Components of a Formal Hypothesis Test

Null Hypothesis: H_0

- **The null hypothesis (denoted by H_0) is a statement that the value of a population parameter (such as *proportion, mean*, or *standard deviation*) is equal to some claimed value.**
- **We test the null hypothesis directly.**
- **Either reject H_0 or fail to reject H_0**

(in other words, accept H_0).

Alternative Hypothesis: H_1

- The alternative hypothesis (denoted by H_1) is the statement that the parameter has a value that somehow differs from the null hypothesis.
- The symbolic form of the alternative hypothesis must use one of these symbols: $\neq$, $<$, $>$.
(not equal, less than, greater than)

General rules:

- **If the null hypothesis is rejected, the alternative hypothesis is accepted.**
- **If the null hypothesis is accepted, the alternative hypothesis is rejected.**
- **Acceptance or rejection of the null hypothesis is an initial conclusion.**
- **Always state the final conclusion expressed in terms of the original claim, not in terms of the null hypothesis or the alternative hypothesis.**

Type I Error

- A **Type I error** is the mistake of rejecting the null hypothesis when it is actually true.
- The symbol α (alpha) is used to represent the probability of a type I error.

Type II Error

- A **Type II error** is the mistake of accepting the null hypothesis when it is actually false.
- The symbol β (beta) is used to represent the probability of a type II error.

Type I and Type II Errors

		True State of Nature	
		The null hypothesis is true	The null hypothesis is false
Decision	We decide to reject the null hypothesis	**Type I error** (rejecting a true null hypothesis) $P(\text{type I error}) = \alpha$	Correct decision
	We fail to reject the null hypothesis	Correct decision	**Type II error** (failing to reject a false null hypothesis) $P(\text{type II error}) = \beta$

Significance Level

The probability of the type I error (denoted by α) is also called the **significance level** of the test.

It characterizes the chances that the test fails (i.e., type I error occurs)

It must be a small number. Typical values used in practice: α = 0.1, 0.05, or 0.01 (in percents, 10%, 5%, or 1%).

Testing hypothesis
Step 1: compute Test Statistic

The **test statistic** is a value used in making a decision about the null hypothesis.

The test statistic is computed by a specific formula depending on the type of the test.

Section 8-3

Testing a Claim About a Proportion

Notation

n = number of trials

$\hat{p} = \frac{x}{n}$ (sample proportion)

p = population proportion (must be specified in the null hypothesis)

$q = 1 - p$

Requirements for Testing Claims About a Population Proportion p

1) The sample observations are simple random samples.

2) The conditions for a binomial distribution are satisfied.

3) The conditions $np \geq 5$ and $nq \geq 5$ are both satisfied, so the binomial distribution of sample proportions can be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$. Note: p is the assumed proportion not the sample proportion.

Test Statistic for Testing a Claim About a Proportion

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Note: *p* is the value specified in the null hypothesis; $q = 1 - p$

Example 1 again:

Claim: the XSORT method of gender selection increases the likelihood of having a baby girl.

Null hypothesis: $H_0 : p = 0.5$

Alternative hypothesis: $H_1 : p > 0.5$

Suppose 14 couples treated by XSORT gave birth to 13 girls and 1 boy.

