

## Chapter 6. Continuous Random Variables

Reminder:

**Continuous random variable**  
takes infinite values

Those values can be associated with measurements on a continuous scale  
(without gaps or interruptions)

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### Example: Uniform Distribution

A continuous random variable has a **uniform distribution** if its values are spread **evenly** over a certain range.

**Example:**

voltage output of an electric generator is between 123 V and 125 V, equally likely. The actual voltage level may be anywhere in this range.

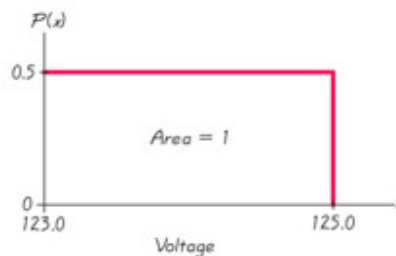


Figure 6-2 Uniform Distribution of Voltage Levels

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## Density Curve

A **density curve** is the graph of a continuous probability distribution. It must satisfy the following properties:

1. The total area under the curve must equal 1.
2. Every point on the curve must have a vertical height that is 0 or greater.  
(That is, the curve cannot fall below the x-axis.)

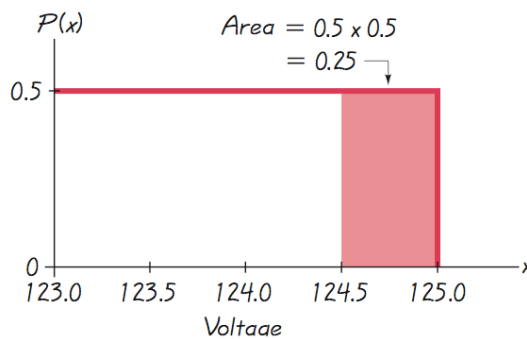
2

## Area and Probability

Because the total area under the density curve is equal to 1, there is a correspondence between **area** and **probability**.

## Using Area to Find Probability

Given the uniform distribution illustrated, find the probability that a randomly selected voltage level is greater than 124.5 volts.



Shaded area represents voltage levels greater than 124.5 volts.

The area corresponds to probability:  
 $P = 0.25$ .

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## Standard Normal Distribution

**Standard normal distribution** has three properties:

1. Its graph is **bell-shaped**.
2. It is **symmetric** about its center.
3. Its mean is equal to 0 ( $\mu = 0$ ).
4. Its standard deviation is equal to 1 ( $\sigma = 1$ ).

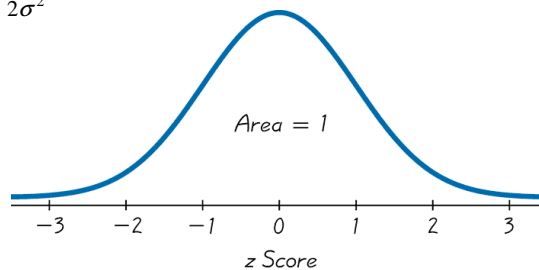
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## Standard Normal Distribution

The **standard normal distribution** is a bell-shaped probability distribution with  $\mu = 0$  and  $\sigma = 1$ . The total area under its density curve is equal to 1.

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



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## Standard Normal Distribution: Areas and Probabilities



**Probability** that the standard normal random variable takes values less than  $z$  is given by the **area** under the curve from the left up to  $z$  (blue area in the figure)

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## Example – Thermometers

Thermometers are supposed to give readings of  $0^{\circ}\text{C}$  at the freezing point of water.

Since these instruments are not perfect, some of them give readings below  $0^{\circ}$  (denoted by negative numbers) and some give readings above  $0^{\circ}$  (denoted by positive numbers).

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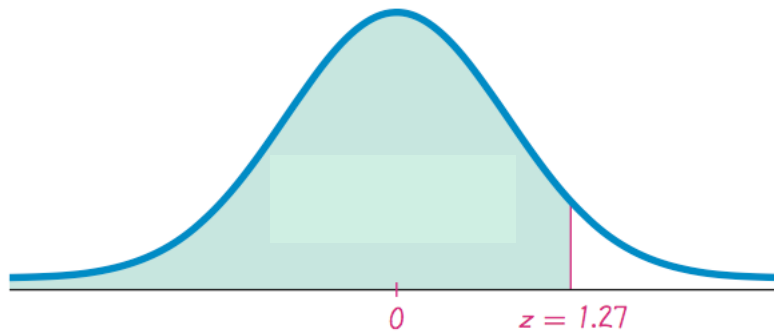
## Example – Thermometers (continued)

Assume that the mean reading is  $0^{\circ}\text{C}$  and the standard deviation of the readings is  $1^{\circ}\text{C}$ . Also assume that the readings are normally distributed.

