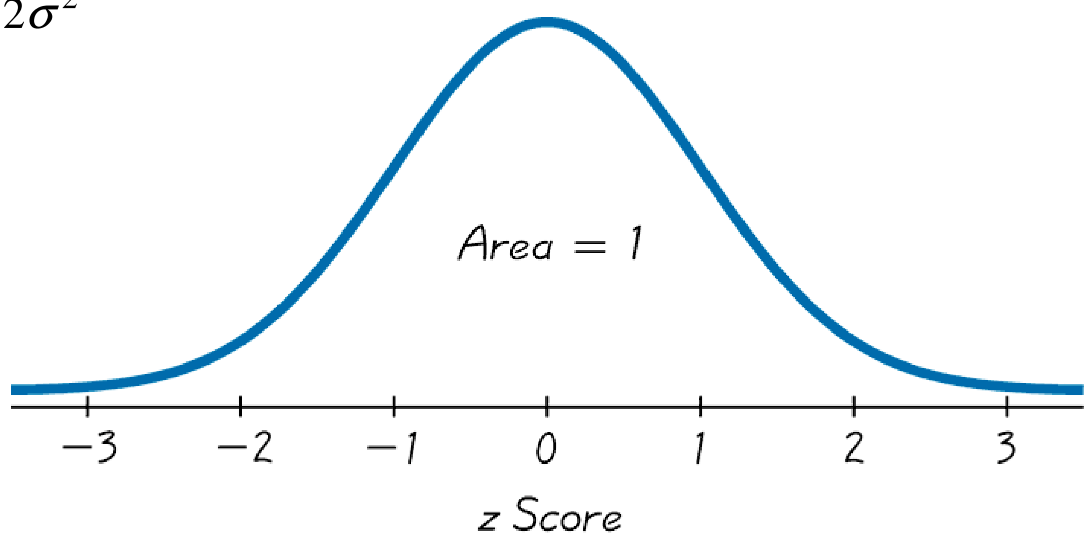
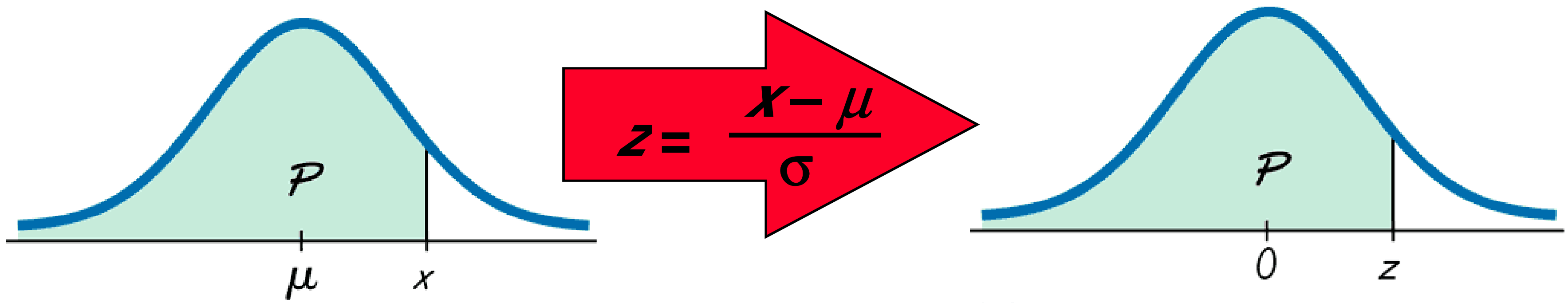
Standard Normal Distribution

The standard normal distribution is a bell-shaped probability distribution with $\mu = 0$ and $\sigma = 1$. The total area under its density curve is equal to 1.

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Converting to a Standard Normal Distribution