Test the claim at a 5% significance level

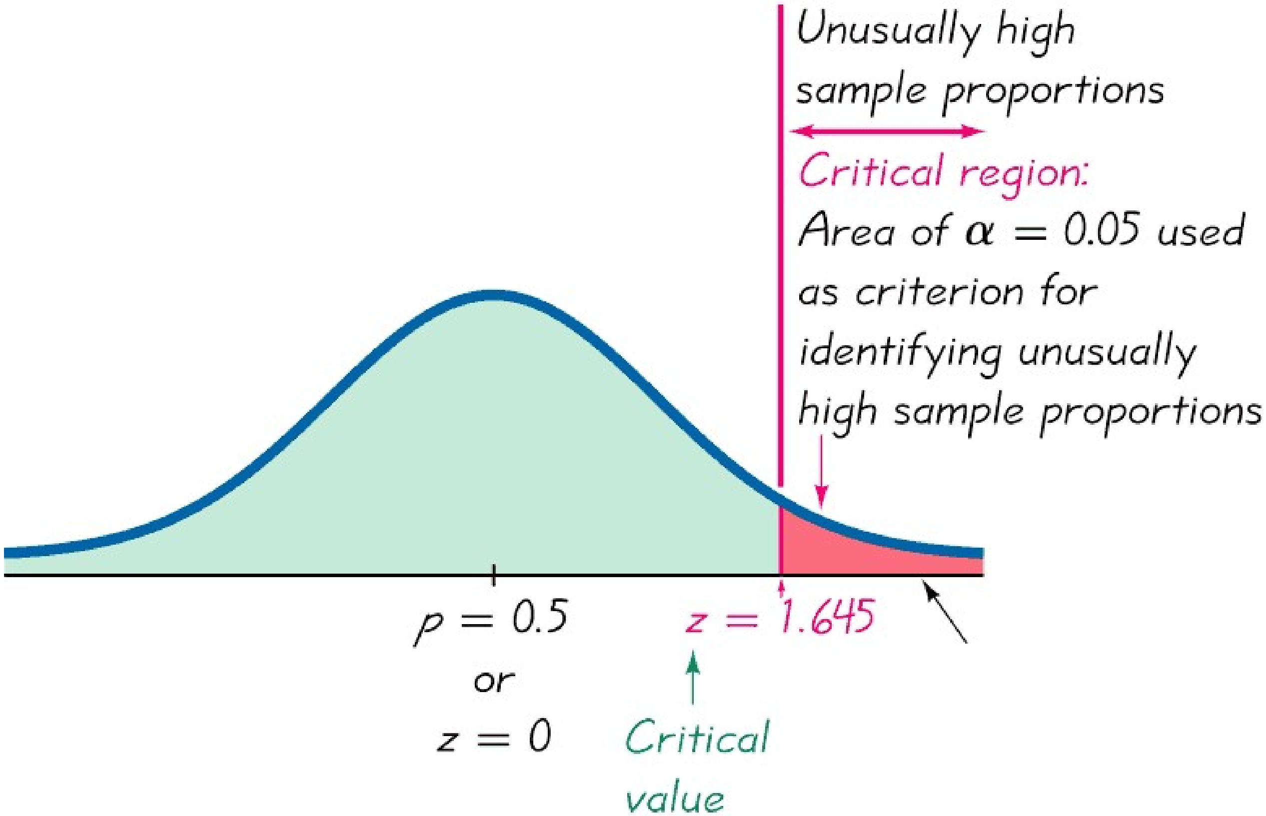
Compute the test statistic:

$$\hat{p} = 13/14 = 0.929$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.929 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{14}}} = 3.21$$

Draw the diagram (the normal curve)

On the diagram, mark a region of extreme values that agree with the alternative hypothesis:



