

Inference approaches in FOL

- Forward-chaining
 - Uses GMP to add new atomic sentences
 - Useful for systems that make inferences as information streams in
 - Requires KB to be in form of first-order definite clauses
- Backward-chaining
 - Works backwards from a query to try to construct a proof
 - Can suffer from repeated states and incompleteness
 - Useful for query-driven inference
- Resolution-based inference (FOL)
 - Refutation-complete for general KB
 - Can be used to confirm or refute a sentence p (but not to generate all entailed sentences)
 - Requires FOL KB to be reduced to CNF
 - Uses generalized version of propositional inference rule

Knowledge Representation Building Knowledge Base in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

Example

Consider the following knowledge base:

- The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.
- Prove that Col. West is a criminal
- We will do it through FC, BC, resolution

Example knowledge base contd.

... it is a crime for an American to sell weapons to hostile nations:

$$\begin{aligned} & \text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x,y,z) \wedge \text{Hostile}(z) \Rightarrow \text{Criminal}(x) \\ & \neg \text{American}(x) \vee \neg \text{Weapon}(y) \vee \neg \text{Sells}(x,y,z) \vee \neg \text{Hostile}(z) \vee \neg \text{Criminal}(x) \end{aligned}$$

Nono ... has some missiles, i.e., $\exists x \text{ Owns}(\text{Nono},x) \wedge \text{Missile}(x)$:

$$\text{Owns}(\text{Nono},M_1) \wedge \text{Missile}(M_1)$$

... all of its missiles were sold to it by Colonel West

$$\text{Missile}(x) \wedge \text{Owns}(\text{Nono},x) \Rightarrow \text{Sells}(\text{West},x,\text{Nono})$$

Missiles are weapons:

$$\text{Missile}(x) \Rightarrow \text{Weapon}(x)$$

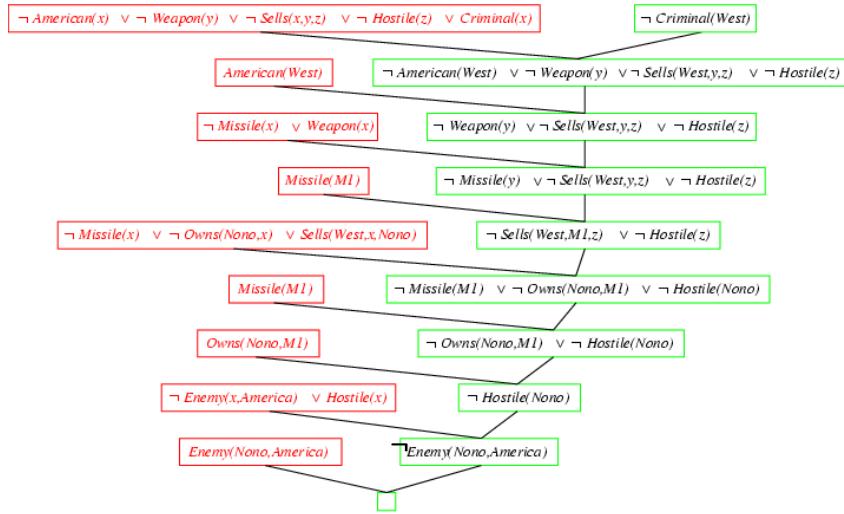
An enemy of America counts as "hostile":

$$\text{Enemy}(x,\text{America}) \Rightarrow \text{Hostile}(x)$$

West, who is American ... $\text{American}(\text{West})$

The country Nono, an enemy of America ... $\text{Enemy}(\text{Nono},\text{America})$