(a) Nonstandard Normal Distribution

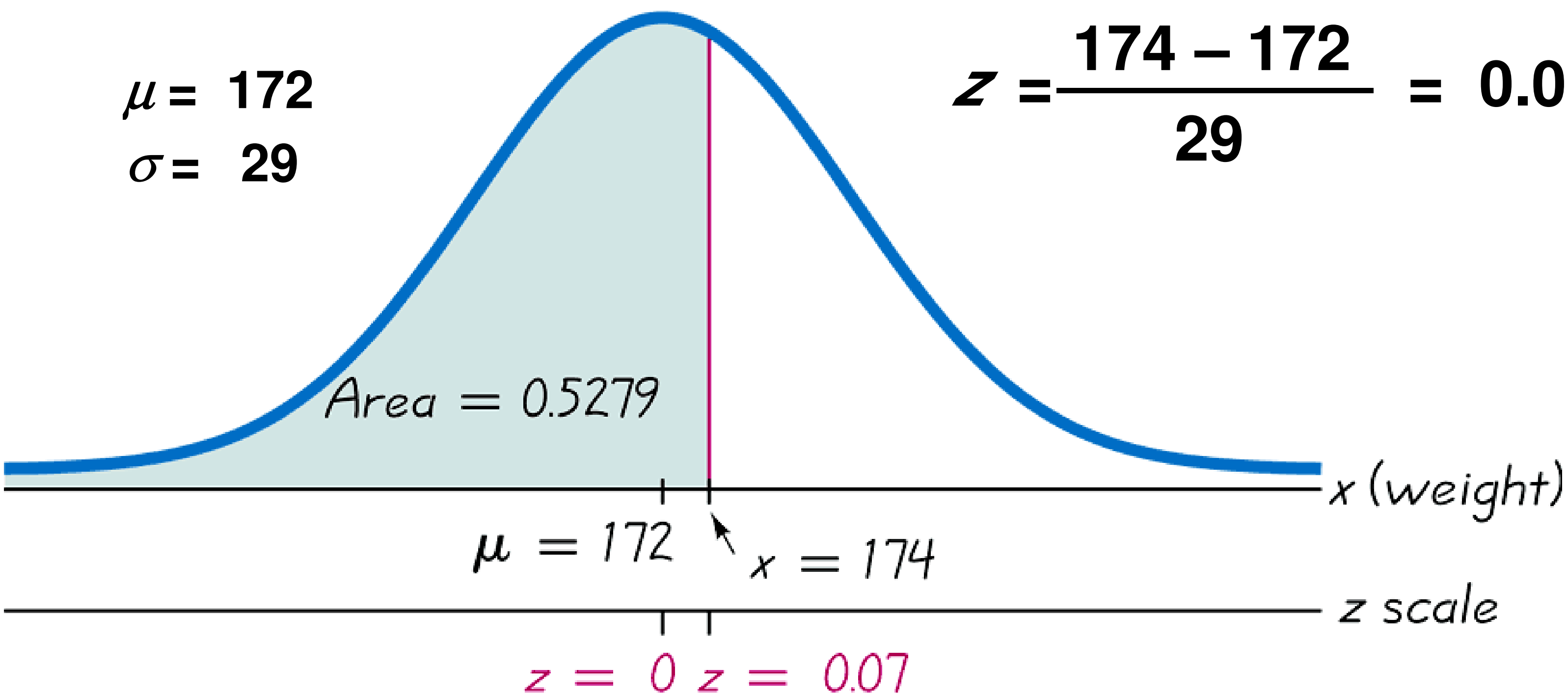
(b) Standard Normal Distribution

Example – Weights of Passengers

Weights of taxi passengers have a normal distribution with mean 172 lb and standard deviation 29 lb. If one passenger is randomly selected, what is the probability he/she weighs less than 174 pounds?

