Key Concept

If the outcome of the first event *A* somehow affects the probability of the second event *B*, it is important to adjust the probability of *B* to reflect the occurrence of event *A*.

Conditional Probability
Important Principle

The probability for the second event *B* should take into account the fact that the first event *A* has already occurred.

Notation for Conditional Probability

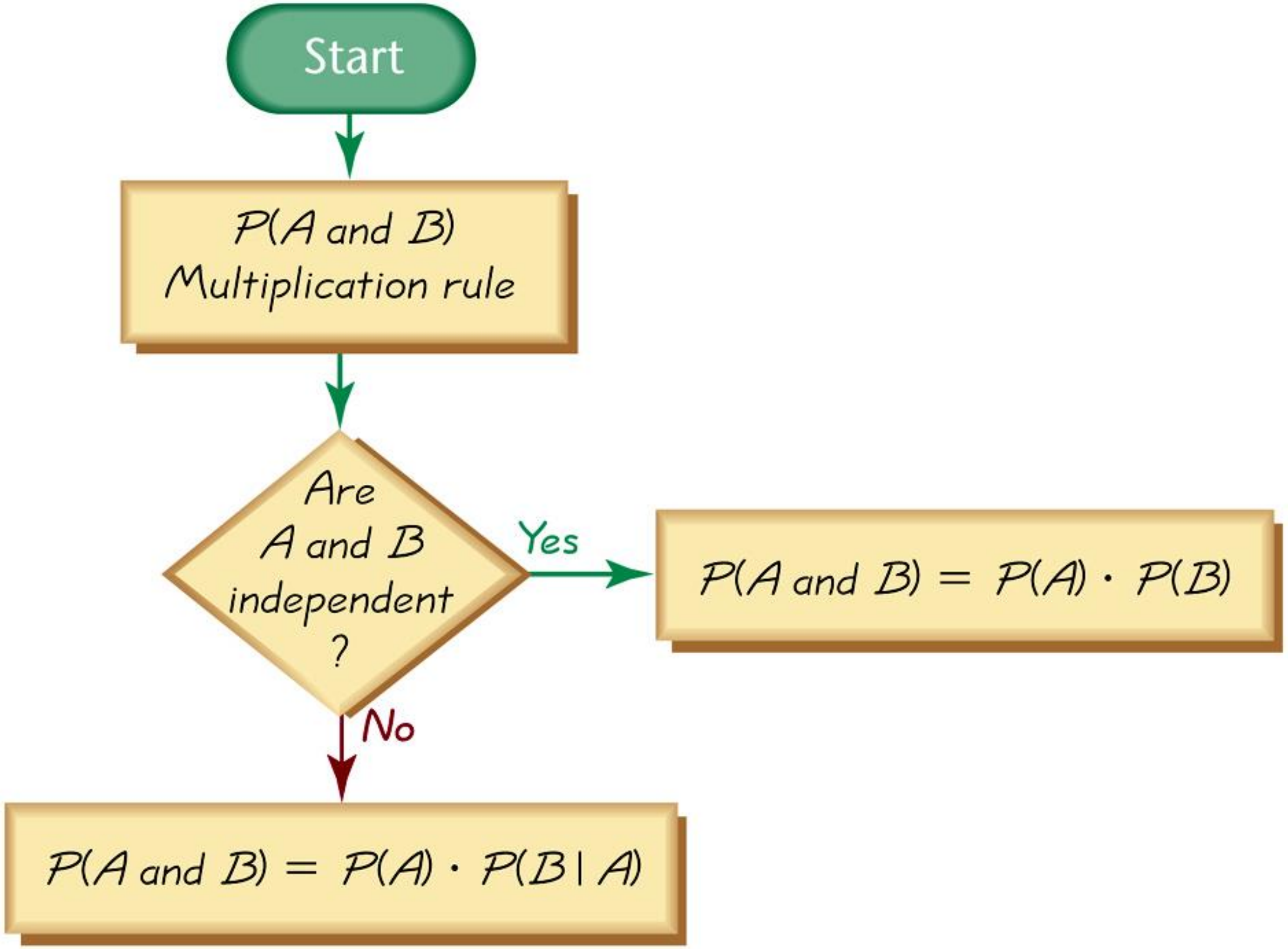
***P*(*B*|*A*) represents the probability of event *B* occurring after it is assumed that event *A* has already occurred**

(read *B*|*A* as "*B* given *A*")

Intuitive Multiplication Rule

When finding the probability that event *A* occurs in one trial and event *B* occurs in the next trial, multiply the probability of event *A* by the probability of event *B*, but be sure that the probability of event *B* takes into account the previous occurrence of event *A*.

