

Recall

Binomial Probability Distribution


1. The procedure must have a **fixed number of trials, n** .
2. The trials must be **independent**.
3. Each trial must have all outcomes classified into **two categories** (commonly, **success** and **failure**).
4. The probability of success **p** remains the same in all trials (the probability of failure is **$q=1-p$**).

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Approximation of a Binomial Distribution with a Normal Distribution

If $np \geq 5$ and $nq \geq 5$

Then $\mu = np$ and $\sigma = \sqrt{npq}$
and the random variable has

a  distribution.
(normal)

Procedure for Using a Normal Distribution to Approximate a Binomial Distribution

1. Verify that both $np \geq 5$ and $nq \geq 5$. If not, you cannot use normal approximation to binomial.
2. Find the values of the parameters μ and σ by calculating $\mu = np$ and $\sigma = \sqrt{npq}$.
3. Identify the discrete whole number x that is relevant to the binomial probability problem. Use the continuity correction (See **continuity corrections** discussion later) Draw a normal curve and enter the values of μ , σ , and either $x - 0.5$ or $x + 0.5$, as appropriate.

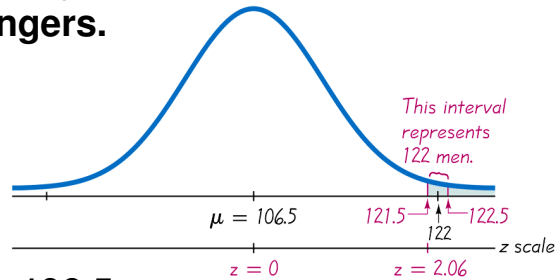
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Procedure for Using a Normal Distribution to Approximate a Binomial Distribution

4. Change x by replacing it with $x - 0.5$ or $x + 0.5$, as appropriate.
5. Using $x - 0.5$ or $x + 0.5$ (as appropriate) in place of x , find the area corresponding to the desired probability by first finding the z -score and finding the area to the left of the adjusted value of x .

Example – Number of Men Among Passengers

Finding the Probability of “At Least 122 Men” Among 213 Passengers.



$$\mu = np = 213 * 0.5 = 106.5$$

$$\text{and } \sigma = \sqrt{npq} = \sqrt{213 * 0.5 * 0.5} = 7.29$$

Consider $x-5$ and $x+5$ for continuity correction

$$Z = (121.5 - 106.5) / 7.29 = 2.06$$

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Definition

When we use the normal distribution (which is a **continuous** probability distribution) as an approximation to the binomial distribution (which is **discrete**), a **continuity correction** is made to a discrete whole number x in the binomial distribution by representing the single value x by the interval from

$$x - 0.5 \text{ to } x + 0.5$$

(that is, adding and subtracting 0.5).

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Procedure for Continuity Corrections

1. When using the normal distribution as an approximation to the binomial distribution, **always** use the continuity correction.
2. In using the continuity correction, first identify the discrete whole number x that is relevant to the binomial probability problem.
3. Draw a normal distribution centered about μ , then draw a **vertical strip area** centered over x . Mark the left side of the strip with the number $x - 0.5$, and mark the right side with $x + 0.5$. For $x = 122$, draw a strip from 121.5 to 122.5. **Consider the area of the strip to represent the probability of discrete whole number x .**