$\mu = 172$
$\sigma = 29$

$$z = \frac{174 - 172}{29} = 0.07$$

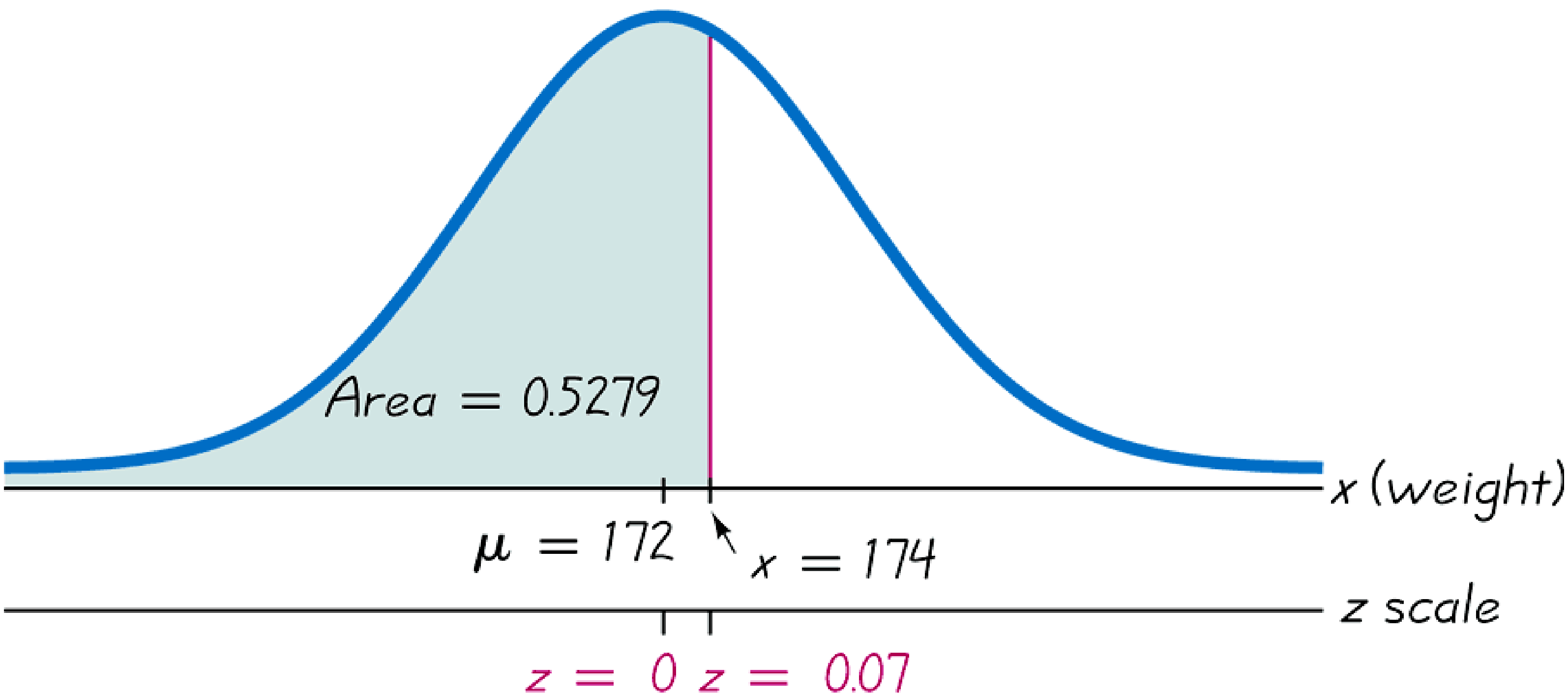


Example - continued

$P(x < 174 \text{ lb.}) = P(z < 0.07) = 0.5279$

$\mu = 172$

$\sigma = 29$

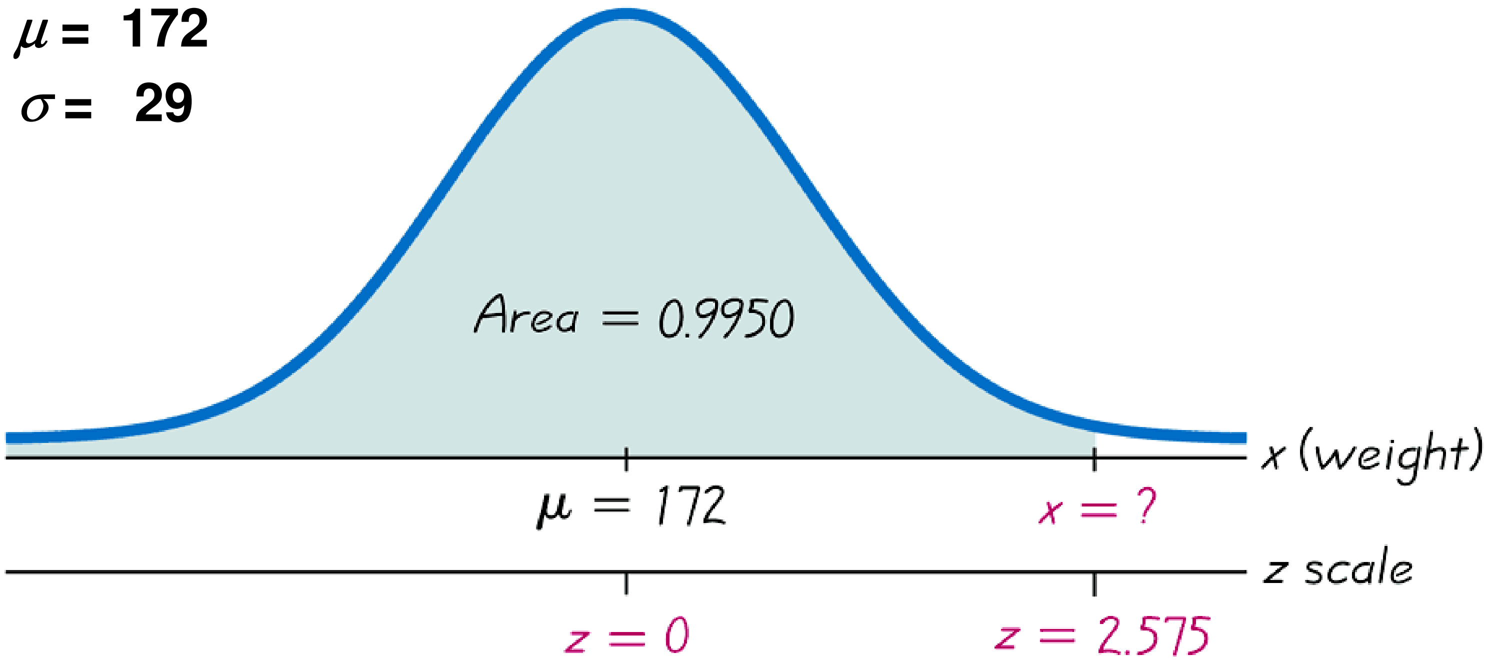


Example – Lightest and Heaviest

Weights of taxi passengers have a normal distribution with mean 172 lb and standard deviation 29 lb. Determine what weight separates the lightest 99.5% from the heaviest 0.5%?

