Soft Computing

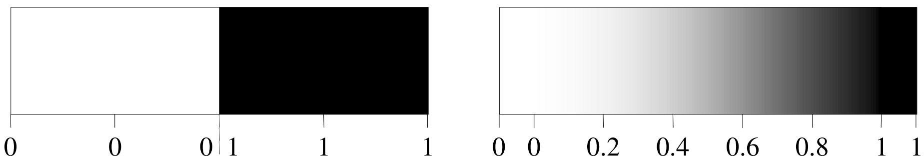
- Soft computing differs from conventional (hard) computing in that:" Its tolerance to imprecision, uncertainty, partial truth, and approximation.
- The model for soft computing is the human mind.
- The guiding principle of soft computing is: Exploit the tolerance for **imprecision**, **uncertainty**, **partial truth**, and **approximation** to achieve tractability, robustness and low solution cost.
- Soft computing is used as an umbrella term for sub-disciplines of computing, including fuzzy logic and fuzzy control, neural networks based computing and machine learning, and genetic algorithms, together with chaos theory in physics.

Introduction to Fuzzy Logic

Fuzzy logic is based on **degrees of membership.**

Unlike two-valued Boolean logic, fuzzy logic is **multi-valued**. It deals with **degrees of membership** and **degrees of truth**. Fuzzy logic uses the continuum of logical values between 0 (completely false) and 1 (completely true). Instead of just black and white, it employs the spectrum of colours, accepting that things can be partly true and partly false at the same time.

Range of logical values in Boolean and fuzzy logic



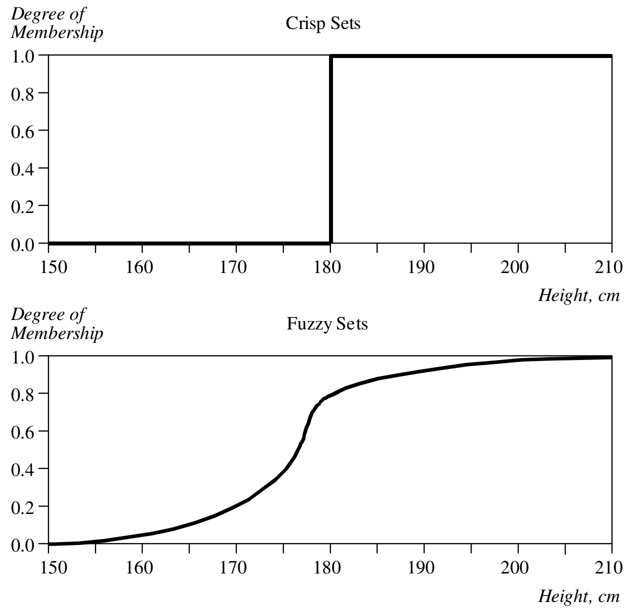
(*a*) Boolean Logic. (*b*) Multi-valued Logic.

Fuzzy sets

- The classical example in fuzzy sets is ***tall men***. The elements of the fuzzy set "tall men" are all men, but their degrees of membership depend on their height.

Name	Height, cm	Degree of Membership	
		Crisp	*Fuzzy*
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00

Crisp and fuzzy sets of “*tall men*”



Fuzzy Logic

“Because **Fuzzy Logic** is similar to the way we talk and think, it is easier for us to adjust” Cynthia Taylor

- Fuzzy Logic is particularly good at handling uncertainty, vagueness and imprecision.
- This is especially useful where a problem can be described linguistically (using words) or, as with neural networks, where there is data and you are looking for relationships or patterns within that data.
- Fuzzy Logic uses imprecision to provide robust, tractable solutions to problems.
- Fuzzy logic relies on the concept of a ***fuzzy set***.

Fuzzy sets, logic, inference, control

Explanation of some terms. The following have become widely accepted:

Fuzzy logic system

anything that uses fuzzy set theory

Fuzzy control

any control system that employs fuzzy logic

Fuzzy associative memory

any system that evaluates a set of fuzzy *if-then* rules uses fuzzy inference. Also known as **fuzzy rule base** or **fuzzy expert system**

Fuzzy inference control

a system that uses fuzzy control and fuzzy inference

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Remark: different authors and researchers use the same term either for the same thing or for different things.