**Q:** If one thermometer is randomly selected, find the probability that, at the freezing point of water, the reading is less than  $1.27^{\circ}$ .

## Example – Thermometers (continued)

$P(z < 1.27) =$   
area under the curve from the left up to 1.27



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## Look at Table A-2

**TABLE A-2** (continued) Cumulative Area from the LEFT

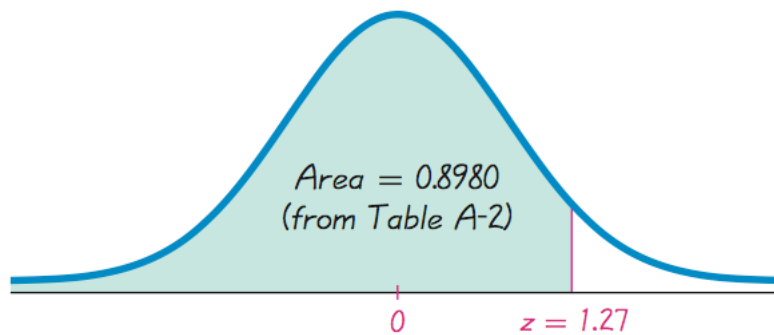
<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064
<hr/>								
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292

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## Example – Thermometers (continued)

$$P(z < 1.27) = 0.8980$$



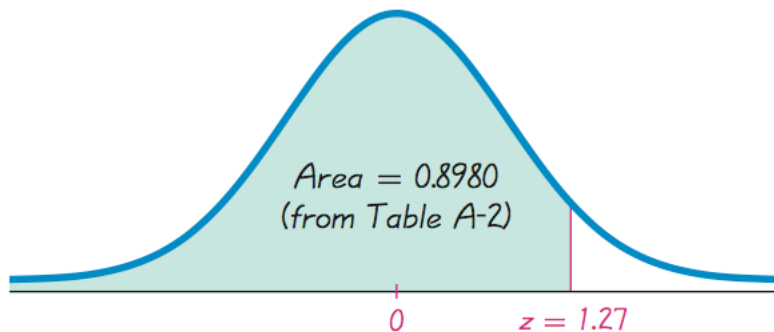
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## Example – Thermometers (continued)

$$P(z < 1.27) = 0.8980$$



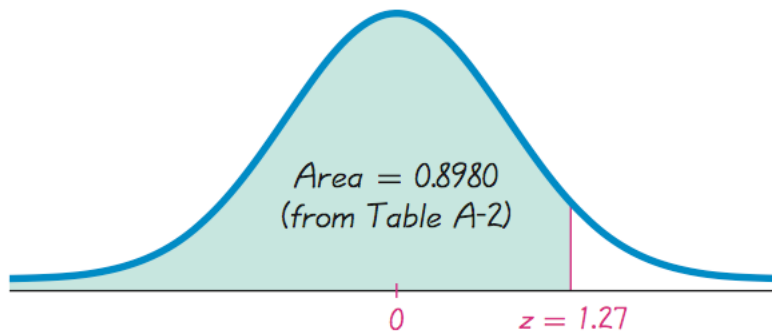
The *probability* of randomly selecting a thermometer with a reading less than  $1.27^\circ$  is 0.8980.

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## Example – Thermometers (continued)

$$P(z < 1.27) = 0.8980$$



Or 89.80% of randomly selected thermometers will have readings below 1.27°.

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## Using Table A-2

1. It is designed only for the **standard normal distribution**, which has a mean of 0 and a standard deviation of 1.
2. It is on two pages, with one page for *negative* z-scores and the other page for *positive* z-scores.
3. Each value in the body of the table is a *cumulative area from the left* up to a vertical boundary above a specific z-score.

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## Using Table A-2

4. When working with a graph, avoid confusion between z-scores and areas.

### z Score

**Distance** along horizontal scale of the standard normal distribution; refer to the leftmost column and top row of Table A-2.