Applying the Multiplication Rule



Multiplication Rule for Several Independent Events

In general, the probability of any sequence of independent events is simply the product of their corresponding probabilities.

Selections with replacement and without replacement

Selections with replacement are always independent.

Selections without replacement are always dependent.

Treating Dependent Events as Independent

Some calculations are awkward, but they can be made manageable by using the common practice of treating events as independent when small samples are drawn from large populations. In such cases, it is rare to select the same item twice.

EXAMPLE 4 **Birthdays** Assume that two people are randomly selected and also assume that birthdays occur on the days of the week with equal frequencies.

a. Find the probability that the two people are born on the same day of the week.

b. Find the probability that the two people are both born on Monday.

- The prob. That the second is born is the same “some” day is just 1/7

- Each one on Mon. 1/7 *1/7

The 5% Guideline for Cumbersome Calculations

If a sample size is no more than 5% of the size of the population, treat the selections as being <u>independent</u> (even if the selections are made without replacement, so they are technically dependent).

Computing the probability of "at least" one event

Find the probability that among several trials, we get at least one of some specified event.

"At least one" is equivalent to "one or more"

The complement of getting at least one event of a particular type is that you get no events of that type

(*either* none *or* at least one)

To find the probability of at least one of something, calculate the probability of none, then subtract that result from 1. That is,

$$P(\text{at least one}) = 1 - P(\text{none})$$

Some Examples

- **Are the following simple events or not?**
  - **Throwing Three coins**

-> All coins show head

Yes

-> Exactly one coin shows head

No

-> At least one coin show head

No

Some Examples

- What is the complement of the following

1- A: Every one failed the exam

Ans: someone passed the exam

2- At most 5 students got an A in the exam

Ans: At least 6 students got an A in the exam

- Two fair 6-sided dice are rolled. What is the probability that their sum is 7 or exactly one die is 2

P(A or B) = P (A) + P(B) – P(A and B)

= 6/36 + 10/36- 2/36 = 7/18

Some Examples

- Suppose that two cards are drown from a standard 52 card deck without replacement. Find the probability

1- both cards are two

Ans: 4/52 * 3/51 = 1/221

2- both cards are hearts

Ans: 13/52 * 12/51 = 156/2652

- The first card is an ace and the second card is 2

Ans: 4/52 * 4/51 = 4/663

Some Examples

- Suppose that two cards are drown from a standard 52 card deck with replacement. Find the probability

1- both cards are two

Ans: 4/52 * 4/52 = 1/169

2- both cards are hearts

Ans: 13/52 * 13/52 = 1/16

- The first card is an ace and the second card is 2

Ans: 4/52 * 4/52 = 1/169

Some Examples

- A card is drawn from a standard 52-cards. Find the probability that the card is

1- A king given that the card is heart

Ans: (# of hearts that are kings)/(# of hears)= 1/13

2- A heart given that the card is a king

Ans: (# of kings that are hearts)/(#of kings)= 1/4

- The first card is 1 and the second card is 2

Ans: 4/52 * 4/52 = 1/169

Random Variables
Chapter 5

- **Random variable**
  a variable (typically represented by *x*) that takes a numerical value by chance.
- **For each outcome of a procedure, *x* takes a certain value, but for different outcomes that value may be different.**

Examples:

- Number of boys in a randomly selected family with three children. Possible values: $x=0,1,2,3$

- The weight of a randomly selected person from a population.
  Possible values: positive numbers, $x>0$

Discrete and Continuous Random Variables

- **Discrete random variable**
  either a finite number of values or countable number of values (resulting from a counting process)

- **Continuous random variable**
  infinite values, and those values can be associated with measurements on a continuous scale without gaps or interruptions

Probability Distributions

- Probability distribution
  a description that gives the probability for <u>each value</u> of the random variable;
- Three ways to describe a distribution
  - Formula
  - Table
  - Graph