$\mu = 172$

$\sigma = 29$

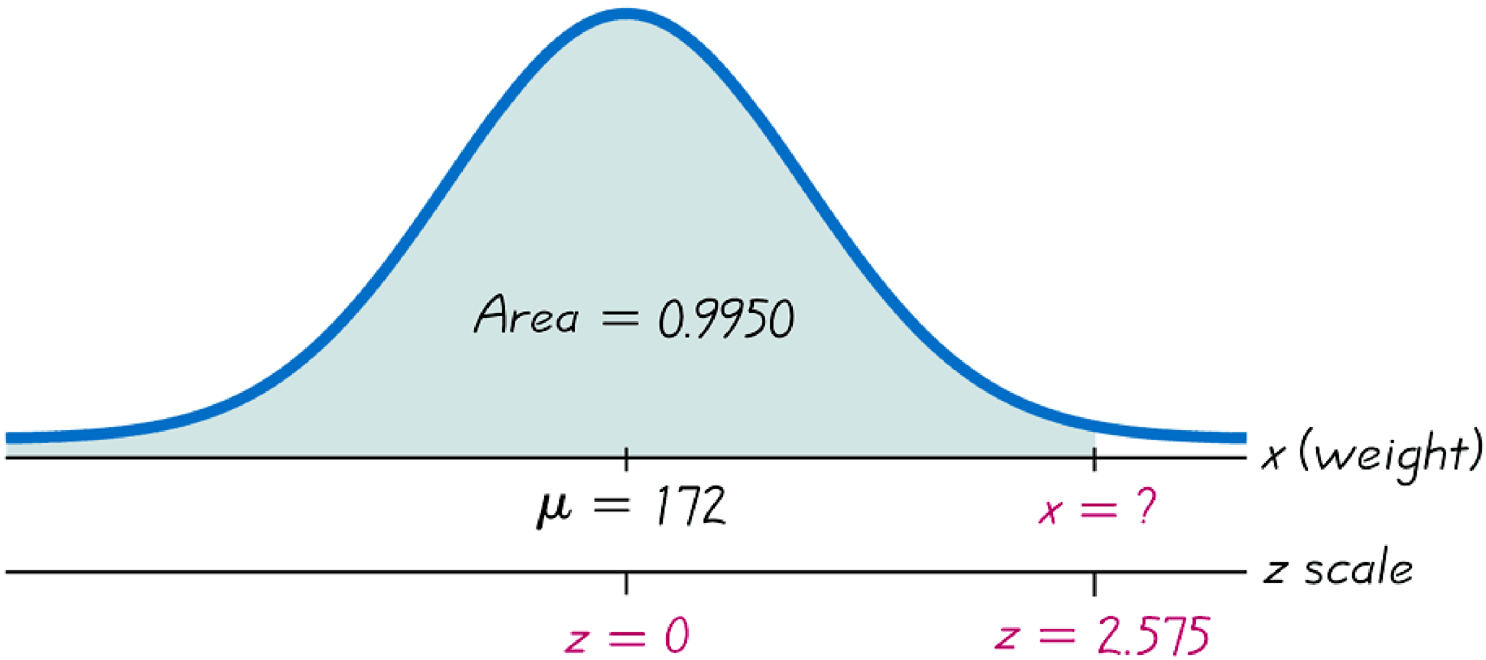


Example – Lightest and Heaviest - cont

$x = \mu + (z \bullet \sigma)$

$x = 172 + (2.575 \bullet 29)$

$x = 246.675$ (247 rounded)

