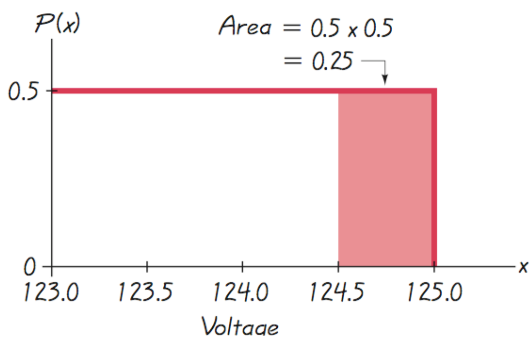
Using Area to Find Probability

Given the uniform distribution illustrated, find the probability that a randomly selected voltage level is greater than 124.5 volts.

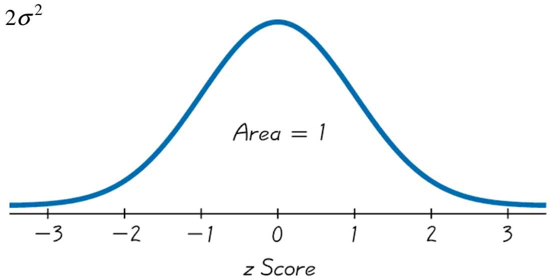


Shaded area represents voltage levels greater than 124.5 volts. The area corresponds to probability: P = 0.25.

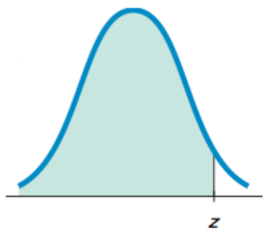
Standard Normal Distribution

The standard normal distribution is a bell-shaped probability distribution with $\mu = 0$ and $\sigma = 1$. The total area under its density curve is equal to 1.

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

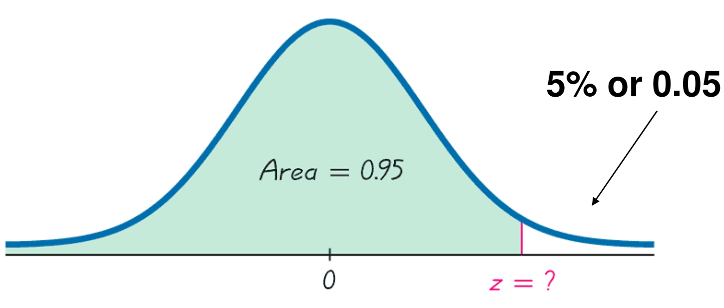


Standard Normal Distribution: Areas and Probabilities



Probability that the standard normal random variable takes values <u>less than *z*</u> is given by the **area** under the curve <u>from the left up to *z*</u> (blue area in the figure)

Finding *z* Scores When Given Probabilities



(*z* score will be positive)

Finding the z-score separating 95% bottom values from 5% top values.

Finding z Scores When Given Probabilities

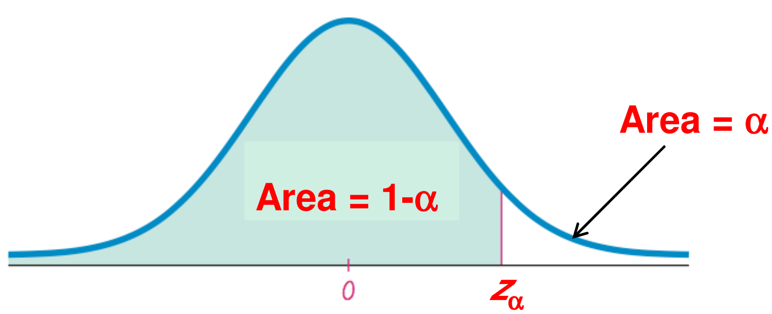
TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

Notation

We use z_α to represent the z-score separating the <u>top</u> α from the <u>bottom</u> **$1-\alpha$**.