### Area

**Region** under the curve; refer to the values in the body of Table A-2.

5. The part of the z-score denoting hundredths is found across the top.

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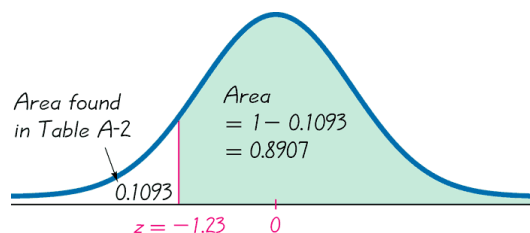
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## Example - Thermometers Again

If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above  $-1.23$  degrees.

$$P(z > -1.23) = 0.8907$$



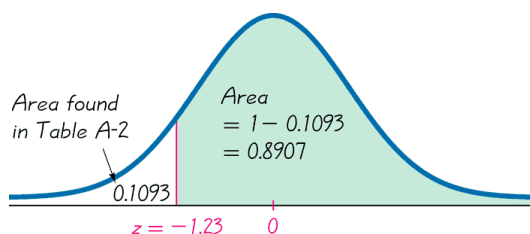
**Probability of randomly selecting a thermometer with a reading above  $-1.23^\circ$  is 0.8907.**

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## Example - cont

$$P(z > -1.23) = 0.8907$$

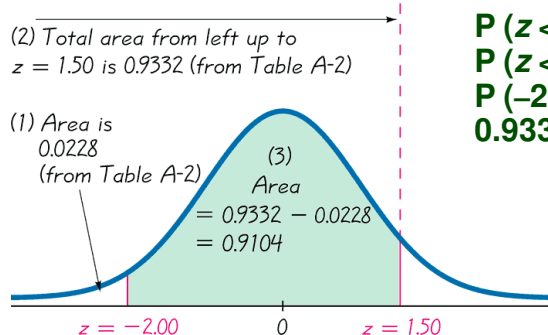


**89.07% of the thermometers have readings above  $-1.23$  degrees.**

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## Example - Thermometers III

A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) **between  $-2.00$  and  $1.50$  degrees.**

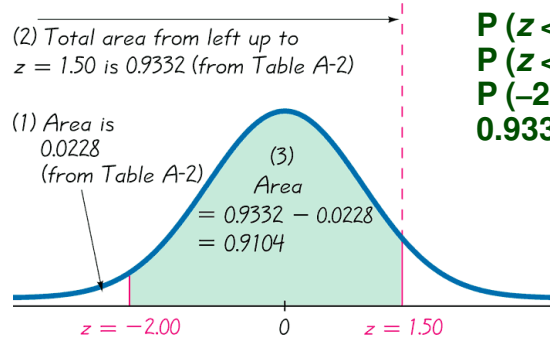


$$\begin{aligned} P(z < -2.00) &= 0.0228 \\ P(z < 1.50) &= 0.9332 \\ P(-2.00 < z < 1.50) &= 0.9332 - 0.0228 = 0.9104 \end{aligned}$$

**The probability that the chosen thermometer has a reading between  $-2.00$  and  $1.50$  degrees is **0.9104**.**

## Example - continued

A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between **-2.00** and **1.50** degrees.



$$\begin{aligned}P(z < -2.00) &= 0.0228 \\P(z < 1.50) &= 0.9332 \\P(-2.00 < z < 1.50) &= \\0.9332 - 0.0228 &= 0.9104\end{aligned}$$

If many thermometers are selected and tested at the freezing point of water, then **91.04%** of them will read between **-2.00** and **1.50** degrees.

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## Notation

$$P(a < z < b)$$

denotes the probability that the  $z$  score is between  $a$  and  $b$ .

$$P(z > a)$$

denotes the probability that the  $z$  score is greater than  $a$ .

$$P(z < a)$$

denotes the probability that the  $z$  score is less than  $a$ .

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## Special Cases in Table A-2

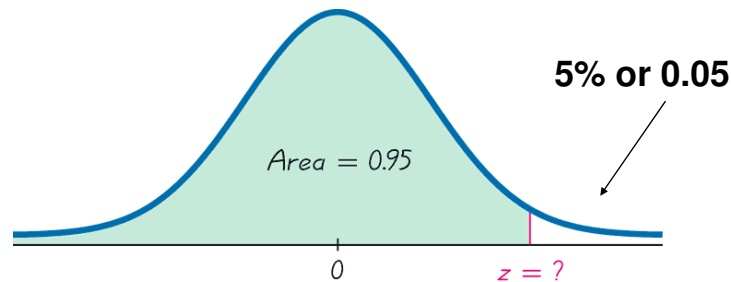
- ❖ If z-score is **above 3.49**, then the area=**0.9999**
- ❖ If z-score is **below -3.49**, then the area=**0.0001**
- ❖ Some special values are marked by stars:
  - z-score=**1.645** corresponds to the area=**0.95**
  - z-score=**2.575** corresponds to the area=**0.995**

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## Finding a z Score When Given a Probability Using Table A-2

1. Draw a bell-shaped curve and identify the region under the curve that corresponds to the given probability. If that region is NOT a cumulative region from the left, work instead with a known region that is a cumulative region from the left.
2. Using the cumulative area from the left, locate the closest probability in the **body** of Table A-2 and identify the corresponding z score.