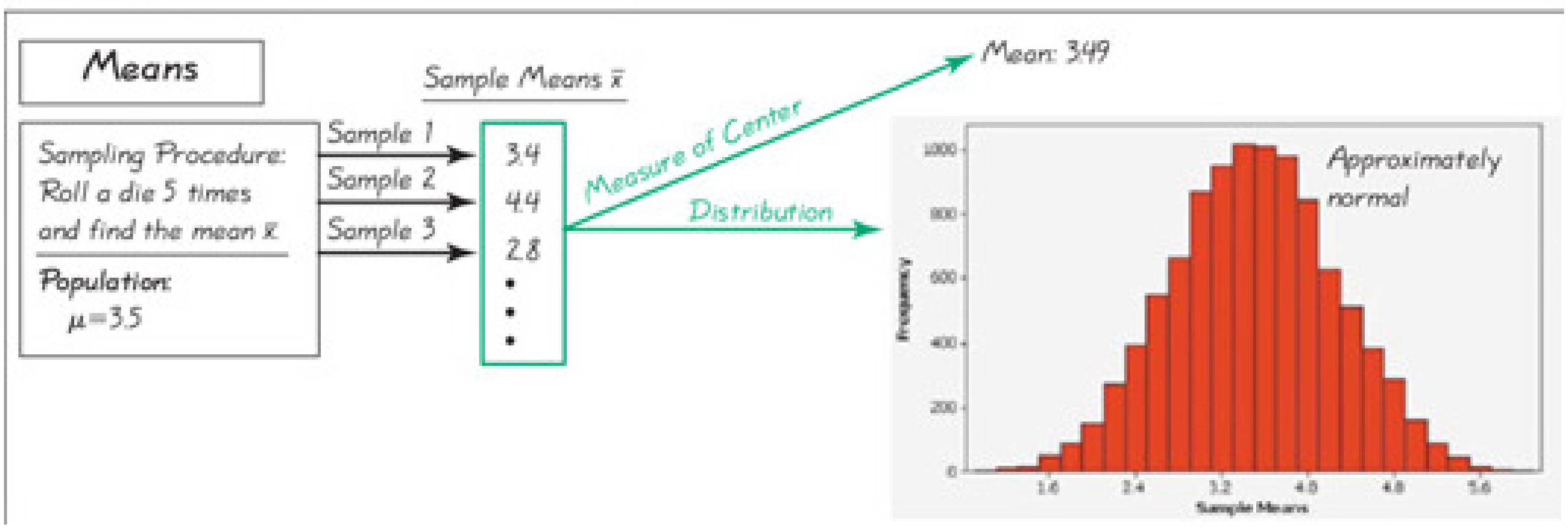


Sampling Distribution of the mean

- The sampling distribution of the mean is the distribution of the sample means, with all samples having the same sample size n taken from the same population

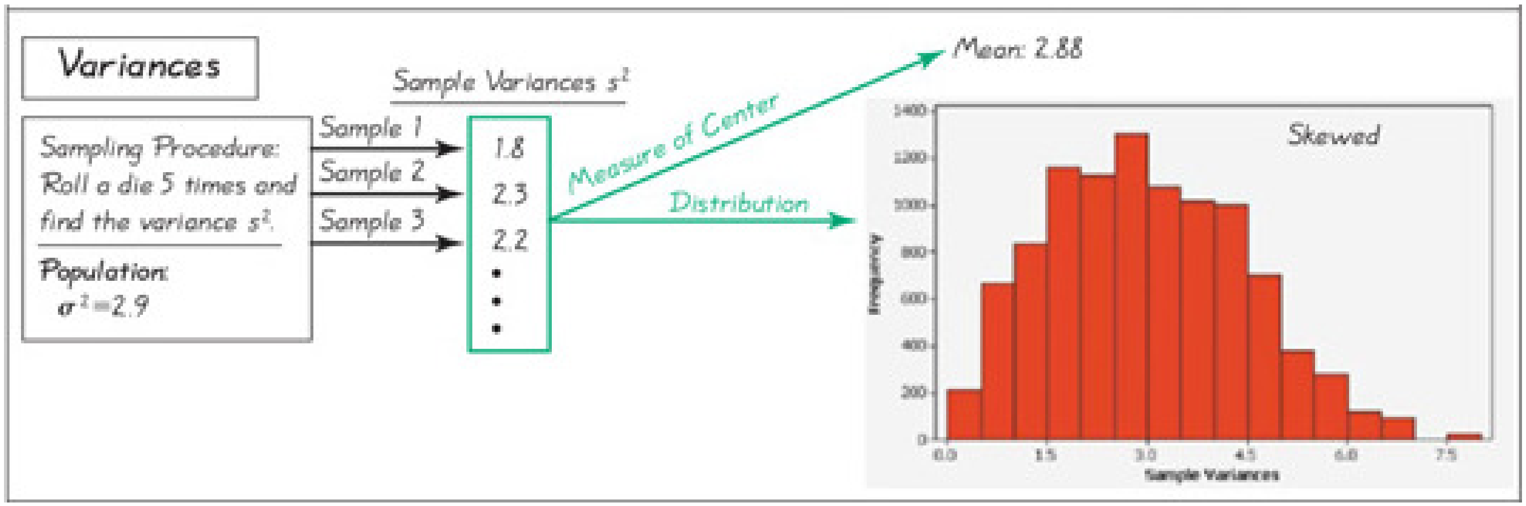
Example

- Consider rolling a die 5 times and find the mean $\bar{x}$ of the results.