Examples: $z_{0.025}$ **= 1.96**, $z_{0.05}$ **= 1.645**



Normal distributions that are not standard

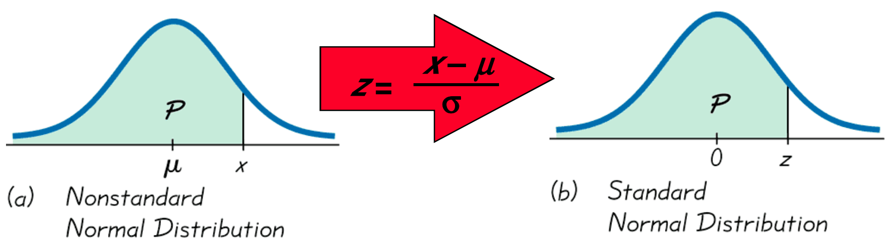
All normal distributions have bell-shaped density curves.

A normal distribution is <u>standard</u> if its mean μ is 0 and its standard deviation σ is 1.

A normal distribution is <u>not standard</u> if its mean μ is not 0, or its standard deviation σ is not 1, or both.

We can use a simple conversion that allows us to <u>standardize</u> any normal distribution so that Table A-2 can be used.

Converting to a Standard Normal Distribution

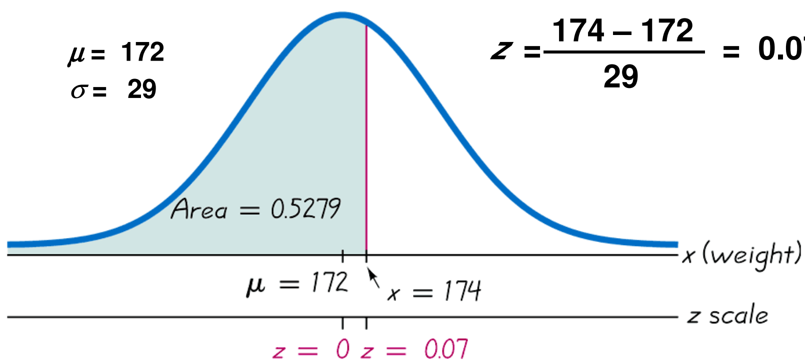


Example – Weights of Passengers

Weights of taxi passengers have a normal distribution with mean 172 lb and standard deviation 29 lb. If one passenger is randomly selected, what is the probability he/she weighs less than 174 pounds?

$\mu = 172$
$\sigma = 29$

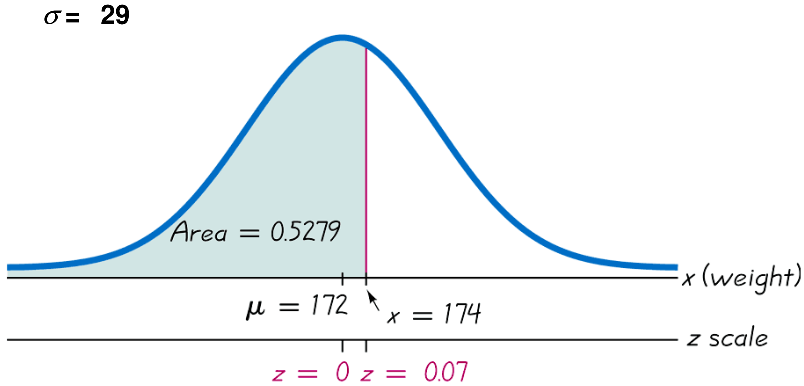
$$z = \frac{174 - 172}{29} = 0.07$$



Example - continued

$\mu = 172$
$\sigma = 29$

P (x < 174 lb.) = P(z < 0.07) = 0.5279



Finding x Scores When Given Probabilities

1. Use Table A-2 to find the z score corresponding to the given probability (the area to the left).

2. Use the values for μ, σ, and the z score found in step 1, to find x:

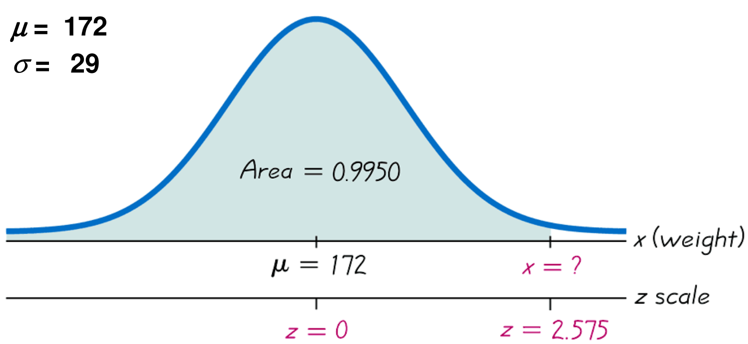
$$x = \mu + (z \cdot \sigma)$$

(If z is located to the left of the mean, be sure that it is a negative number.)