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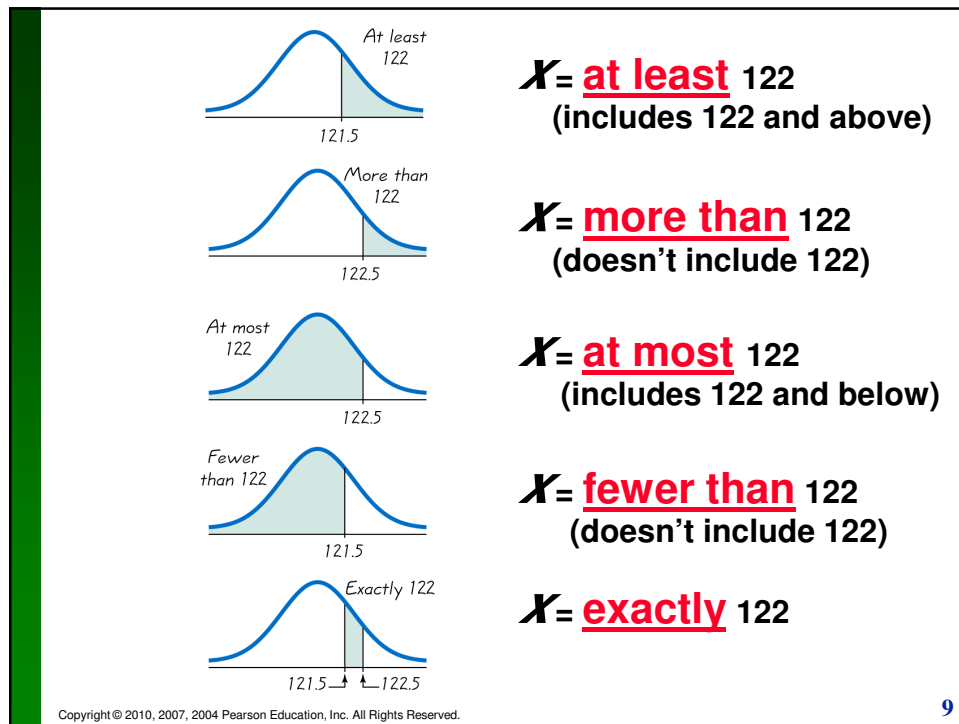
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Procedure for Continuity Corrections

4. Now determine whether the value of x itself should be included in the probability you want. Next, determine whether you want the probability of at least x , at most x , more than x , fewer than x , or exactly x . Shade the area to the right or left of the strip, as appropriate; also shade the interior of the strip itself **if and only if x itself** is to be included. The total shaded region corresponds to the probability being sought.

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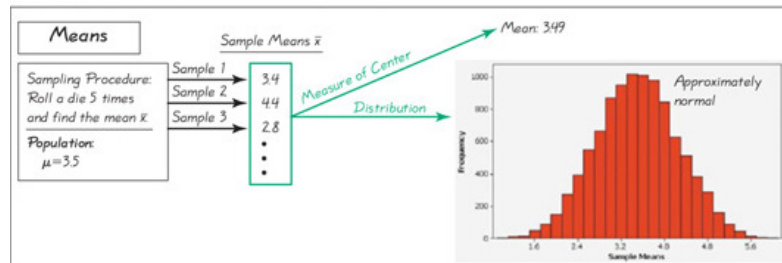
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Sampling Distribution of the mean

- ❖ The sampling distribution of the mean is the distribution of the sample means, with all samples having the same sample size n taken from the same population

Example

- ❖ Consider rolling a die 5 times and find the mean \bar{x} of the results.



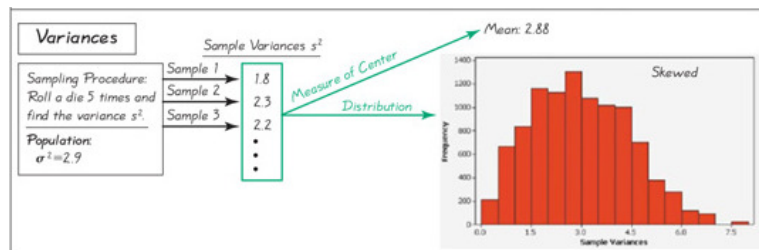
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Sampling distribution of the variance

- ❖ The sampling distribution of the variance is the distribution of the sample variances, with all samples having the same size n taken from the population



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Sampling diistribution

The main objective of this section is to understand the concept of a **sampling distribution of a statistic**, which is the distribution of all values of that statistic when all possible samples of the same size are taken from the same population.

We will also see that some statistics are better than others for estimating population parameters.

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Properties

- ❖ Sample proportions tend to target the value of the population proportion. (That is, all possible sample proportions have a mean equal to the population proportion.)
- ❖ Under certain conditions, the distribution of the sample proportion can be approximated by a normal distribution.

