Some Examples

- **Are the following simple events or not?**
  - **Throwing Three coins**

-> All coins show head

Yes

-> Exactly one coin shows head

No

-> At least one coin show head

NO

Some Examples

- What is the complement of the following

1- A: Every one failed exam

Ans: someone passed the exam

2- At most 5 students got an A in the exam

Ans: At least 6 students got an A in the exam

- Two fair 6-sided dice are rolled. What is the probability that their sum is 7 or exactly one die is 2

P(A or B) = P (A) + P(B) – P(A and B)

= 6/36 + 10/36- 2/36 = 7/18

Some Examples

- Suppose that two cards are drown from a standard 52 card deck without replacement. Find the probability

1- both cards are two

Ans: 4/52 * 3/51 = 1/221

2- both cards are hearts

Ans: 13/52 * 12/52 = 1/17

- The first card is an ace and the second card is 2

Ans: 4/52 * 4/51 = 4/663

Some Examples

- Suppose that two cards are drown from a standard 52 card deck with replacement. Find the probability

1- both cards are two

Ans: 4/52 * 4/52 = 1/169

2- both cards are hearts

Ans: 13/52 * 13/52 = 1/16

- The first card is an ace and the second card is 2

Ans: 4/52 * 4/52 = 1/169

Some Examples

- A card is drawn from a standard 52-cards. Find the probability that the card is

1- A king given that the card is heart

Ans: (# of hearts that are kings)/(# of hears)= 1/13

2- A heart given that the card is a king

Ans: (# of kings)/(#of kings that are hears)= 1/4

- The first card is an ace and the second card is 2

Ans: 4/52 * 4/52 = 1/169

Random Variables
Chapter 5

- **Random variable**
  a variable (typically represented by *x*) that takes a numerical value by chance.
- **For each outcome of a procedure, *x* takes a certain value, but for different outcomes that value may be different.**

Examples:

- Number of boys in a randomly selected family with three children.
  Possible values: x=0,1,2,3

- The weight of a randomly selected person from a population.
  Possible values: positive numbers, x>0

Discrete and Continuous Random Variables

- Discrete random variable
  either a finite number of values or countable number of values (resulting from a counting process)

- Continuous random variable
  infinitely many values, and those values can be associated with measurements on a continuous scale without gaps or interruptions

Probability Distributions

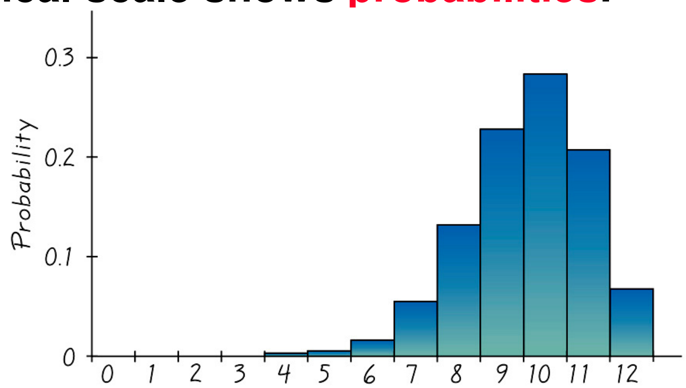
- **Probability distribution**
  a description that gives the probability for <u>each value</u> of the random variable; often expressed in the format of a table, graph, or formula
- Three ways to describe a distribution
  - Formula
  - Table
  - Graph

Tables

Values: x	Probabilities: P(x)
0	1/8
1	3/8
2	3/8
3	1/8

Graphs

The probability histogram is very similar to a relative frequency histogram, but the vertical scale shows probabilities.



Probability Histogram for Number of Mexican-American Jurors Among 12

Requirements for Probability Distribution

$$\sum P(x) = 1$$

where *x* assumes all possible values.

$$0 \leq P(x) \leq 1$$

for every individual value of *x*.

