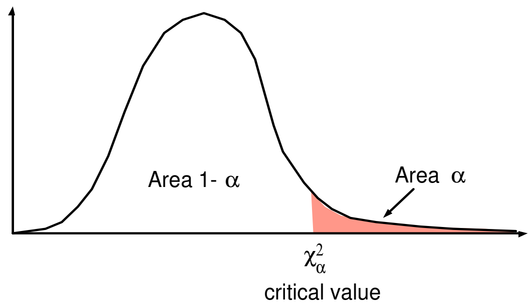
Critical values

Right-tailed test: needs <u>one</u> critical value

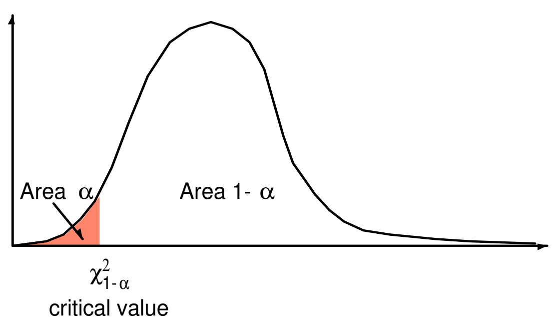
Because the area to the right of the critical value is 0.05, locate 0.05 at the top of Table A-4.



Critical values

Left-tailed test: needs <u>one</u> critical value

With a left-tailed area of 0.05, the area to the right of the critical value is 0.95, so locate 0.95 at the top of Table A-4.



Critical values

Two-tailed test: **needs two critical values**

Critical values are two different positive numbers, both taken from Table A-4

Divide a significance level of 0.05 between the left and right tails, so the areas to the right of the two critical values are 0.975 and 0.025, respectively.

Locate 0.975 and 0.025 at top of Table A-4

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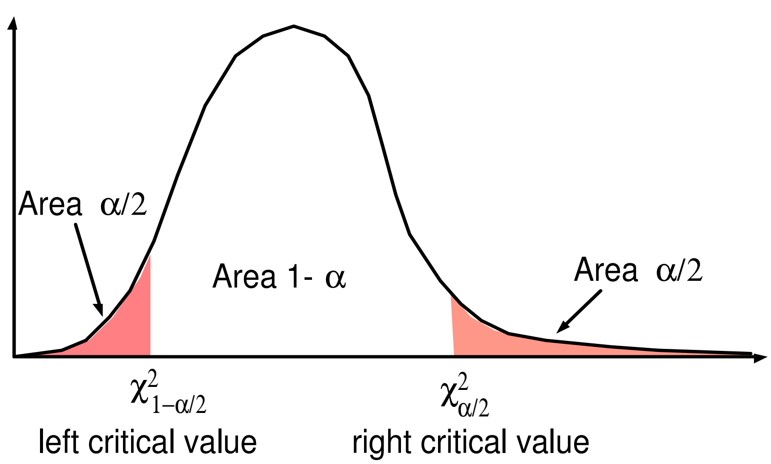
Critical values for a two-tailed test



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Examples:

Right-tailed test: If the test statistic χ^2 is between critical values corresponding to the areas α_1 and α_2, then your *P*-value is between α_1 and α_2.

Left-tailed test: If the test statistic χ^2 is between critical values corresponding to the areas $1-\alpha_1$ and $1-\alpha_2$, then your *P*-value is between α_1 and α_2.

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