## Finding z Scores When Given Probabilities



(z score will be positive)

Finding the z-score separating 95% bottom values from 5% top values.

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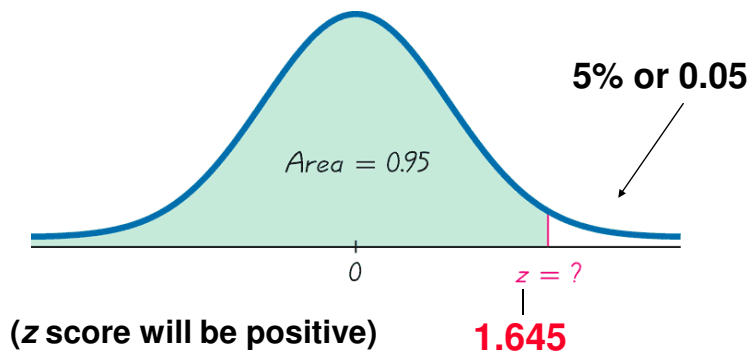
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## Finding z Scores When Given Probabilities

TABLE A-2 (continued) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

## Finding z Scores When Given Probabilities



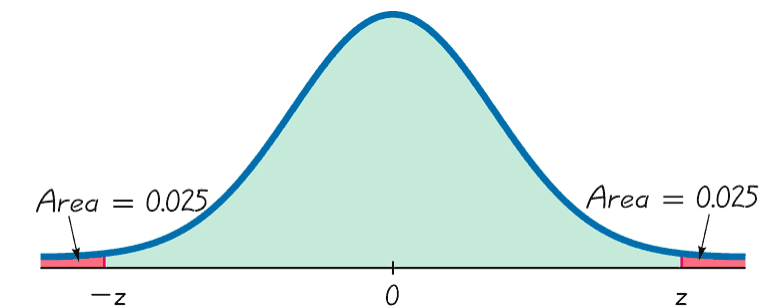
Finding the z-score separating 95% bottom values from 5% top values.

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## Finding z Scores When Given Probabilities - cont



(One z score will be negative and the other positive)

Finding the Bottom 2.5% and Upper 2.5%

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**TABLE A-2** Standard Normal (z) Distribution: Cumulative Area from the LEFT

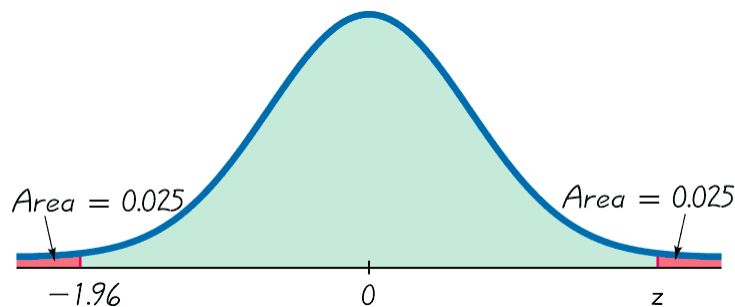
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.50 and lower	.0001									
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294

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## Finding z Scores When Given Probabilities - cont



(One z score will be negative and the other positive)

**Finding the Bottom 2.5% and Upper 2.5%**

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$$1 - .025 = .975$$

**TABLE A-2** (continued) Cumulative Area from the LEFT

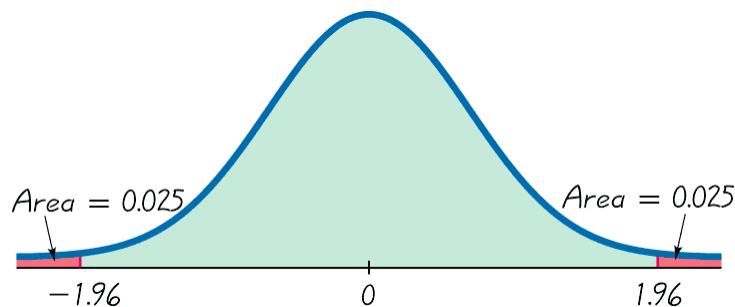
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