Method 1: Using the Binomial Probability Formula

$$P(x) = \frac{n!}{(n - x)!x!} \cdot p^x \cdot q^{n-x}$$

for $x = 0, 1, 2, . . ., n$

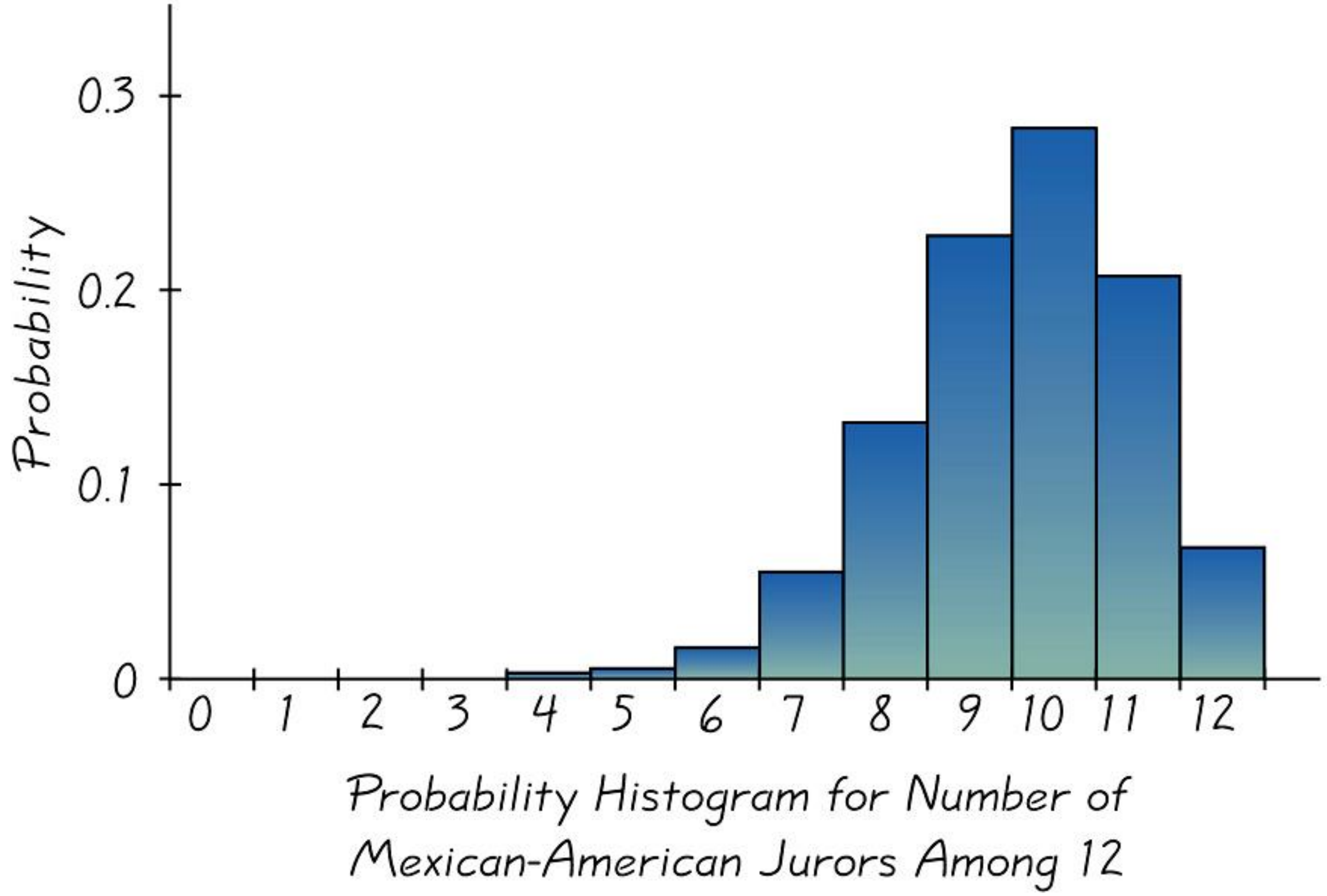
Tables

Values: Probabilities:

x	P(x)
0	1/8
1	3/8
2	3/8
3	1/8

Graphs

The probability histogram is very similar to a relative frequency histogram, but the vertical scale shows probabilities.



Requirements for Probability Distribution

$$\sum P(x) = 1$$

where *x* assumes all possible values.

$$0 \leq P(x) \leq 1$$

for every individual value of *x*.

Mean, Variance and Standard Deviation of a Probability Distribution

$\mu = \Sigma [x \cdot P(x)]$ Mean

$\sigma^2 = \Sigma [(x - \mu)^2 \cdot P(x)]$ Variance

$\sigma^2 = \Sigma [x^2 \cdot P(x)] - \mu^2$ Variance (shortcut)

$\sigma = \sqrt{\Sigma [x^2 \cdot P(x)] - \mu^2}$ Standard Deviation

Example

- Find the mean and Standard deviation of the following discrete probability distribution

x	P(x)		
0	0		
1	.15		
2	.45		
3	.20		
4	.20		

- $\sigma^2 = \Sigma\,[x^2 \cdot P(x)] - \mu^2$
- $\mu = \Sigma\,[x \cdot P(x)]$

Example

- Find the mean and Standard deviation of the following discrete probability distribution

x	P(x)	xP(x)	
0	0	0	
1	.15	.15	
2	.45	.9	
3	.20	.6	
4	.20	.4	

Example

- Find the mean and Standard deviation of the following discrete probability distribution

x	P(x)	xP(x)	$x^2P(x)$
0	0	0	0
1	.15	.15	.15
2	.45	.9	1.8
3	.20	.6	1.8
4	.20	.4	1.6