Sampling distribution of the variance

- The sampling distribution of the variance is the distribution of the sample variances, with all samples having the same size n taken from the population



Sampling distribution

The main objective of this section is to understand the concept of a sampling distribution of a statistic, which is the distribution of all values of that statistic when all possible samples of the same size are taken from the same population.

We will also see that some statistics are better than others for estimating population parameters.

Properties

- Sample proportions tend to target the value of the population proportion. (That is, all possible sample proportions have a mean equal to the population proportion.)
- Under certain conditions, the distribution of the sample proportion can be approximated by a normal distribution.

Definition

- The value of a statistic, such as the sample mean $\overline{x}$, depends on the particular values included in the sample, and generally varies from sample to sample. This variability of a statistic is called **sampling variability**.

Central Limit Theorem

The *Central Limit Theorem* tells us that the distribution of the sample mean $\bar{x}$ for a sample of size n approaches a normal distribution, as the sample size n increases.

Central Limit Theorem

Given:

1. The random variable x has a distribution (which may or may not be normal) with mean μ and standard deviation σ.
2. A random sample of size n is selected from the population.

Central Limit Theorem – cont.

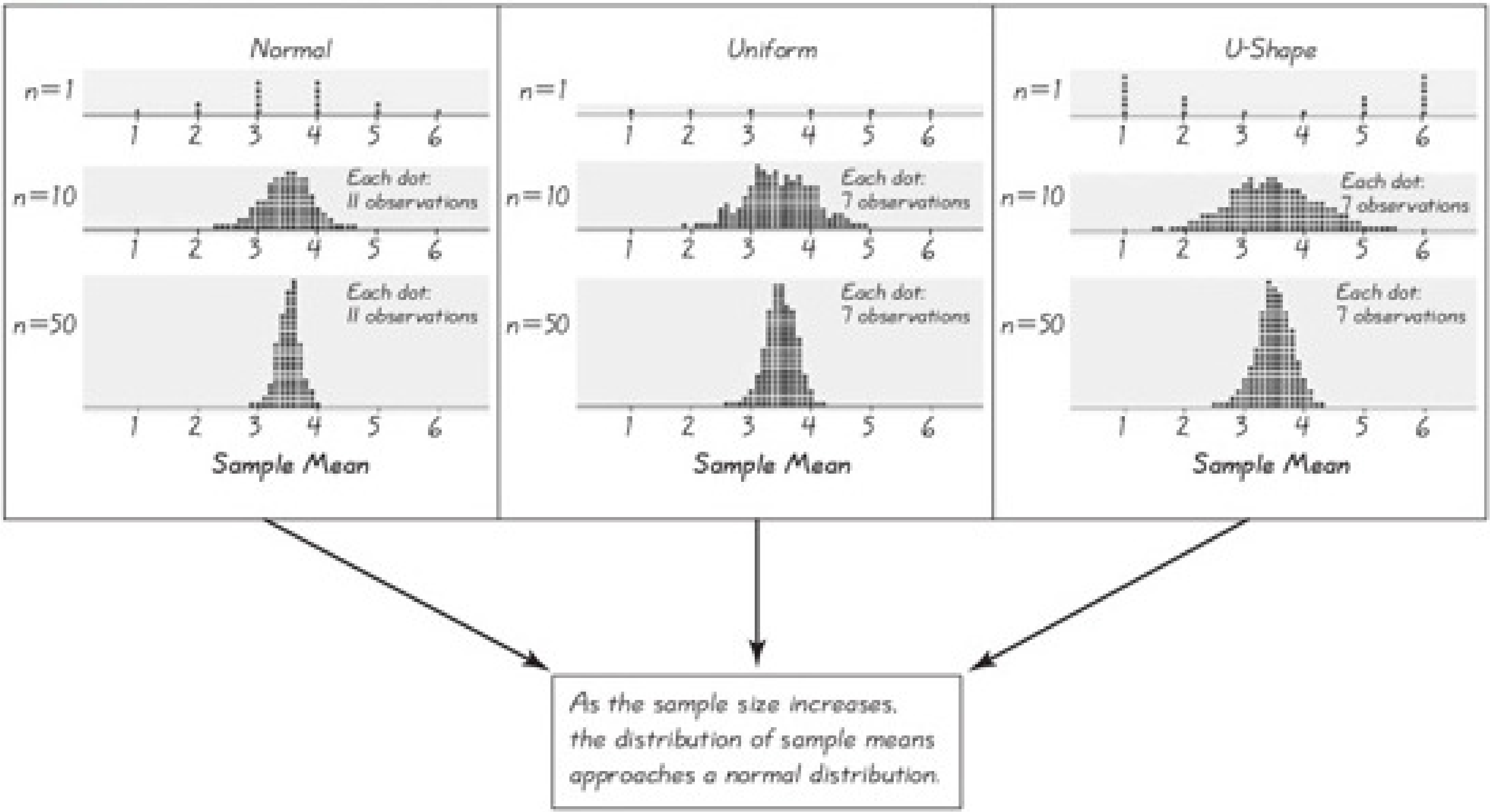
Conclusions:

1. The distribution of the sample mean $\overline{x}$ will, as the sample size increases, approach a normal distribution.