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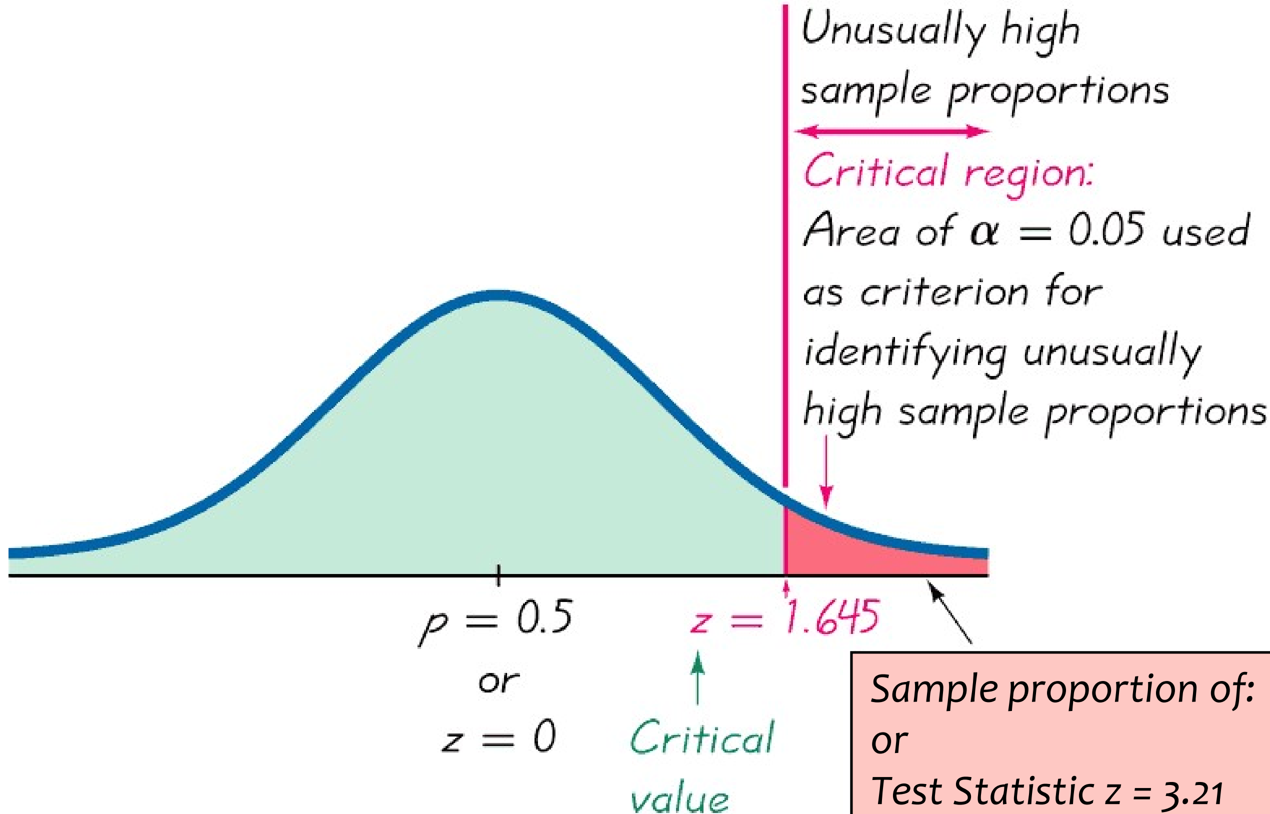

Sample proportion of: $\hat{p} = 0.929$
or
Test Statistic z = 3.21

Critical Region

The critical region (or rejection region) is the set of all values of the test statistic that cause us to reject the null hypothesis.

For example, see the red-shaded region in the previous figure.

Critical Value

A critical value is a value that separates the critical region (where we reject the null hypothesis) from the values of the test statistic that do not lead to rejection of the null hypothesis.

See the previous figure where the critical value is $z = 1.645$. It corresponds to a significance level of $\alpha = 0.05$.

Significance Level

The significance level (denoted by α) is the probability that the test statistic will fall in the critical region (when the null hypothesis is actually true).

Conclusion of the test

Since the test statistic ($z=3.21$) falls in the critical region ($z>1.645$), we **reject the null hypothesis($p=.5$)**.

Final conclusion: the original claim is accepted, the XSORT method of gender selection indeed increases the likelihood of having a baby girl.

Section 8-4

Testing a Claim About a Mean: σ Known

Notation

n = sample size

$\bar{x}$ = sample mean

μ = claimed population mean (from H_0)

σ = known value of the population standard deviation

Requirements for Testing Claims About a Population Mean (with σ Known)

1) The sample is a simple random sample.

2) The value of the population standard deviation σ is known.

3) Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$.

Test Statistic for Testing a Claim About a Mean (with σ Known)

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Example:

People have died in boat accidents because an obsolete estimate of the mean weight of men (166.3 lb) was used.

A random sample of *n* = 40 men yielded the mean $\bar{x}$ = 172.55 lb. Research from other sources suggests that the population of weights of men has a standard deviation given by σ = 26 lb.

Test the claim that men have a mean weight greater than 166.3 lb.

Example:

Requirements are satisfied: σ is known (26 lb), sample size is 40 (n > 30)

We can express claim as μ > 166.3 lb

It does not contain equality, so it is the alternative hypothesis.

H_0: μ = 166.3 lb null hypothesis

H_1: μ > 166.3 lb alternative hypothesis (and original claim)

Example:

For a significance level to α = 0.05

Next we calculate *z*

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}} = \frac{172.55 - 166.3}{\frac{26}{\sqrt{40}}} = 1.52$$

It is a right-tailed test, so *the critical region* is the area is to the right of *z* = 1.52;

Example:

The critical value $z = 1.645$ And
Test statistic $z = 1.52$
i.e. the test statistic is to the left of the critical value

For the significance level of $\alpha = 0.05$, we fail to reject the null hypothesis.
i.e. we reject H_1; alternative hypothesis (and original claim)