Cairo University
Faculty of science
Mathematics Department
180 min.

Final Exam (Fall 2015)
Selected topics in Artificial Intelligence
(The Theory of Fuzzy Sets)
COMP 792-a

Full credit is 100. Start each question in a new page.
The exam is in **three** pages.

Question (1) (25 Points) Fuzzy Sets and Fuzzy Logic:

A. **(14 Points)** Set up your own continuous membership functions for modeling linguistic values about the temperature: "hot", "warm", and "cold". According to your definition, what is the membership function for

i. "warm or hot"
ii. "not cold"
iii. "warm and not cold"

B. **(9 Points)** Consider these two fuzzy sets and *min* and *max* fuzzy set operations

$A = \frac{0.4}{1} + \frac{0.6}{2} + \frac{0.7}{3} + \frac{0.8}{4} + \frac{1.0}{5}$ and $B = \frac{0.3}{1} + \frac{0.65}{2} + \frac{0.4}{3} + \frac{0.1}{4}$

a. Find the intersection of *A* and its complement. Comment on the result with respect to the difference between crisp and fuzzy sets
b. Can the above two sets be fuzzy numbers? Why?
c. Find the alpha-cuts of A

C. **(4 Points)** Prove that for any fuzzy sets $A,B \in F(X)$ on a universe of discourse *X*, we have

$$(A \wedge B^c) \vee (A^c \wedge B) \geq 0.5 \wedge (A \vee B) \wedge (A^c \vee B^c)$$

Where B^c is the complement of B.

Question (2) (25 points) T-Norm and Conorm:

i. Given any *t-norm T* and *t-conorm S*, Is it necessary to have $T(x,0)= 0$ and $S(x,1) = 1$, for all $x \in [0, 1]$? why?
ii. Show that the algebraic product $T(x,y) = xy$, and the algebraic sum $S(x,y)= x+y-xy$ are *t-norm* and *t-conorm*.
iii. Decide whether, they (in ii) form DeMorgan triplet together with the standard negation.
iv. According to the definition in (ii) , determine if the following are true or false
   i. $S(A, T(B,C)) = T(S(A,B), S(A,C))$
   ii. $S(A, T(A,B)) = A$
v. Let *T be a t-norm* such that $T(a,b+c)=T(a,b)+T(a,c)$ for all $a,b,c \in [0,1]$, $b+c \leq 1$. Show that *T must be the algebraic product defined in (ii).*
vi. For any *t-norm* T and *t-conorm* S, we have $T(x,y) \geq x \wedge y$ and $S(x,y) \geq x \vee \mathbf{y}$
vii. Show that the *t-norm* in (ii) is not idemponent ($T(a,a)=a$). For extra credit, show that the only idemponent *t-norm* is the standard one (min).
viii. According to the above, comment of the popularity of the definitions in (ii) compared with the standard ones (min and max ones).

Question (3) (20 points) Algebraic properties:

A. Let A and B be two fuzzy sets defined on X such that

$$A(x)=\begin{cases}0 & x<2,\\ \dfrac{x-2}{5} & 2\le x<7\\ \dfrac{10-x}{3} & 7\le x<10\\ 0 & 10\le x\end{cases},\qquad B(x)=\begin{cases}0 & x<3,\\ x-3 & 2\le x<4\\ 1 & 4\le x<5\\ \dfrac{7-x}{2} & 5\le x<7\\ 0 & 7\le x\end{cases}$$

And consider the fuzzy implications

$$I_1(x,y)=\max\{1-x,y\},\qquad I_2(x,y)=\begin{cases}1 & x\le y,\\ y & y<x\end{cases}$$

a. Choose one of the above implications to calculate and graph.
b. Does any of them reduces to the crisp (Boolean) implication (show that)?
c. According to your Boolean background, which one of them is closer to the Boolean one?
d. Why does the implication function needs to decreasing in its first variable and increasing in its second variable?
e. If I is a fuzzy implication, what can you say about I', defined by $I'(x,y)=I(\sim y, \sim x)$

Question (4) (15 points) Fuzzy Numbers and Interval Arithmetic:

A. **(3 points)** Calculate the following

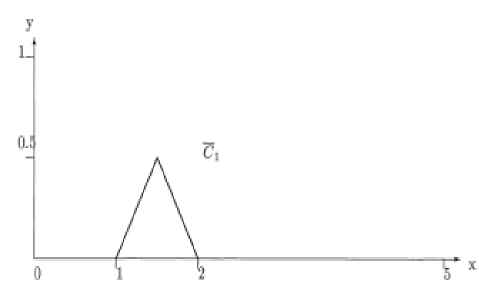
i. [-2,4]-[3,6]
ii. [-3,4]*[-3,4]
iii. [-4,6]/[1,2]

B. **(12 points)** Determine which fuzzy sets defined here are fuzzy numbers. If it is not a fuzzy number, explain why.

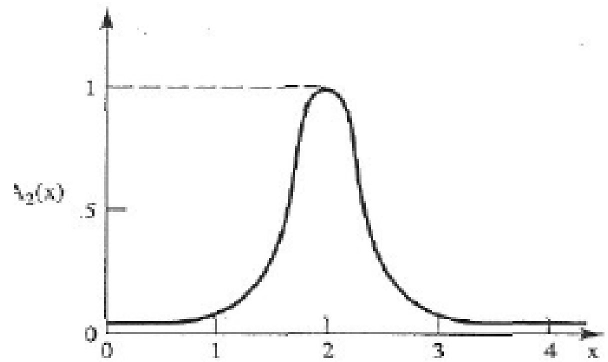
i. $A=\frac{0.4}{1}+\frac{0.6}{2}+\frac{1}{3}+\frac{1}{4}$

ii. $A(x)=\begin{cases}\sin(x) & 0\le x\le\pi,\\ 0 & \textit{otherwise}\end{cases}$

iii.



iv.



v. $B(x) = \begin{cases} x & 0 \le x \le 1, \\ 0 & otherwise \end{cases}$

vi. $C(x) = \begin{cases} 1 & 0 \le x \le 10, \\ 0 & otherwise \end{cases}$

Question (5) (15 points) Extension Principle:

A. **(5 points)** Explain the importance and the limitations of the Extension Principle in the theory of fuzzy sets. And explain how can we overcome its limitations.

B. **(10 points)** Consider the following two fuzzy numbers
$A = \{ 1/2 , 0.5/3\}$, $B = \{ 1/3, 0.8/4\}$
Add them using the alpha-cuts and the extension principle, then comment on your answer.

Best Wishes
Areeg Abdalla