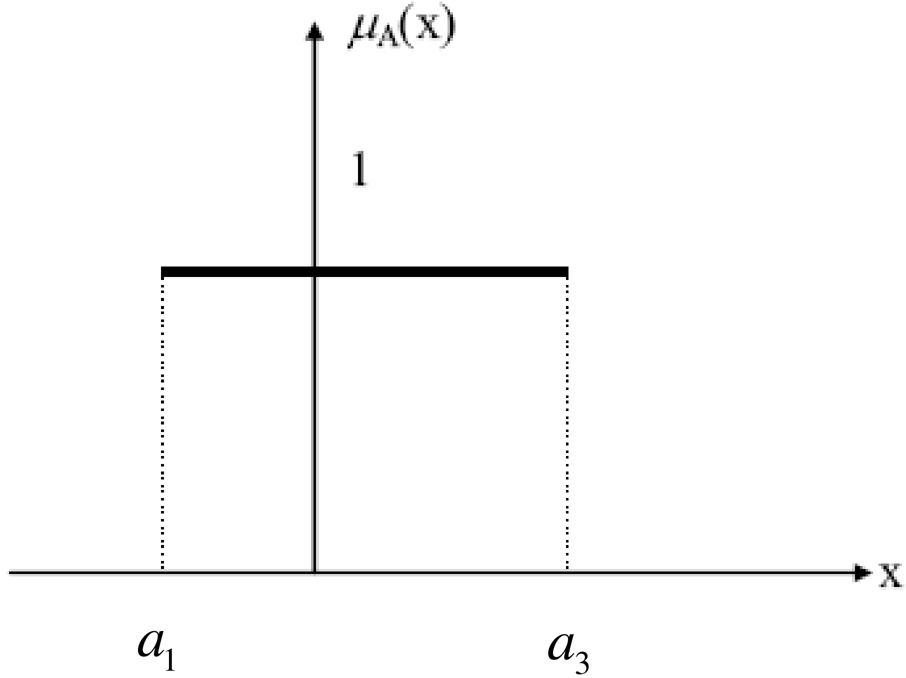
Interval

$[a_1, a_3]$

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ 1, & a_1 \le x \le a_3 \\ 0, & x > a_3 \end{cases}$$