2. The mean of that normal distribution is the same as the population mean μ.

3. The standard deviation of that normal distribution is $\sigma/\sqrt{n}$. (So it is smaller than the standard deviation of the population.)

Table 6-6 Sampling Distributions



Formulas

the mean

$$\mu_{\bar{x}} = \mu$$

the standard deviation

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Practical Rules:

1. For samples of size n larger than 30, the distribution of the sample mean can be approximated by a normal distribution.
2. If the original population is *normally distributed*, then for any sample size n, the sample means will be normally distributed.

3. We can apply Central Limit Theorem if either $n > 30$ or the original population is normal.

Example: Water Taxi Passengers

Assume the population of taxi passengers is normally distributed with a mean of 172 lb and a standard deviation of 29 lb.

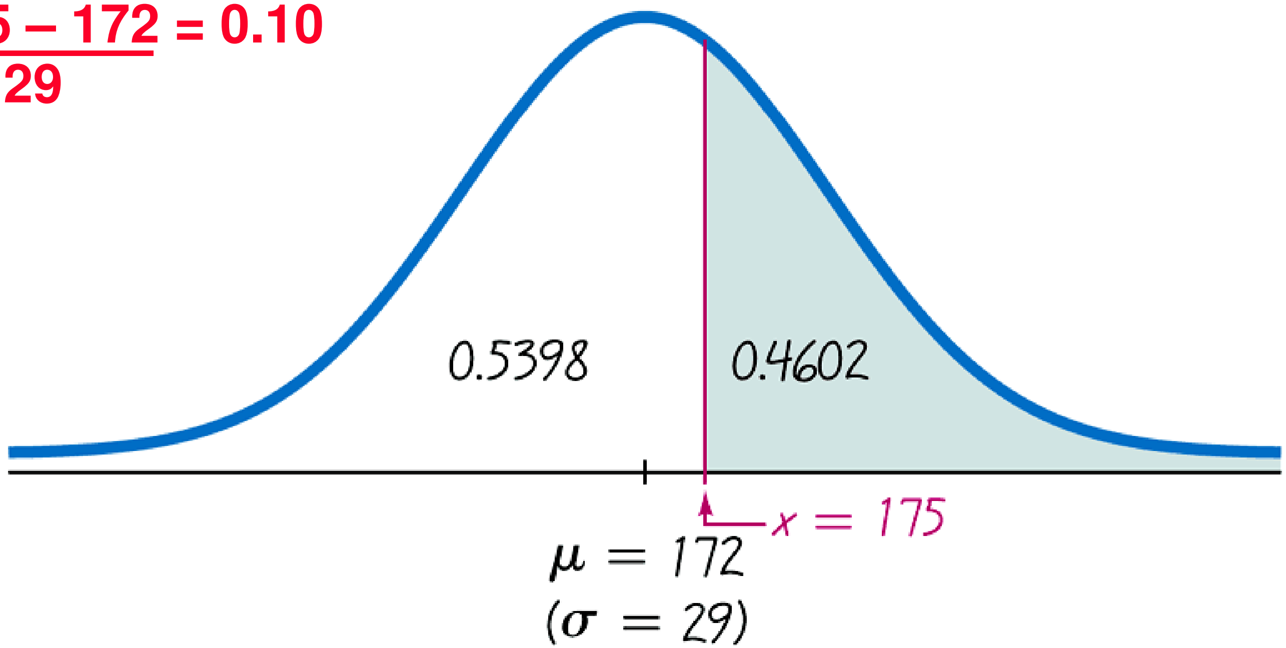
a) **Find the probability that if an *individual* passenger is randomly selected, his weight is greater than 175 lb.**

b) **Find the probability that *20 randomly selected passengers* will have a mean weight that is greater than 175 lb.**

Example – cont

a) Find the probability that if an *individual* man is randomly selected, his weight is greater than 175 lb.

$$z = \frac{175 - 172}{29} = 0.10$$



(a)

