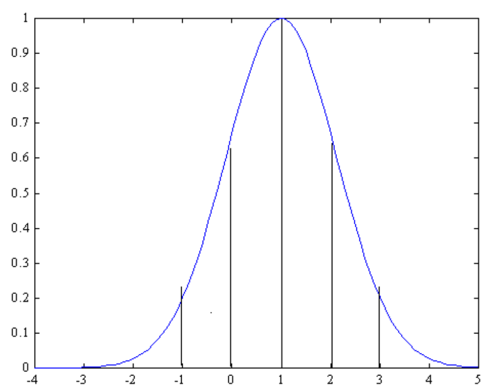
Fuzzy Number

N is called a fuzzy number if:

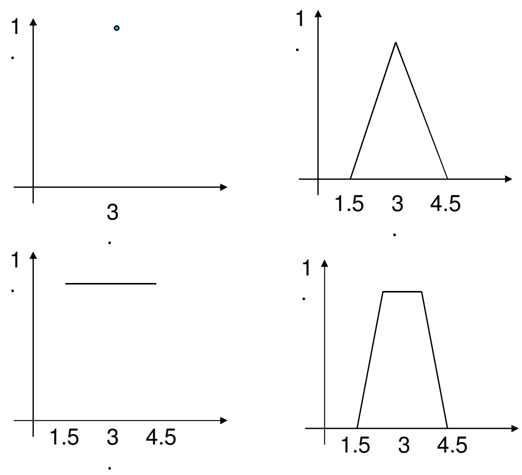
1. The core of *N* is not empty
2. All α-cuts of *N* are closed, bounded intervals
3. The support of N is bounded

Ex. Fuzzy number 1

Example: Discrete and Continuous fuzzy sets to represent the fuzzy number 1



Examples of fuzzy number 3



Examples

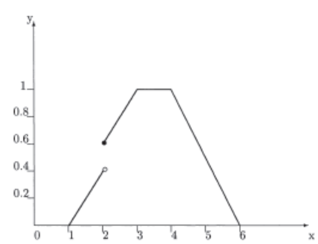
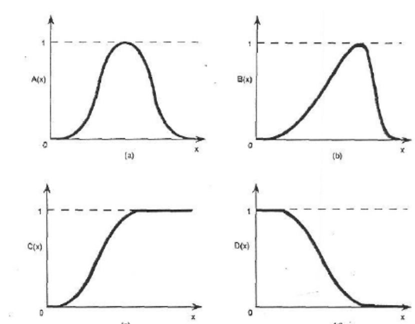
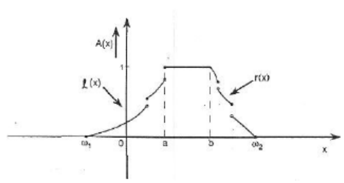
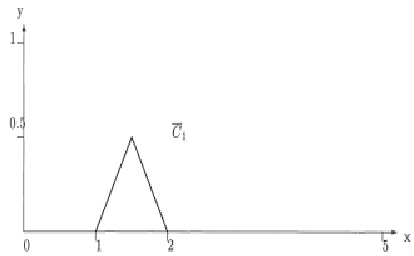


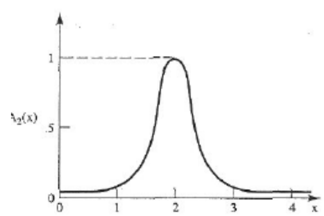
Figure 4.3: A General Fuzzy Number





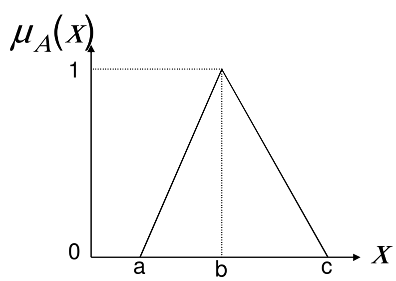
Examples: Not Fuzzy Numbers





Triangular Fuzzy Number

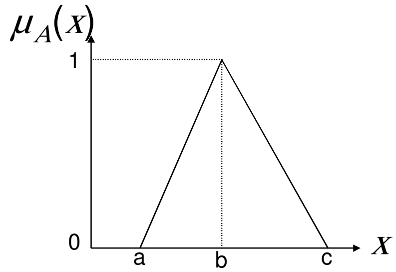
a, b and c represent the *x* coordinates of the three vertices of $\mu_A(x)$ in a fuzzy set A (a: lower boundary and c: upper boundary where membership degree is zero, b: the centre where membership degree is 1)