α-cut

$$A_\alpha = \left\{ x \in X \,\middle|\, \mu_A(x) \geq \alpha \right\}$$

$$A_{\alpha^+} = \left\{ x \in X \,\middle|\, \mu_A(x) > \alpha \right\}$$

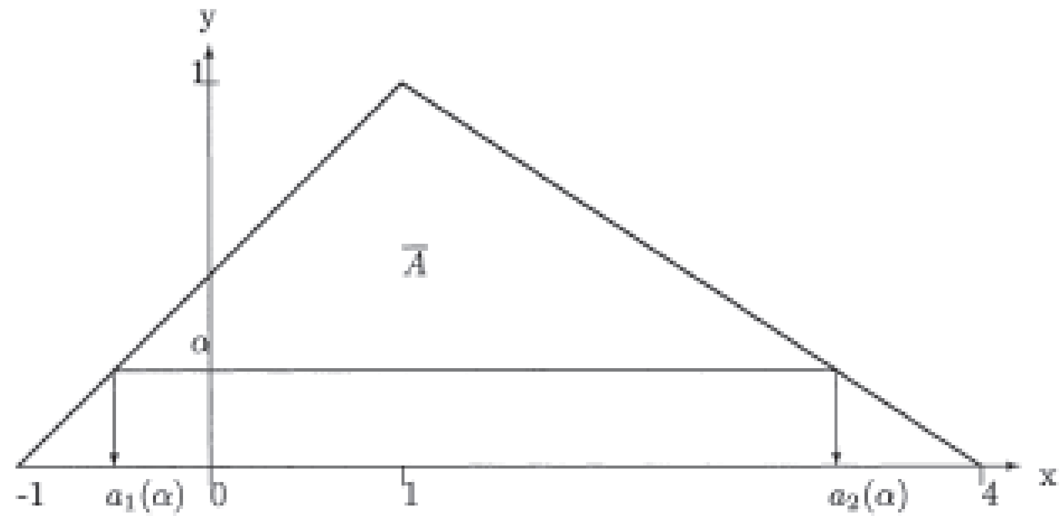


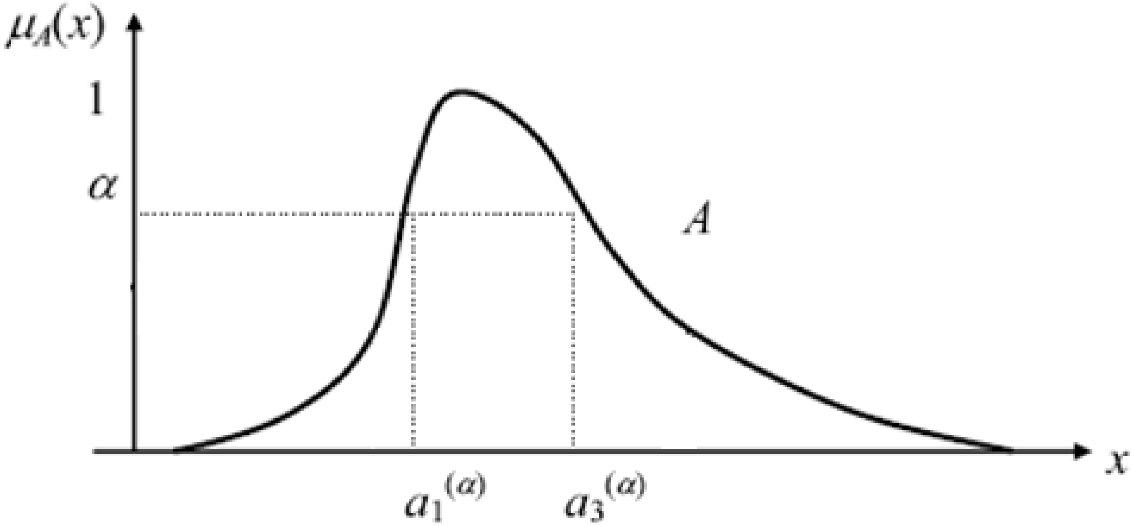
Figure 3.9: Continuous Fuzzy Set

Definitions

A_{0^+} is called the support of A

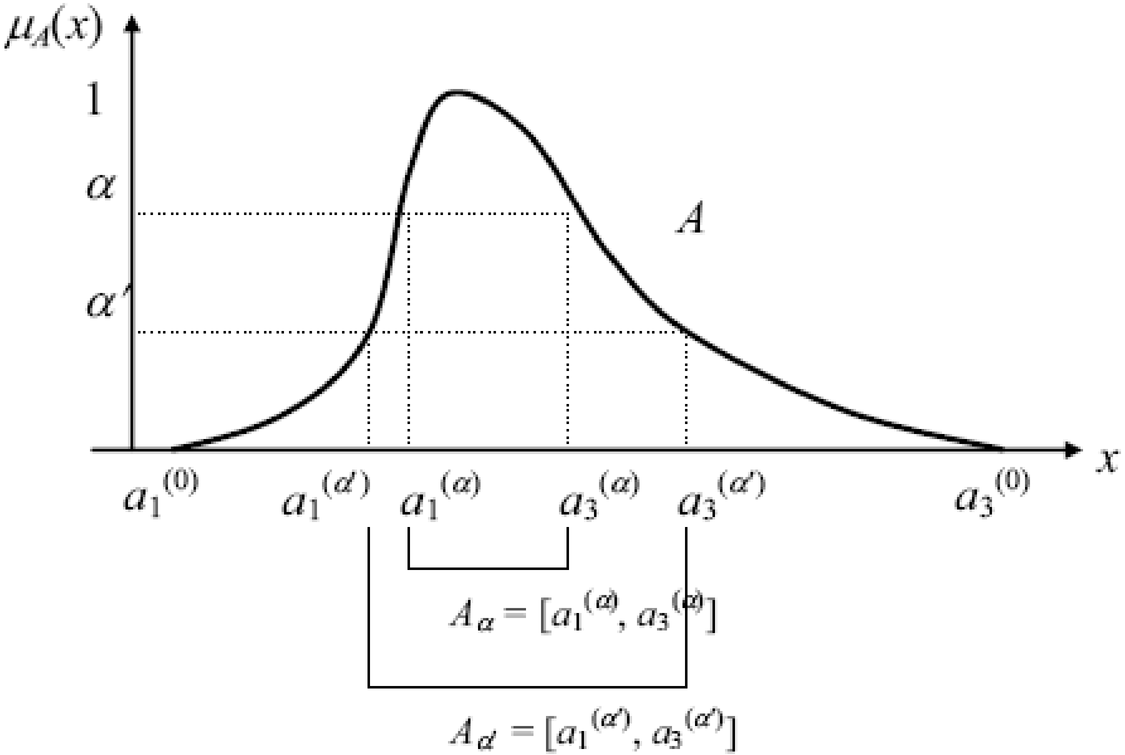
A_1 is called the core of A

α-cut



$$A_\alpha = \left[a_1^{(\alpha)}, a_3^{(\alpha)} \right]$$

α-cut



$\mu_A(x)$
1
α
α'
A
x
$a_1^{(0)}$
$a_1^{(\alpha')}$
$a_1^{(\alpha)}$
$a_3^{(\alpha)}$
$a_3^{(\alpha')}$
$a_3^{(0)}$
$A_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}]$
$A_{\alpha'} = [a_1^{(\alpha')}, a_3^{(\alpha')}]$