Definition

- ❖ The value of a statistic, such as the sample mean \bar{x} , depends on the particular values included in the sample, and generally varies from sample to sample. This variability of a statistic is called **sampling variability**.

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Estimators

Some statistics work much better than others as estimators of the population. The example that follows shows this.

Example - Sampling Distributions

A population consists of the values 1, 2, and 5. We randomly select samples of size 2 with replacement. There are 9 possible samples.

- a. For each sample, find the mean, median, range, variance, and standard deviation.
- b. For each statistic, find the mean from part (a)

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Sampling Distributions

A population consists of the values 1, 2, and 5. We randomly select samples of size 2 with replacement. There are 9 possible samples.

- a. For each sample, find the mean, median, range, variance, and standard deviation.

Table 6-7 Sampling Distributions of Statistics (for Samples of Size 2 Drawn with Replacement from the Population 1, 2, 5)

Sample	Mean \bar{x}	Median	Range	Variance s^2	Standard Deviation s	Proportion of Odd Numbers	Probability
1, 1	1.0	1.0	0	0.0	0.000	1	1/9
1, 2	1.5	1.5	1	0.5	0.707	0.5	1/9
1, 5	3.0	3.0	4	8.0	2.828	1	1/9
2, 1	1.5	1.5	1	0.5	0.707	0.5	1/9
2, 2	2.0	2.0	0	0.0	0.000	0	1/9
2, 5	3.5	3.5	3	4.5	2.121	0.5	1/9
5, 1	3.0	3.0	4	8.0	2.828	1	1/9
5, 2	3.5	3.5	3	4.5	2.121	0.5	1/9
5, 5	5.0	5.0	0	0.0	0.000	1	1/9
Mean of Statistic Values	8/3	8/3	16/9	26/9	1.3	2/3	
Population Parameter	8/3	2	4	26/9	1.7	2/3	
Does the sample statistic target the population parameter?	Yes	No	No	Yes	No	Yes	

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Sampling Distributions

A population consists of the values 1, 2, and 5. We randomly select samples of size 2 with replacement. There are 9 possible samples.

b. For each statistic, find the mean from part (a)

The means are found near the bottom of Table 6-7.

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Interpretation of Sampling Distributions

We can see that when using a sample statistic to estimate a population parameter, some statistics are good in the sense that they target the population parameter and are therefore likely to yield good results. Such statistics are called **unbiased estimators**.

Statistics that target population parameters: mean, variance, proportion

Statistics that do not target population parameters: median, range, standard deviation

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Poisson Distribution

The Poisson distribution is important because it is often used for describing the behavior of rare events (with small probabilities).

Definition

The **Poisson distribution** is a discrete probability distribution that applies to occurrences of some event **over a specified interval**. The random variable x is the number of occurrences of the event in an interval. The interval can be time, distance, area, volume, or some similar unit.

Formula

$$P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!} \text{ where } e \approx 2.71828$$

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Poisson Distribution Requirements

- ❖ The random variable x is the number of occurrences of an event **over some interval**.
- ❖ The occurrences must be **random**.
- ❖ The occurrences must be **independent** of each other.
- ❖ The occurrences must be **uniformly distributed** over the interval being used.

Parameters

- ❖ The mean is μ .
- ❖ The standard deviation is $\sigma = \sqrt{\mu}$.

Difference from a Binomial Distribution

The Poisson distribution differs from the binomial distribution in these fundamental ways:

- ❖ The binomial distribution is affected by the sample size n and the probability p , whereas the Poisson distribution is affected only by the mean μ .
- ❖ In a binomial distribution the possible values of the random variable x are $0, 1, \dots, n$, but a Poisson distribution has possible x values of $0, 1, \dots$, with no upper limit.

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Poisson as Approximation to Binomial

The Poisson distribution is sometimes used to approximate the binomial distribution when n is large and p is small.

Rule of Thumb

❖ $n \geq 100$

❖ $np \leq 10$

Poisson as Approximation to Binomial - μ

❖ $n \geq 100$

❖ $np \leq 10$

Value for μ

$$\mu = n \cdot p$$