Game Playing

A (pure) strategy:

a complete set of advance instructions that specifies a definite choice for every conceivable situation in which the player may be required to act.

In a two-player game, a strategy allows the player to have a response to every move of the opponent.

Game-playing programs implement a strategy as a software mechanism that supplies the right move on request.

Two-Person Perfect Information Deterministic Game

- Two players take turns making moves
- Call one Min and the other Max
- Deterministic moves: Board state fully known,
- One player wins by defeating the other (or else there is a tie)
- Want a strategy to win, assuming the other person plays rationally

A logic-based approach to games

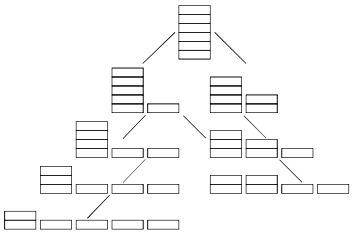
Find a winning strategy by ***proving*** that the game can be won -- use backward chaining.

A very simple game: nim.

- initially, there is one stack of chips;
- a move: select a stack and divide it in two unequal non-empty stacks;
- a player who cannot move loses the game.

(The player who moves first can win.)

Nim



Static evaluation

A **static evaluation function** returns the value of a move without trying to play (which would mean simulating the rest of the game but not playing it).

"Usually" a static evaluation function returns positive values for positions advantageous to **Player 1**, negative values for positions advantageous to **Player 2.**

If **Player 1** is rational, he will choose the maximal value of a leaf.
Then, **Player 2** will choose the minimal value.

Static evaluation

If we can have (guess or calculate) the value of an internal node **N**, we can treat it as if it were a leaf. This is the basis of the minimax procedure.

No tree would be necessary if we could evaluate the initial position statically. Normally we need a tree, and we need to look-ahead into it. Further positions can be evaluated more precisely, because there is more information, and a more focussed search.

Minimax Tree

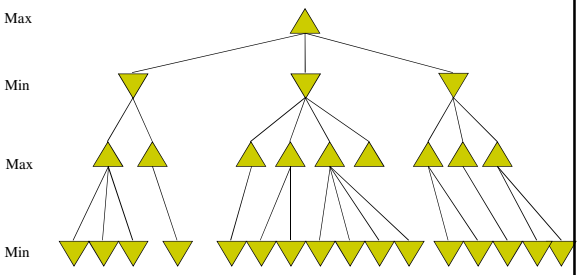
- Create a utility function
  - Evaluation of board/game state to determine how strong the position of each player.
  - Player 1 wants to maximize the utility function
  - Player 2 wants to minimize the utility function
- Minimax tree
  - Generate a new level for each move
  - Levels alternate between “max” (player 1 moves) and “min” (player 2 moves)

Minimax Tree Evaluation

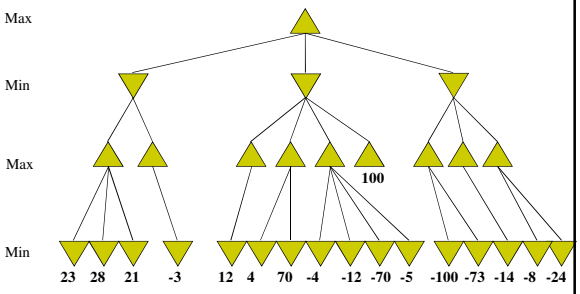
•Assign utility values to leaves

- If leaf is a “final” state, assign the maximum or minimum possible utility value (depending on who would win)
- If leaf is not a “final” state, must use some other heuristic, specific to the game, to evaluate how good/bad the state is at that point