Triangular Fuzzy Number

a, b and c represent the *x* coordinates of the three vertices of $\mu_A(x)$ in a fuzzy set A (a: lower boundary and c: upper boundary where membership degree is zero, b: the centre where membership degree is 1)

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x-a}{b-a} & \text{if } a \le x \le b \\ \dfrac{c-x}{c-b} & \text{if } b \le x \le c \\ 0 & \text{if } x \ge c \end{cases}$$



Triangular/trapezoidal Shaped Buckley

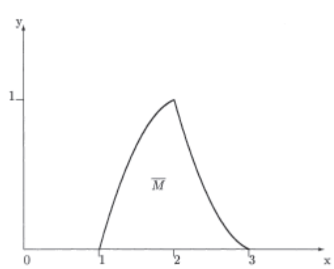


Figure 4.4: Triangular Shaped Fuzzy Number

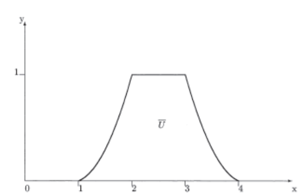


Figure 4.5: Trapezoidal Shaped Fuzzy Number

Operations of Fuzzy Numbers

Operations on fuzzy numbers can be done through two ways:

- Interval Arithmetic
- Extension Principle

Operations of Fuzzy Numbers

α-cut $A(+)B = \bigcup_{\alpha \in [0,1]} \alpha(A_\alpha (+) B_\alpha)$

Addition:

$$A = [a_1, a_3] \quad a_1, a_3 \in \Re$$

$$A_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}], \forall \alpha \in [0, 1], a_1^{(\alpha)}, a_3^{(\alpha)} \in \Re$$

$$B = [b_1, b_3], \quad b_1, b_3, \in \Re$$

$$B_\alpha = [b_1^{(\alpha)}, b_3^{(\alpha)}], \forall \alpha \in [0, 1], b_1^{(\alpha)}, b_3^{(\alpha)} \in \Re$$

operations between A_α and B_α can be described as follows :

$$[a_1^{(\alpha)}, a_3^{(\alpha)}] (+) [b_1^{(\alpha)}, b_3^{(\alpha)}] = [a_1^{(\alpha)} + b_1^{(\alpha)}, a_3^{(\alpha)} + b_3^{(\alpha)}]$$

Operations of Fuzzy Number

α-cut

$$A(-)B = \bigcup_{\alpha\in[0,1]} \alpha(A_\alpha(-)B_\alpha)$$

Subtraction:

$$A(\cdot)B = \bigcup_{\alpha\in[0,1]} \alpha(A_\alpha(\cdot)B_\alpha)$$

Multiplication:

$$A(/)B = \bigcup_{\alpha\in[0,1]} \alpha(A_\alpha(/)B_\alpha)$$

Division:

$$A(\wedge)B = \bigcup_{\alpha\in[0,1]} \alpha(A_\alpha(\wedge)B_\alpha)$$

Minimum:

$$A(\vee)B = \bigcup_{\alpha\in[0,1]} \alpha(A_\alpha(\vee)B_\alpha)$$

Maximum:

Operations of Fuzzy Number

α-cut

α = 0.5

$$A_{0.5} = [2,3],\ B_{0.5} = [3,4]$$

$$A_{0.5}(+)B_{0.5} = [5,7]$$

α = 1.0

$$A_{1.0} = 2,\ B_{1.0} = 3$$

$$A_{1.0}(+)B_{1.0} = 5$$