Mean, Variance and Standard Deviation of a Probability Distribution

$\mu = \Sigma [x \cdot P(x)]$ **Mean**

$\sigma^2 = \Sigma [(x - \mu)^2 \cdot P(x)]$ **Variance**

$\sigma^2 = \Sigma [x^2 \cdot P(x)] - \mu^2$ **Variance (shortcut)**

$\sigma = \sqrt{\Sigma [x^2 \cdot P(x)] - \mu^2}$ **Standard Deviation**

Example

- Find the mean and Standard deviation of the following discrete probability distribution

x	P(x)		
0	.10		
1	.15		
2	.45		
3	.20		
4	.20		

Example

- Find the mean and Standard deviation of the following discrete probability distribution

x	P(x)	xP(x)	$x^2P(x)$
0	.10	0	0
1	.15	.15	.15
2	.45	.9	1.8
3	.20	.6	1.8
4	.20	.4	1.6

$$\mu = \Sigma [x \cdot P(x)] = 2.05$$

$$\sigma^2 = \Sigma [x^2 \cdot P(x)] - \mu^2$$
$$= 1.07$$

Roundoff Rule for μ, σ, and σ^2

Round results by carrying one more decimal place than the number of decimal places used for the random variable *x*.

If the values of *x* are integers, round μ, σ, and σ^2 to one <u>(better two)</u>decimal place.

Identifying *Unusual* Results
Range Rule of Thumb

According to the range rule of thumb, most values should lie within 2 standard deviations of the mean.

We can therefore identify “unusual” values by determining if they lie outside these limits:

Maximum usual value = $\mu + 2\sigma$

Minimum usual value = $\mu - 2\sigma$

Identifying *Unusual* Results
By Probabilities

Using Probabilities to Determine When Results Are Unusual:

- Unusually high: a particular value *x* is unusually high if P(*x* or more) ≤ 0.05.
- Unusually low: a particular value *x* is unusually low if P(*x* or fewer) ≤ 0.05.

Binomial Probability Distribution

A **binomial probability distribution** results from a procedure that meets all the following requirements:

1. The procedure has a **fixed number of trials**.
2. The trials must be **independent**. (The outcome of any individual trial doesn't affect the probabilities in the other trials.)
3. Each trial must have all outcomes classified into **two categories** (commonly referred to as **success** and **failure**).
4. The probability of a success remains the same in all trials.

Notation for Binomial Probability Distributions

S and F (success and failure) denote the two possible categories of all outcomes; p and q denote the probabilities of S and F, respectively:

$P(S) = p$ (p = probability of success)

$P(F) = 1 - p = q$ (q = probability of failure)

Notation (continued)

n	denotes the fixed number of trials.
x	denotes a specific number of successes in n trials, so x can be any whole number between 0 and n, inclusive.
p	denotes the probability of success in one of the n trials.
q	denotes the probability of failure in one of the n trials.
$P(x)$	denotes the probability of getting exactly x successes among the n trials.

Methods for Finding Probabilities

We will now discuss two methods for finding the probabilities corresponding to the random variable x in a binomial distribution.

Method 1: Using the Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x}$$

for $x = 0, 1, 2, \ldots, n$

where

n = number of trials

x = number of successes among n trials

p = probability of success in any one trial

q = probability of failure in any one trial ($q = 1 - p$)

Rationale for the Binomial Probability Formula

$$P(x) = \underbrace{\frac{n!}{(n-x)!x!}}_{\text{Number of outcomes with exactly } x \text{ successes among } n \text{ trials}} \cdot p^x \cdot q^{n-x}$$

Number of outcomes with exactly x successes among n trials

Rationale for the Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x}$$

$\frac{n!}{(n-x)!x!}$: Number of outcomes with exactly x successes among n trials

$p^x \cdot q^{n-x}$: The probability of x successes among n trials for any one particular order

Binomial Distribution: Formulas

Mean $\mu = n \cdot p$

Variance $\sigma^2 = n \cdot p \cdot q$

Std. Dev. $\sigma = \sqrt{n \cdot p \cdot q}$

Where

n = number of fixed trials

p = probability of success in one of the n trials

q = probability of failure in one of the n trials

Interpretation of Results

It is especially important to interpret results. The range rule of thumb suggests that values are unusual if they lie outside of these limits:

Maximum usual values $= \mu + 2\sigma$

Minimum usual values $= \mu - 2\sigma$