Minimax tree



Minimax tree



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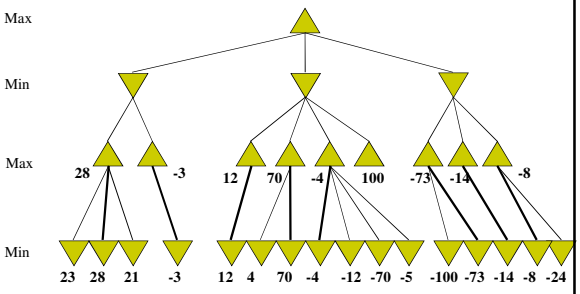
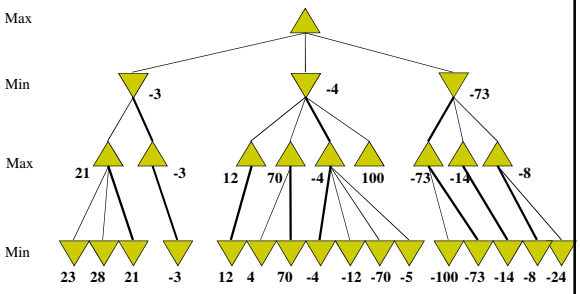
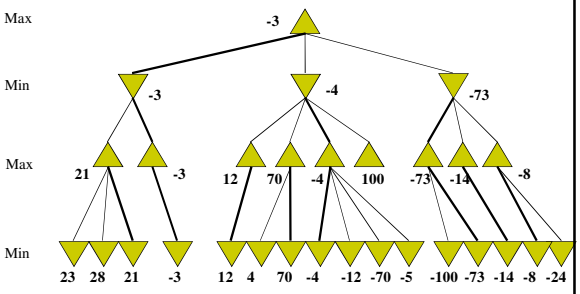


Minimax tree



Minimax tree



Tic-Tac-Toe

Let player **A** be x and let open(x), open(o) mean the number of lines open to x and o. There are 8 lines. An evaluation function for position P:

f(P) = -∞ if o wins

f(P) = +∞if x wins, otherwise

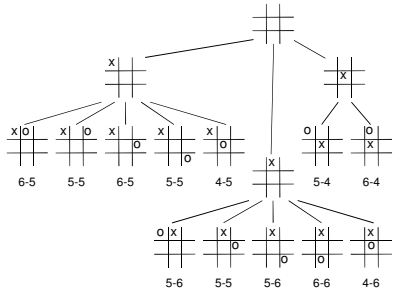
f(P) = open(x) - open(o)

Example:

open(x) - open(o) = 4 - 6

Assumptions:

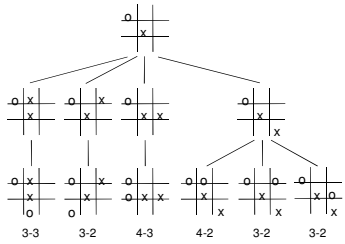
only one of symmetrical positions is generated;

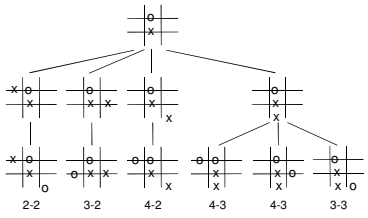


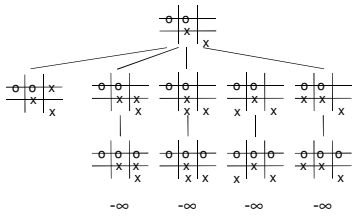
Player **B** chooses the minimal backed-up value among level 1 nodes.

Player **A** chooses the maximal value, and makes the move.

Player **B**, as a rational agent, selects the optimal response.







Building complete piles is usually not necessary. If we evaluate a position when it is generated, we may save a lot.

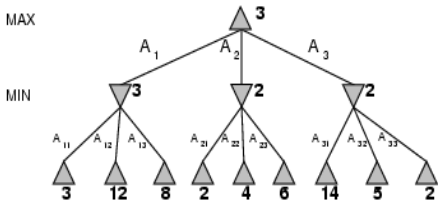
Assume that we are at a minimizing level. If the evaluation function returns -∞, we do not need to consider other positions:
-∞ will be the minimum.

The same applies to +∞ at a maximizing level.