Example – Lightest and Heaviest

Weights of taxi passengers have a normal distribution with mean 172 lb and standard deviation 29 lb. Determine what weight separates the lightest 99.5% from the heaviest 0.5%?

μ = 172
σ = 29

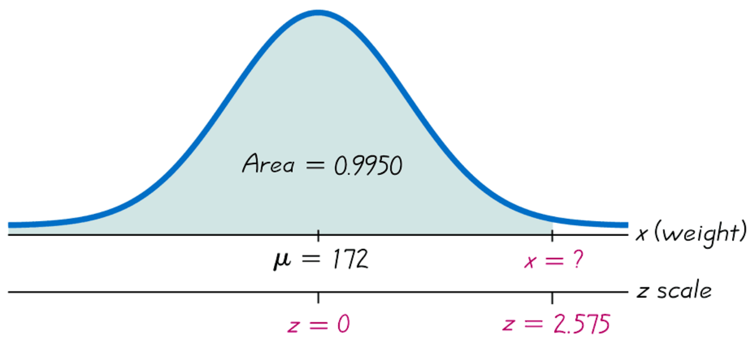


Example – Lightest and Heaviest - cont

$x = \mu + (z \bullet \sigma)$

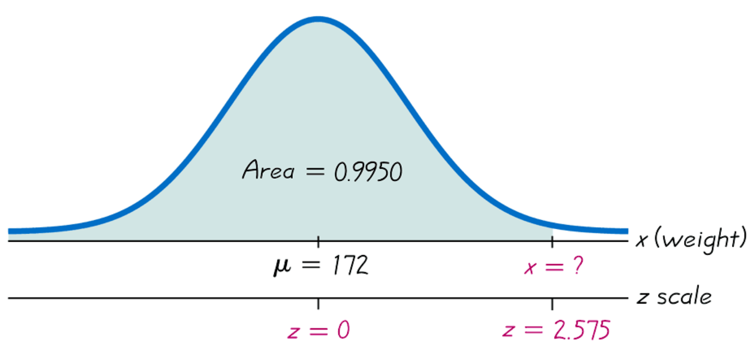
$x = 172 + (2.575 \bullet 29)$

$x = 246.675$ (247 rounded)



Example – Lightest and Heaviest - cont

The weight of 247 pounds separates the lightest 99.5% from the heaviest 0.5%

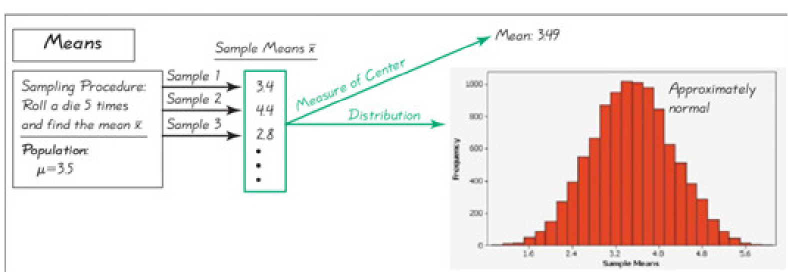


Sampling Distribution of the mean

❖ The sampling distribution of the mean is the distribution of the sample means, with all samples having the same sample size n taken from the same population

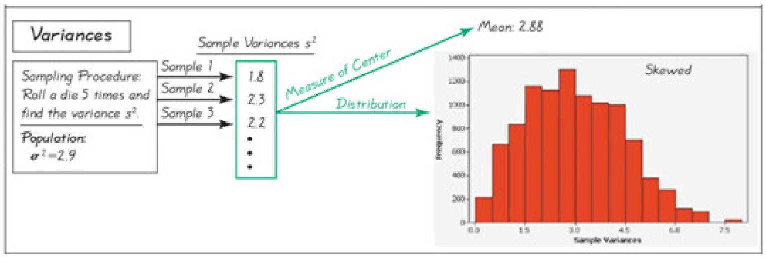
Example

❖ Consider rolling a die 5 times and find the mean x of the results.