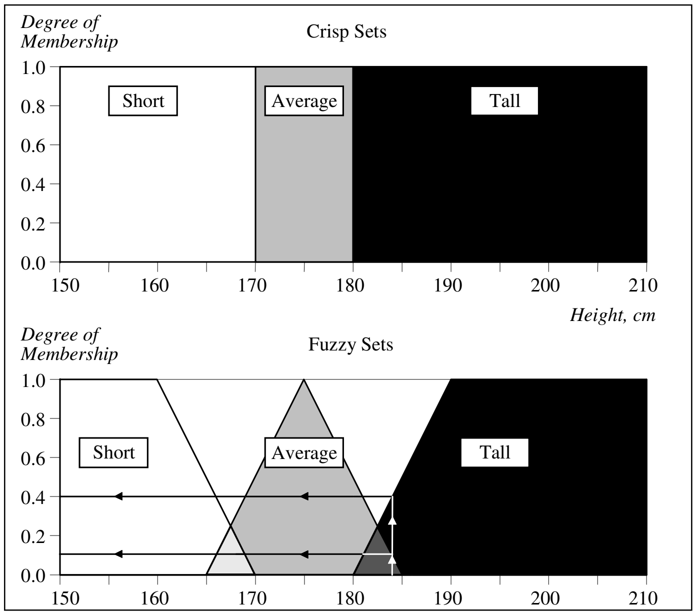
Fuzzy Logic

- It is hard to characterize the truth of "John is old" as unambiguously true or false if John is 60 years old.
- In some respects he is old, being eligible for senior citizen benefits at many establishments,
- But in other respects he is not old since he still can work.
- ***The truth value of the statement could be tv(John is old) = 70***
- ***The negation of the statement may be***
  $tv(\neg p) = 1 - tv(p)$
- ***We need also to find*** $tv(p \wedge q)$ ***and*** $tv(p \wedge q)$

How to represent a fuzzy set in a computer?

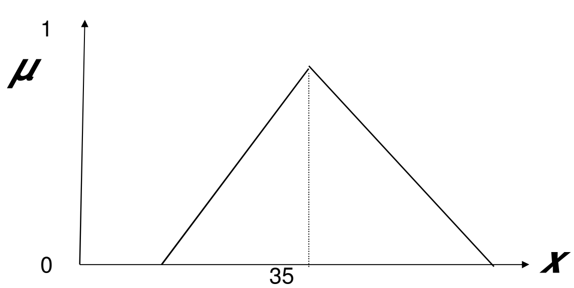
- First, we determine the membership functions. In our "***tall men***" example, we can obtain fuzzy sets of ***tall***, ***short*** and ***average*** men.
- The universe of discourse – the men's heights – consists of three sets: ***short***, ***average*** and ***tall men***. As you will see, a man who is 184 cm tall is a member of the ***average men*** set with a degree of membership of 0.1, and at the same time, he is also a member of the ***tall men*** set with a degree of 0.4.

Crisp and fuzzy sets of short, average and tall men



Imprecision / Vagueness / Uncertainty

In many physical systems, measurements are never precise Fuzzy numbers are one way of capturing this imprecision by having a fuzzy set representing a real number where the numbers in an interval near to the number are in the fuzzy set to some degree.



The fuzzy number 'About 35'

Fuzzy sets

- The ***x***-axis represents the **universe of discourse** – the range of all possible values applicable to a chosen variable. In our case, the variable is the man height. According to this representation, the universe of men's heights consists of all tall men.
- The ***y***-axis represents the **membership value of the fuzzy set**. In our case, the fuzzy set of "***tall men***" maps height values into corresponding membership values.

Fuzzy sets

A fuzzy set is a set with fuzzy boundaries

- Let X be the universe of discourse and its elements be denoted as x. In the classical set theory, crisp set A of X is defined as function $\psi_A(x)$ called the **characteristic function of A**

$$\psi_A(x): X \rightarrow \{0,1\}$$, where $$\psi_A(x) = \begin{cases} 1, \text{if } x \in A \\ 0, \text{if } x \notin A \end{cases}$$

This set maps universe X to a set of two elements. For any element x of universe X, characteristic function is $\psi_A(x)$ equal to 1 if x is an element of set A, and is equal to 0 if x is not an element of A.