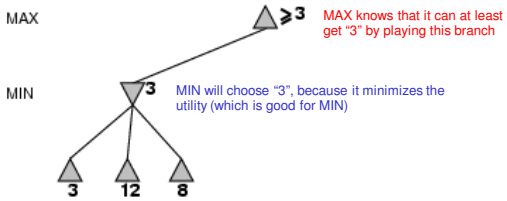
Pruning the Minimax Tree

- Minimax works best for large trees, but it can be useful even in mini-games such as tic-tac-toe.
- Since we have limited time available, we want to avoid unnecessary computation in the minimax tree.
- Pruning: ways of determining that certain branches will not be useful. Then cut of these branches

pruning



α pruning



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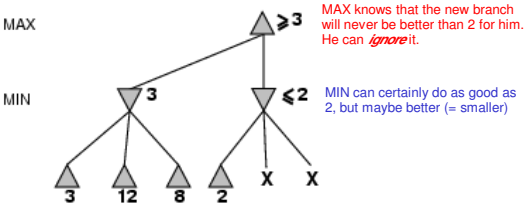

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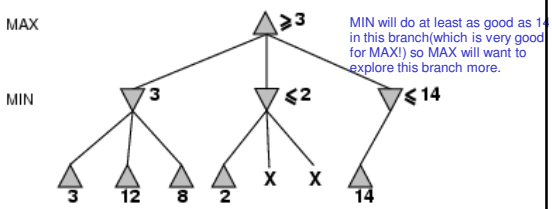
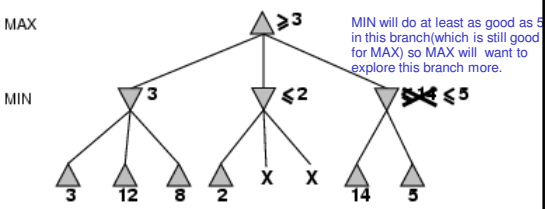
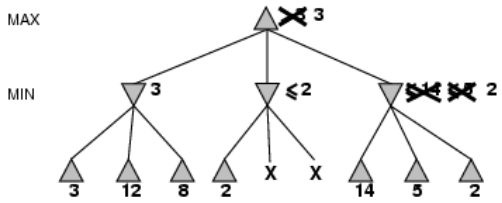


α pruning



α pruning

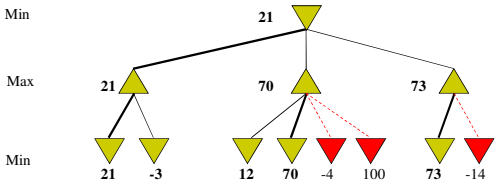


MIN will be able to play this last branch and get 2. This is worse than 3, so MAX will play 3.

β pruning

- Similar idea to α pruning, but the other way around
- If the current minimum is less than the successor's max value, don't look down that max tree any more

β pruning example



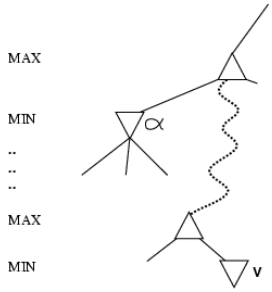
- Some subtrees at second level already have values > min from previous, so we can stop evaluating them.

Why is it called α-β?

- α is the value of the best (i.e., highest-value) choice found so far at any choice point along the path for ***max***
- If ***v*** is worse than α, ***max*** will avoid v

→ prune that branch

- Define β similarly for ***min***



α-β Pruning properties

- Pruning by these cuts does not affect final result
  - May allow you to go much deeper in tree
- Properties:
  - Evaluating "best" branch first yields better likelihood of pruning later branches
  - Perfect ordering reduces time to $b^{m/2}$

Properties of minimax

- Complete? Yes (if tree is finite)
- Optimal? Yes (against an rational opponent)
- Time complexity? $O(b^m)$
- Space complexity? O(bm) (depth-first exploration)

- For chess, b ≈ 35, m ≈100 for "reasonable" games

→ exact solution completely infeasible