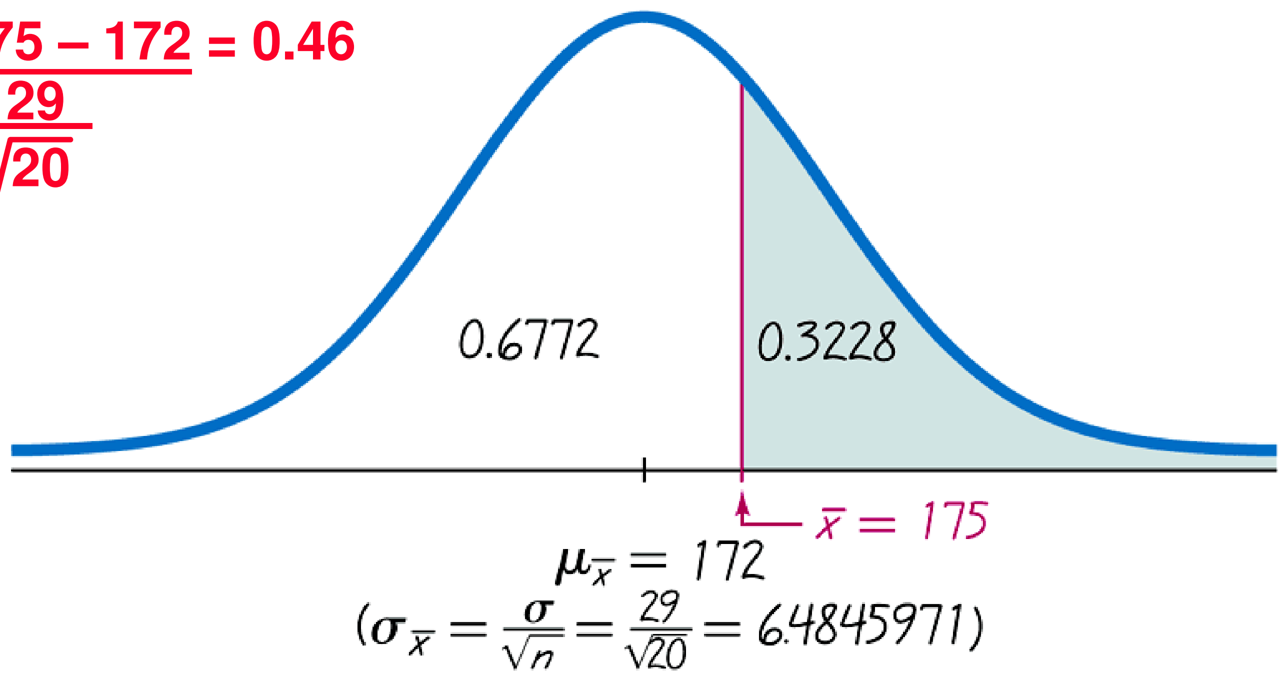
TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495 *	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

Example – cont

b) Find the probability that *20 randomly selected men* will have a mean weight that is greater than 175 lb (so that their total weight exceeds the safe capacity of 3500 pounds).

$$z = \frac{175 - 172}{\frac{29}{\sqrt{20}}} = 0.46$$



(b)

Finding *z* Scores When Given Probabilities

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495 *	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

Example - cont

a) Find the probability that if an *individual* passenger is randomly selected, his weight is greater than 175 lb.

$$P(x > 175) = 0.4602$$

b) Find the probability that *20 randomly selected passengers* will have a mean weight that is greater than 175 lb.

$$P(\bar{x} > 175) = 0.3228$$

It is much easier for an individual to deviate from the mean than it is for a group of 20 to deviate from the mean.

Recall

Binomial Probability Distribution

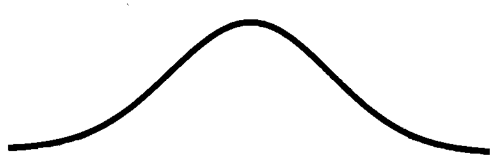
1. The procedure must have a fixed number of trials, n.
2. The trials must be independent.
3. Each trial must have all outcomes classified into two categories (commonly, success and failure).
4. The probability of success p remains the same in all trials (the probability of failure is $q=1-p$)

Approximation of a Binomial Distribution with a Normal Distribution

If $np \geq 5$ and $nq \geq 5$

Then $\mu = np$ and $\sigma = \sqrt{npq}$

and the random variable has

a  distribution.

(normal)

Procedure for Using a Normal Distribution to Approximate a Binomial Distribution

1. Verify that <u>both</u> $np \geq 5$ and $nq \geq 5$. If not, you cannot use normal approximation to binomial.

2. Find the values of the parameters μ and σ by calculating $\mu = np$ and $\sigma = \sqrt{npq}$.

3. Identify the discrete whole number x that is relevant to the binomial probability problem. Use the continuity correction (see next).