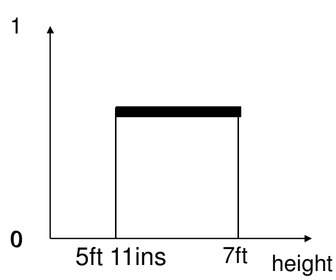
Fuzzy sets

For any fuzzy set, (let's say) A, the function μ_A represents the membership function for which $\mu_A(x)$ indicates the degree of membership that x, of the universal "crisp" set X, belongs to set A and is, usually, expressed as a number between 0 and 1

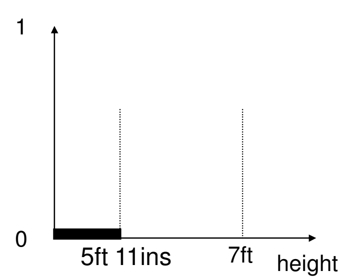
$$\mu_A(x) : X \rightarrow [0,1]$$

Fuzzy sets can be either discrete or continuous

Let's consider the first example (How tall/short we are?)

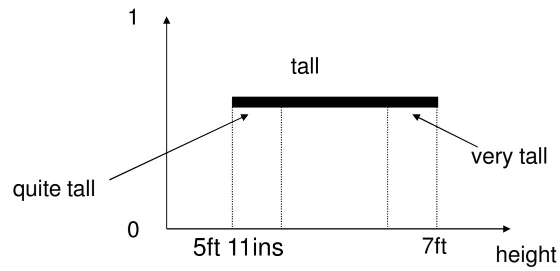


A crisp way of modelling tallness



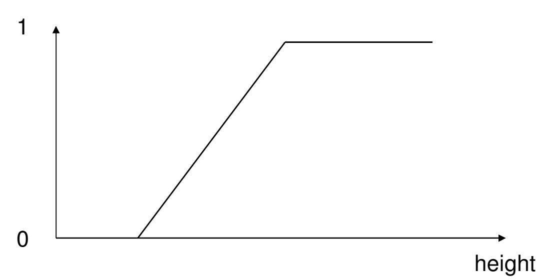
A crisp version of short

Let's consider the first example (How tall/short we are?)

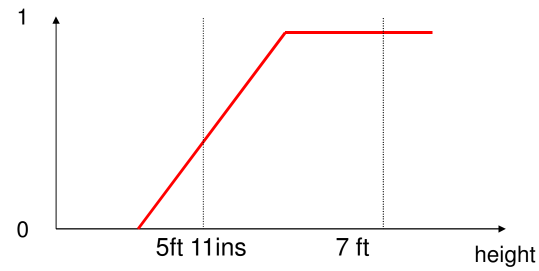


crisp definitions for tallness

Definition in a Fuzzy Set (How tall/short we are?)

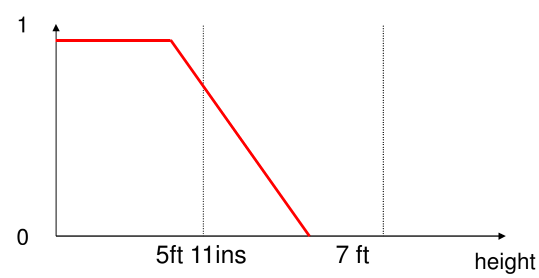


Definition in a Fuzzy Set (How tall/short we are?)



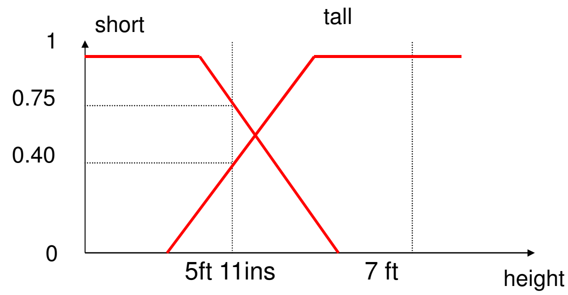
A possible fuzzy set tall

Definition in a Fuzzy Set (How tall/short we are?)