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## Finding z Scores When Given Probabilities - cont



(One z score will be negative and the other positive)

Finding the Bottom 2.5% and Upper 2.5%

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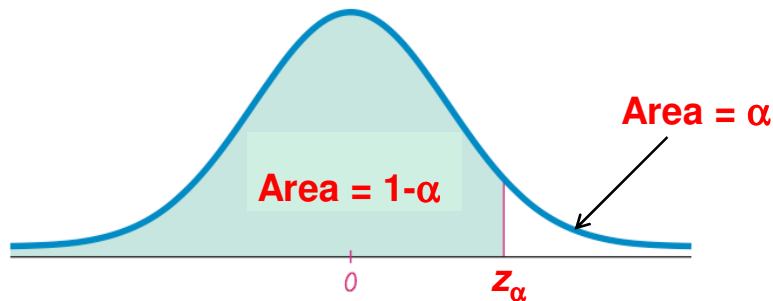
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## Notation

We use  $z_\alpha$  to represent the z-score separating the top  $\alpha$  from the bottom  $1-\alpha$ .

Examples:  $z_{0.025} = 1.96$ ,  $z_{0.05} = 1.645$



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## Normal distributions that are not standard

All **normal distributions** have **bell-shaped** density curves.

A normal distribution is standard if its mean  $\mu$  is 0 and its standard deviation  $\sigma$  is 1.

A normal distribution is not standard if its mean  $\mu$  is not 0, or its standard deviation  $\sigma$  is not 1, or both.

We can use a simple conversion that allows us to standardize any normal distribution so that Table A-2 can be used.

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## Conversion Formula

Let  $x$  be a score for a normal distribution with mean  $\mu$  and standard deviation  $\sigma$

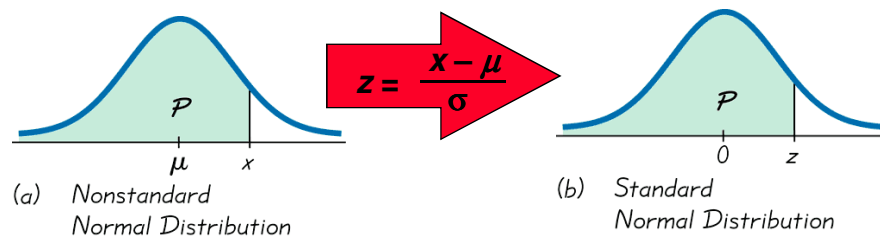
We convert it to a  $z$  score by this formula:

$$z = \frac{x - \mu}{\sigma}$$

(round  $z$  scores to 2 decimal places)

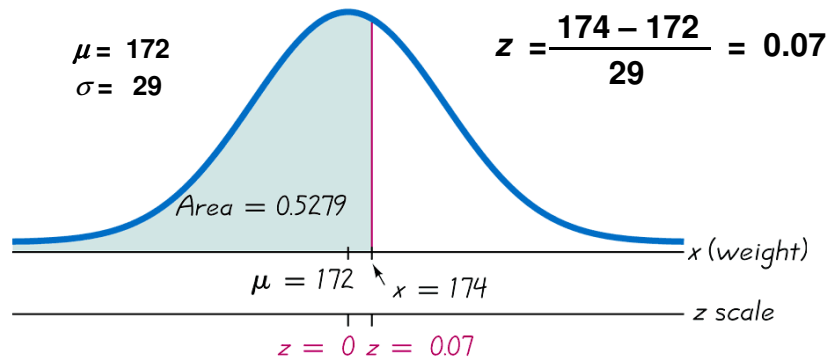
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## Converting to a Standard Normal Distribution



## Example – Weights of Passengers

Weights of taxi passengers have a normal distribution with mean 172 lb and standard deviation 29 lb. If one passenger is randomly selected, what is the probability he/she weighs less than 174 pounds?