Sampling distribution of the variance

❖ The sampling distribution of the variance is the distribution of the sample variances, with all samples having the same size n taken from the population



Central Limit Theorem

The *Central Limit Theorem* tells us that the distribution of the sample mean $\bar{x}$ for a sample of size n approaches a normal distribution, as the sample size n increases.

Central Limit Theorem

Given:

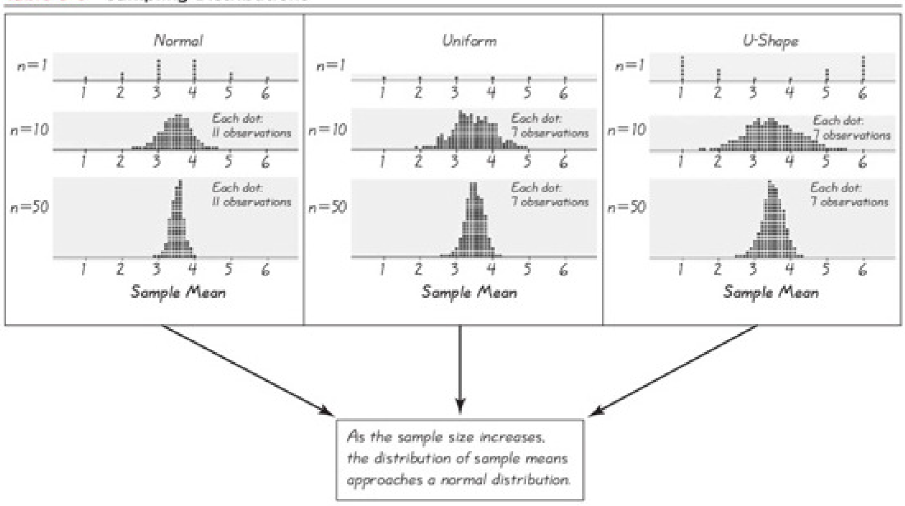
1. The random variable x has a distribution (which may or may not be normal) with mean μ and standard deviation σ.
2. A random sample of size n is selected from the population.

Central Limit Theorem – cont.

Conclusions:

1. The distribution of the sample mean $\bar{x}$ will, as the sample size increases, approach a normal distribution.
2. The mean of that normal distribution is the same as the population mean μ.
3. The standard deviation of that normal distribution is $\sigma/\sqrt{n}$. (So it is smaller than the standard deviation of the population.)

Table 6-6 Sampling Distributions



Formulas

the mean

$$\mu_{\bar{x}} = \mu$$

the standard deviation

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Practical Rules:

1. For samples of size *n* larger than 30, the distribution of the sample mean can be approximated by a normal distribution.
2. If the original population is *normally distributed*, then for any sample size *n*, the sample means will be normally distributed.

3. We can apply Central Limit Theorem if either $n > 30$ or the original population is normal.

Example: Water Taxi Passengers

Assume the population of taxi passengers is normally distributed with a mean of 172 lb and a standard deviation of 29 lb.

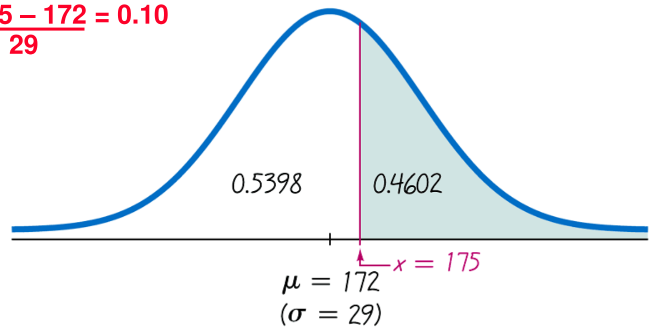
a) Find the probability that if an *individual* passenger is randomly selected, his weight is greater than 175 lb.

b) Find the probability that *20 randomly selected passengers* will have a mean weight that is greater than 175 lb.