A possible fuzzy set short

Definition in a Fuzzy Set (How tall/short we are?)



Membership functions that represent tallness and short

Membership function

Membership functions can

- either be chosen by the user arbitrarily, based on the user's experience (MF chosen by two users could be different depending upon their experiences, perspectives, etc.)
- Or be designed using machine learning methods (e.g., artificial neural networks, genetic algorithms, etc.)

There are different shapes of membership functions; triangular, trapezoidal, piecewise-linear, Gaussian, bell-shaped, etc.

Discrete Fuzzy sets

The notation for fuzzy sets: for the member, *x*, of a discrete set with membership *μ*, x is a member of the set to degree *μ*.

Discrete sets are defined as:

$$A = \frac{\mu_1}{x_1} + \frac{\mu_2}{x_2} + \ldots + \frac{\mu_n}{x_n}$$

or (in a more compact form)

$$A = \sum_{i=1}^{n} \frac{\mu_i}{x_i}$$

$x_1, x_2, \ldots x_n$:members of the set *A*

$\mu_1, \mu_2, \ldots \mu_n : x_1, x_2, \ldots x_n$'s degree of membership.

Example Discrete case

Suppose we have the following (discrete) fuzzy sets:

$$A = 0.4/1 + 0.6/2 + 0.7/3 + 0.8/4$$

$$B = 0.3/1 + 0.65/2 + 0.4/3 + 0.1/4$$

The union of the fuzzy sets *A* and *B*

$= 0.4/1 + 0.65/2 + 0.7/3 + 0.8/4$

The intersection of the fuzzy sets *A* and *B*

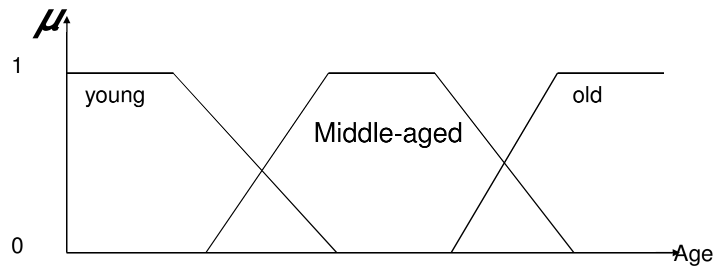
$= 0.3/1 + 0.6/2 + 0.4/3 + 0.1/4$

The complement of the fuzzy set A

$= 0.6/1 + 0.4/2 + 0.3/3 + 0.2/4$

Continuous Fuzzy sets

Example: describing people as "young", "middle-aged", and "old"



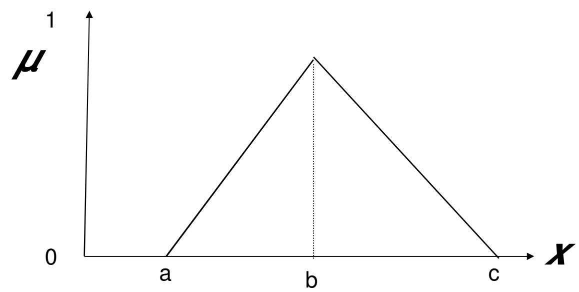
Fuzzy Logic allows modelling of linguistic terms using linguistic variables and linguistic values. The fuzzy sets "young", "middle-aged", and "old" are fully defined by their membership functions. The linguistic variable "Age" can then take linguistic values.