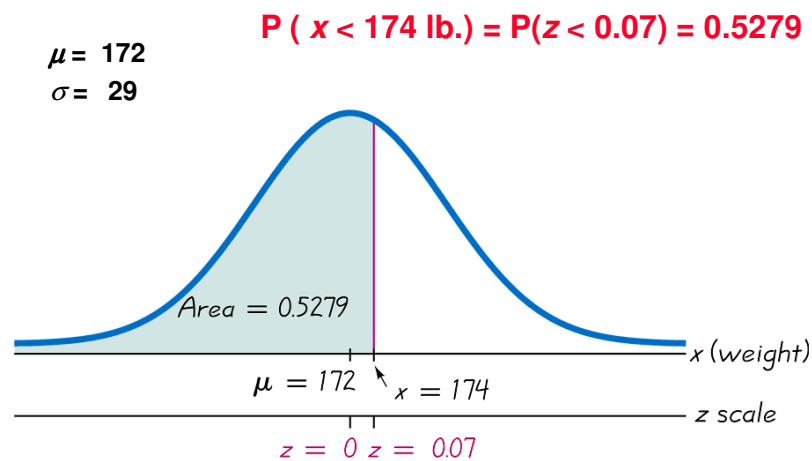


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## Example - continued



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## Finding $x$ Scores When Given Probabilities

1. Use Table A-2 to find the  $z$  score corresponding to the given probability (the area to the left).
2. Use the values for  $\mu$ ,  $\sigma$ , and the  $z$  score found in step 1, to find  $x$ :

$$x = \mu + (z \cdot \sigma)$$

(If  $z$  is located to the left of the mean, be sure that it is a negative number.)

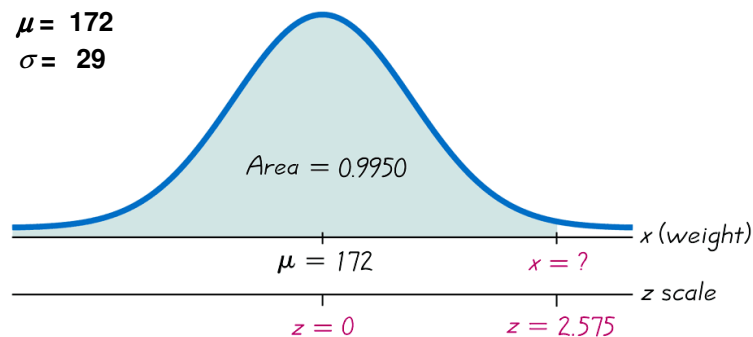
20

## Example – Lightest and Heaviest

Weights of taxi passengers have a normal distribution with mean 172 lb and standard deviation 29 lb. Determine what weight separates the lightest 99.5% from the heaviest 0.5%?

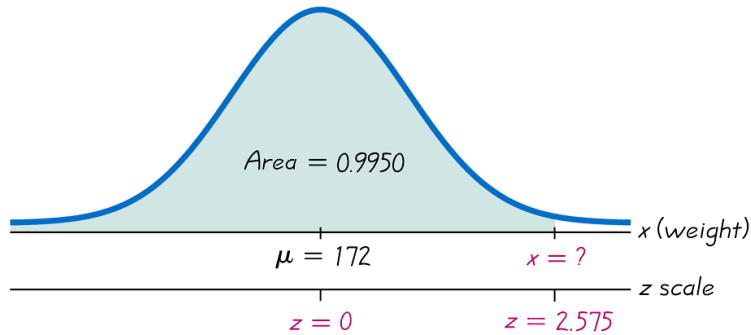
$$\mu = 172$$

$$\sigma = 29$$



## Example – Lightest and Heaviest - cont

$$\begin{aligned}x &= \mu + (z \cdot \sigma) \\x &= 172 + (2.575 \cdot 29) \\x &= 246.675 \text{ (247 rounded)}\end{aligned}$$



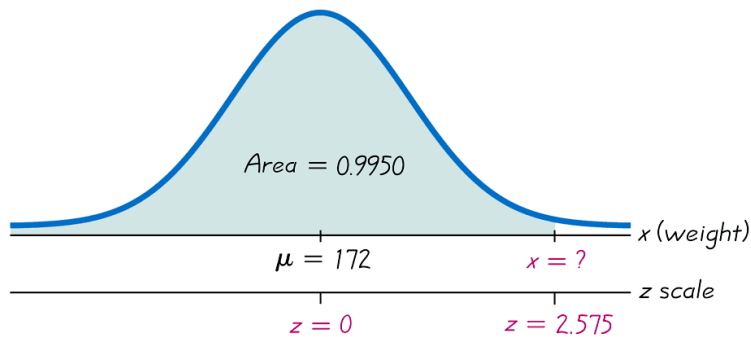
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## Example – Lightest and Heaviest - cont

The weight of 247 pounds separates the  
lightest 99.5% from the heaviest 0.5%



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