α-cut

In case of discrete fuzzy set

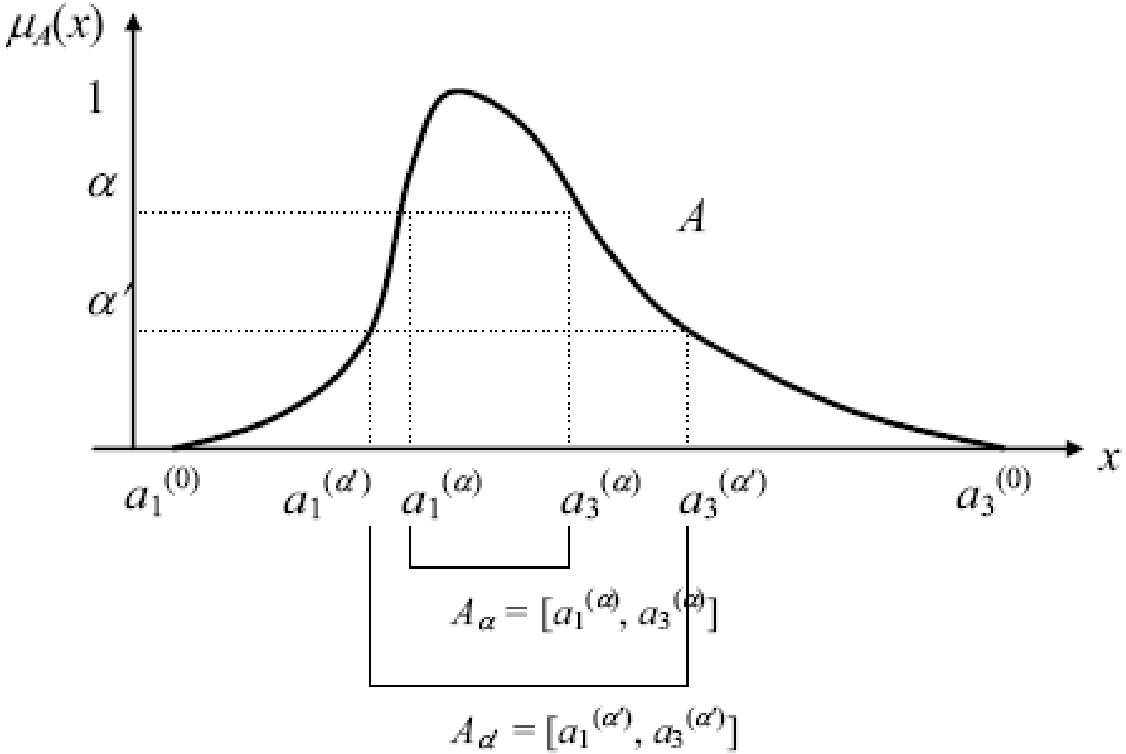
Find α-cuts of

$$\overline{A} = \{\frac{0}{x_1}, \frac{.7}{x_2}, \frac{1}{x_3}, \frac{.5}{x_4}, \frac{.2}{x_5}\},$$

where $X = \{x_1, \ldots, x_5\}$. Then

$$\overline{A}[\alpha] = \{x_2, x_3, x_4, x_5\}, 0 < \alpha \leq 0.2,$$
$$\overline{A}[\alpha] = \{x_2, x_3, x_4\}, 0.2 < \alpha \leq 0.5,$$
$$\overline{A}[\alpha] = \{x_2, x_3\}, 0.5 < \alpha \leq 0.7,$$
$$\overline{A}[\alpha] = \{x_3\}, 0.7 < \alpha \leq 1.$$

Properties of α-cut



$$A_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}]$$

$$(\alpha' < \alpha) \Rightarrow (a_1^{(\alpha')} \leq a_1^{(\alpha)}, a_3^{(\alpha')} \geq a_3^{(\alpha)}) \quad \Rightarrow (A_\alpha \subset A_{\alpha'})$$

Interval Arithmetic

$A = [\, a_1, a_2\,]\,,\quad B = [\, b_1, b_2\,]$

Interval Operations

Addition

$$[a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2]$$

Subtractio n

$$[a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1]$$

Multiplica tion

$$[a_1, a_2] * [b_1, b_2] = [\min\{ a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2 \},\ \max\{ a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2 \}]$$

Division

$$[a_1, a_2] / [b_1, b_2] = [a_1, a_2] * [1/b_2, 1/b_1]$$

$$0 \notin [b_1, b_2]$$

Examples

Addition

[2,5]+[1,3]=[3,8]   [0,1]+[-6,5]=[-6,6]

Subtraction

[2,5]-[1,3]=[-1,4]   [0,1]-[-6,5]=[-5,7]

Multiplication

[-1,1]*[-2,-0.5]=[-2,2]   [3,4]*[2,2]=[6,8]

Division

[-1,1]/[-2,-0.5]=[-2,2]   [4,10]*[1,2]=[2,10]

Properties of Interval Operations

Commutative

$A + B = B + A \qquad A \cdot B = B \cdot A$

Assocoative

$(A + B) + C = A + (B + C) \quad (A \cdot B) \cdot C = A \cdot (B \cdot C)$

Identity $\quad 0 = [0,0] \quad 1 = [1,1]$

$A = A + 0 = 0 + A \qquad A = A \cdot 1 = 1 \cdot A$

Subdistributive

$A \cdot (B + C) \subseteq A \cdot B + A \cdot C$

Inverse

$0 \in A\text{-}A \quad 1 \in A/A$

Monotonicity for any operations *

If $A \subseteq E$ and $B \subseteq F$ then $A * B \subseteq E * F$