Most Common Membership functions

- TRIANGULAR: $\text{tri}(x; a, b, c) = \max\left\{\min\left\{\frac{x-a}{b-a}, \frac{c-x}{c-b}\right\}, 0\right\}$
- TRAPEZOIDAL: $\text{trap}(x; a, b, c, d) = \max\left\{\min\left\{\frac{x-a}{b-a}, \frac{d-x}{d-c}, 1\right\}, 0\right\}$
- GAUSSIAN: $\text{gauss}(x; c, \sigma) = \exp\left[-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2\right]$
- GENERALISED BELL: $\text{gbell}(x; a, b) = \frac{1}{1 + \left|\frac{x-b}{a}\right|^{2a}}$

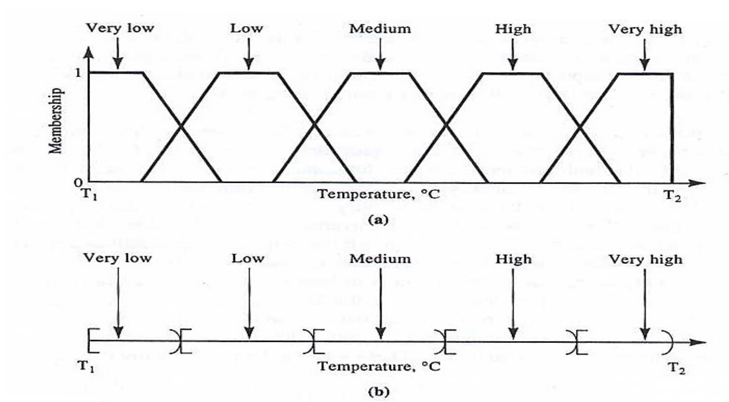
Triangular Fuzzy set

$$\text{tri}(x;a,b,c) = \max\left\{\min\left\{\frac{x-a}{b-a}, \frac{c-x}{c-b}\right\}, 0\right\}$$



Trapezoidal Fuzzy Sets

$$\text{trap}(x;a,b,c,d) = \max\left\{\min\left\{\frac{x-a}{b-a}, \frac{d-x}{d-c}, 1\right\}, 0\right\}$$



Temperature in the range $[T_1, T_2]$ conceived as: (a) a fuzzy variable; (b) a traditional (crisp) variable.

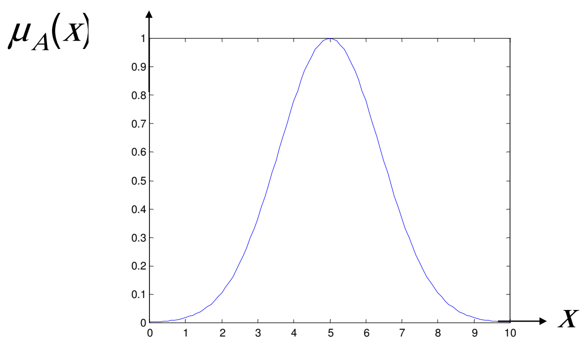
figure from G. Klir "Fuzzy Sets and Fuzzy Logic, theory and applications

Gaussian Fuzzy membership fn

Gaussian membership function

$$\mu_A(x, c, s, m) = \exp\left[-\frac{1}{2}\left|\frac{x-c}{s}\right|^m\right]$$

- *c:* centre
- *s:* width
- *m:* fuzzification factor (e.g., *m*=2)