Example

- Find the mean and Standard deviation of the following discrete probability distribution

x	P(x)	xP(x)	$x^2P(x)$
0	0	0	0
1	.15	.15	.15
2	.45	.9	1.8
3	.20	.6	1.8
4	.20	.4	1.6

$$\mu = \Sigma\,[x \bullet P(x)] = 2.05$$

$$\sigma^2 = \Sigma\,[x^2 \bullet P(x)] - \mu^2$$

$$= 1.07$$

Roundoff Rule for μ, σ, and σ^2

Round results by carrying one more decimal place than the number of decimal places used for the random variable x.

If the values of x are integers, round μ, σ, and σ^2 to one <u>(better two)</u>decimal place.

Identifying *Unusual* Results

Range Rule of Thumb

According to the range rule of thumb, most values should lie within 2 standard deviations of the mean.

We can therefore identify "unusual" values by determining if they lie outside these limits:

Maximum usual value = $\mu + 2\sigma$

Minimum usual value = $\mu - 2\sigma$

Identifying *Unusual* Results By Probabilities

Using Probabilities to Determine When Results Are Unusual:

- **Unusually high: a particular value x is unusually high if $P(x \text{ or more}) \leq 0.05$.**

- **Unusually low: a particular value x is unusually low if $P(x \text{ or fewer}) \leq 0.05$.**

Bernoulli Trials

Any simple trial with two possible outcomes. p and q

EX: Tossing a coin, repeat, with counting # of success p “ the number of heads”,

Then # of failure q=(1-p) , “ # of tails”

P(HHT) =p.p.q

P(TTT)=q.q.q

If we have k as the number of successes and n-k failures

Then the probability is

$$p^k q^{n-k}$$

Binomial Random Variable

If we have $X : S \to \{0,1,2,3\}$

Where X is the number of successes

$$X(sss) = 3$$

$$X(sfs) = X(ssf) = 2$$

X now is a random variable.

X is named Binomial random variable resulted from n Bernoulli trials denoted:

Binomial Random Variable

Now the probability that $X = k$ $0 \leq k \leq n$

that is all strings with k success and $n - k$ fails

there are $\binom{n}{k}$ different ways

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

Binomial Probability Distribution

A binomial probability distribution results from a procedure that meets all the following requirements:

1. **The procedure has a fixed number of trials.**
2. **The trials must be independent. (The outcome of any individual trial doesn't affect the probabilities in the other trials.)**
3. **Each trial must have all outcomes classified into two categories (commonly referred to as success and failure).**
4. **The probability of a success remains the same in all trials.**

Notation for Binomial Probability Distributions

S and F (success and failure) denote the two possible categories of all outcomes; *p* and *q* denote the probabilities of S and F, respectively:

$P(S) = p$ (*p* = probability of success)

$P(F) = 1 - p = q$ (*q* = probability of failure)

Notation (continued)

n	**denotes the fixed number of trials.**
x	**denotes a specific number of successes in n trials, so x can be any whole number between 0 and n, inclusive.**
p	**denotes the probability of success in one of the n trials.**
q	**denotes the probability of failure in one of the n trials.**
$P(x)$	**denotes the probability of getting exactly x successes among the n trials.**

Methods for Finding Probabilities

We will now discuss one methods for finding the probabilities corresponding to the random variable *x* in a binomial distribution.

Other methods like Table at the end of the book , or the calculator

Method 1: Using the Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x}$$

for $x = 0, 1, 2, \ldots, n$

where

n = number of trials

x = number of successes among n trials

p = probability of success in any one trial

q = probability of failure in any one trial ($q = 1 - p$)

Rationale for the Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x}$$

Number of outcomes with exactly *x* successes among *n* trials

Rationale for the Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x}$$

Number of outcomes with exactly *x* successes among *n* trials (for $\frac{n!}{(n-x)!x!}$)

The probability of *x* successes among *n* trials for any one particular order (for $p^x \cdot q^{n-x}$)

Binomial Distribution: Formulas

Mean $\mu = n \cdot p$

Variance $\sigma^2 = n \cdot p \cdot q$

Std. Dev. $\sigma = \sqrt{n \cdot p \cdot q}$

Where

n **= number of fixed trials**

p **= probability of success in one of the *n* trials**

q **= probability of failure in one of the *n* trials**

Interpretation of Results

It is especially important to interpret results. The range rule of thumb suggests that values are unusual if they lie outside of these limits:

Maximum usual values $= \mu + 2\sigma$

Minimum usual values $= \mu - 2\sigma$