Resolution proof: definite clauses



$American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$

$\exists x \ Owns(Nono,x) \wedge Missile(x)$

$Owns(Nono,M1) \wedge Missile(M1)$

$Missile(x) \wedge Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$

$Missile(x) \Rightarrow Weapon(x)$

$Enemy(x,America) \Rightarrow Hostile(x)$

$American(West)$

$Enemy(Nono,America)$

F C proof

$American(West)$

$Missile(M1)$

$Owns(Nono,M1)$

$Enemy(Nono,America)$

$American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$

$\exists x \text{ Owns}(Nono,x) \wedge \text{Missile}(x)$

$\text{Owns}(Nono,M_1) \wedge \text{Missile}(M_1)$

$\text{Missile}(x) \wedge \text{Owns}(Nono,x) \Rightarrow Sells(West,x,Nono)$

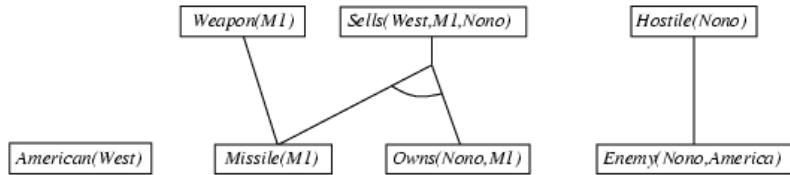
$\text{Missile}(x) \Rightarrow Weapon(x)$

$\text{Enemy}(x,America) \Rightarrow Hostile(x)$

$American(West)$

$\text{Enemy}(Nono,America)$

F C proof



$American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$

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$\text{Owns}(Nono,M_1) \wedge \text{Missile}(M_1)$

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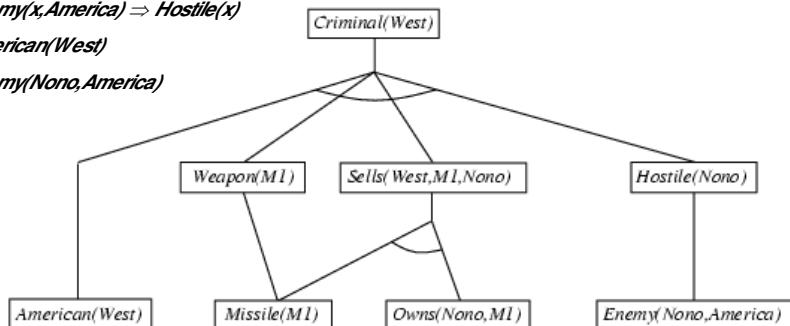
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 $\text{Missile}(x) \Rightarrow \text{Weapon}(x)$
 $\text{Enemy}(x,America) \Rightarrow \text{Hostile}(x)$ Criminal(West)
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 $\text{Enemy}(Nono,America)$

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 $American(West)$
 $\text{Enemy}(Nono,America)$

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graph TD
    Root["Criminal(West)"] --- Node1["American(x)"]
    Root --- Node2["Weapon(y)"]
    Root --- Node3["Sells(x,y,z)"]
    Root --- Node4["Hostile(z)"]
  
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$American(West)$

$\text{Enemy}(Nono,America)$

$American(West)$

$Weapon(y)$

$Sells(x,y,z)$

$Hostile(z)$

B C proof

$(x/West)$

$Criminal(West)$

{ }

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$Weapon(y)$

$Sells(x,y,z)$

$Hostile(z)$

B C proof

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$\text{Enemy}(Nono,America)$

$American(West)$

$Weapon(y)$

$Sells(x,y,z)$

$Hostile(z)$

$Criminal(West)$

$\{x/West, y/M1\}$

$\text{Missile}(y)$

$\{y/M1\}$

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$American(West)$

$Weapon(y)$

$Sells(West,M1,z)$

$Hostile(z)$

$Criminal(West)$

$\{x/West, y/M1, z/Nono\}$

$\text{Missile}(y)$

$\{y/M1\}$

$\text{Missile}(M1)$

$\text{Owns}(Nono,M1)$

American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)

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Missile(x) \Rightarrow Weapon(x)

Enemy(x,America) \Rightarrow Hostile(x)

American(West)

Enemy(Nono,America)

American(West)

{ }

Weapon(y)

{ }

Sells(West,M₁,z)

{ }

z/Nono

{x/West, y/M₁, z/Nono}

Hostile(Nono)

{ }

Missile(y)

{y/M₁}

Missile(M₁)

{ }

Owes(Nono,M₁)

{ }

Enemy(Nono,America)

{ }

B C proof

Criminal(West)

{ }