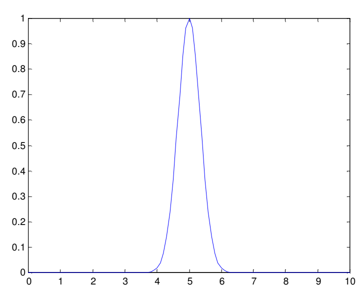


c=5

s=2

m=2

Gaussian Fuzzy membership fn

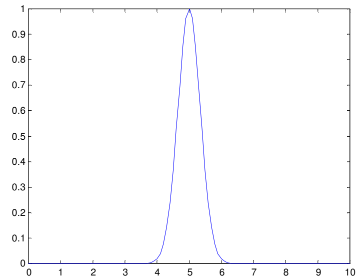


c=5

s=0.5

m=2

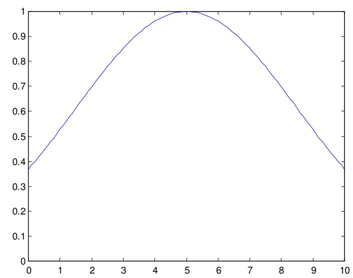
Gaussian Fuzzy membership fn



c=5

s=0.5

m=2

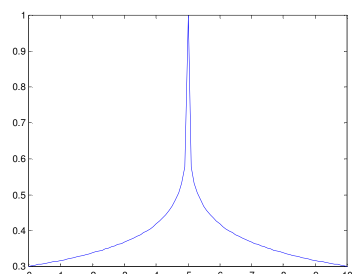


c=5

s=5

m=2

Gaussian Fuzzy membership fn

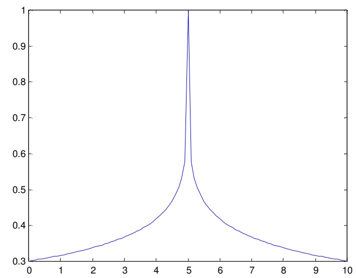


c=5

s=2

m=0.2

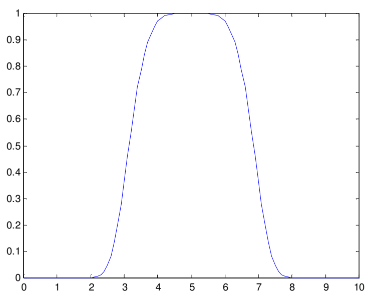
Gaussian Fuzzy membership fn



c=5

s=2

m=0.2



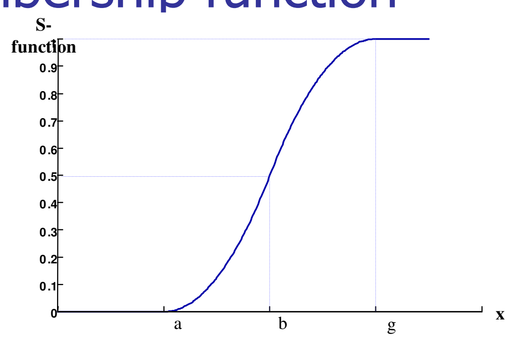
c=5

s=5

m=5

Sigmoid Membership function

$$S(x;a,b,g)=\begin{cases} 0\, for x \le \alpha \\ 2\left(\dfrac{x-a}{g-a}\right)^2 fora\langle x \le b \\ 1-2\left(\dfrac{x-a}{g-a}\right)^2 forb\langle x \le g \\ 1\, forx\rangle\gamma \end{cases}$$



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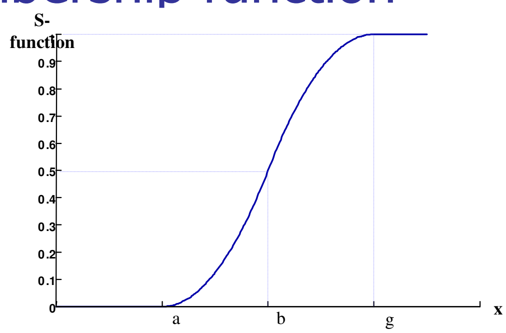
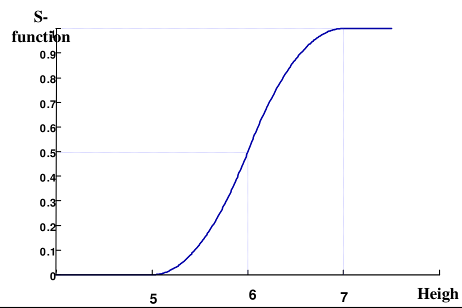


$$S(x;5,6,7)=\begin{cases} 0\, for\, x \le 5 \\ \left(\dfrac{x-5}{2}\right)^2 for\, 5\langle x \le 6 \\ 1-\left(\dfrac{x-7}{2}\right)^2 for\, 6\langle x \le 7 \\ 1\, for\, x\rangle 7 \end{cases}$$



P function

$$\prod(x;b,g)=\begin{cases} S(x;\, g-b,\, g-\dfrac{b}{2},\, g) \text{ for x} \le g \\ 1-S(x;\, g,\, g+\dfrac{b}{2},\, g+b) \text{ for x} \ge g \end{cases}$$

The P-function goes to zero at $\gamma < \beta$, and the 0.5 point is at $\gamma = (\beta/2)$. Notice that the β parameter represents the bandwidth of the 0.5 points.