Example – cont

a) Find the probability that if an *individual* man is randomly selected, his weight is greater than 175 lb.

$$z = \frac{175 - 172}{29} = 0.10$$

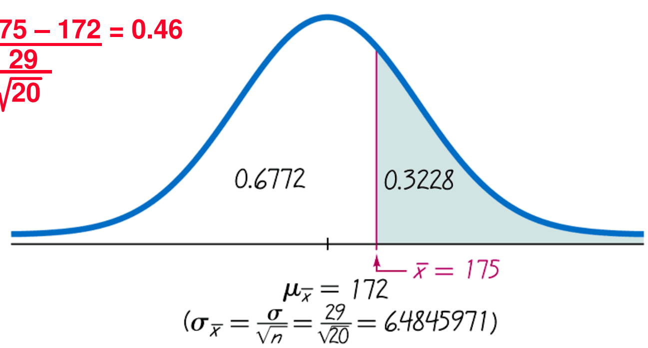


(a)

Example – cont

b) Find the probability that *20 randomly selected men* will have a mean weight that is greater than 175 lb (so that their total weight exceeds the safe capacity of 3500 pounds).

$$z = \frac{175 - 172}{\frac{29}{\sqrt{20}}} = 0.46$$



(b)

Example - cont

a) Find the probability that if an *individual* passenger is randomly selected, his weight is greater than 175 lb.

$$P(x > 175) = 0.4602$$

b) Find the probability that *20 randomly selected passengers* will have a mean weight that is greater than 175 lb.

$$P(\bar{x} > 175) = 0.3228$$

It is much easier for an individual to deviate from the mean than it is for a group of 20 to deviate from the mean.

Recall

Binomial Probability Distribution

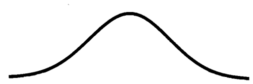
1. The procedure must have a fixed number of trials, n.
2. The trials must be independent.
3. Each trial must have all outcomes classified into two categories (commonly, success and failure).
4. The probability of success p remains the same in all trials (the probability of failure is $q=1-p$)

Approximation of a Binomial Distribution with a Normal Distribution

If $np \geq 5$ and $nq \geq 5$

Then $\mu = np$ and $\sigma = \sqrt{npq}$

and the random variable has

a  distribution.

(normal)

Procedure for Using a Normal Distribution to Approximate a Binomial Distribution

1. Verify that both $np \geq 5$ and $nq \geq 5$. If not, you cannot use normal approximation to binomial.
2. Find the values of the parameters μ and σ by calculating $\mu = np$ and $\sigma = \sqrt{npq}$.
3. Identify the discrete whole number x that is relevant to the binomial probability problem. Use the continuity correction (see next).

Continuity Correction

When we use the normal distribution (which is a continuous probability distribution) as an approximation to the binomial distribution (which is discrete), a continuity correction is made to a whole number x in the binomial distribution by representing the number x by the interval from

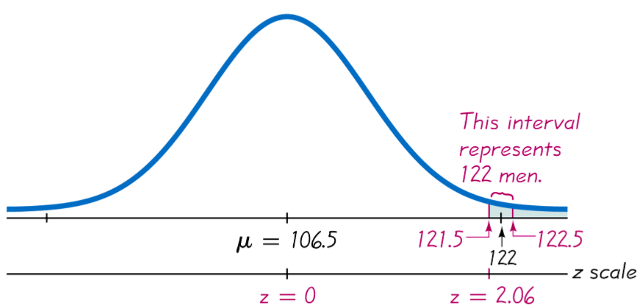
$$x - 0.5 \text{ to } x + 0.5$$

(that is, adding and subtracting 0.5).

31

Example:

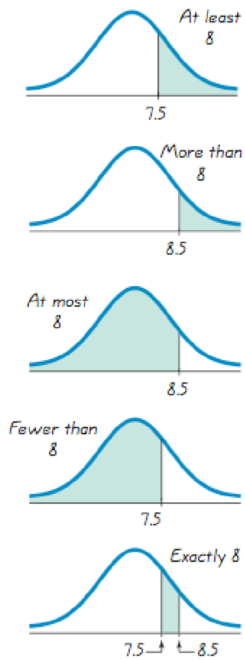
Finding the Probability of "At Least 122 Men" Among 213 Passengers



The value 122 is represented by the interval (121.5,122.5)

The values "at least 122 men" are represented by the interval starting at 121.5.

32



at least 8
(includes 8 and above)

more than 8
(doesn't include 8)

at most 8
(includes 8 and below)

fewer than 8
(doesn't include 8)

exactly 8