Informed Search

- Blind search - no notion concept of the "right direction"
  - can only recognize goal once it's achieved
- Heuristic search – we have rough idea of how good various states are, and use this knowledge to guide our search
- Can find solutions more efficient than uninformed
- General approach is *best-first-search*
- A node is selected based on an *evaluation function f(n)*
- A node that **seems** to be best is picked and it may not be the actual best

Best First Search

- The Idea:
  - use an *evaluation function* for each node... estimate of ``desirability''
  - Expand most desirable unexpanded node

Implementation

- Fringe: is a queue sorted in decreasing order of desirability

Special cases

Greedy

A*

Cost function $f(n)$

- A function f is maintained for each node

$f(n) = g(n) + h(n)$, n is the node in the open list

- "Node chosen" for expansion is the one with least f value
- $g(n)$ is the cost from root S to node n
- $h(n)$ is the estimated cost from node n to a goal
- For BFS: $f = 0$,
- For DFS: $f = 0$,
- For greedy g =0

Greedy search

- Expands a node it sees closest to a the goal
- $f(n) = h(n)$
- Resembles DFS in that it prefers to follow a single path all the way to the goal
- Also suffers from the same defects of DFS, it may stuck in a loop i.e. not complete As well as it is not optimal.

Hill climbing

This is a *greedy* algorithm

Expands a node it sees closest to a goal

f(n) =h(n)

The algorithm

select a heuristic function;

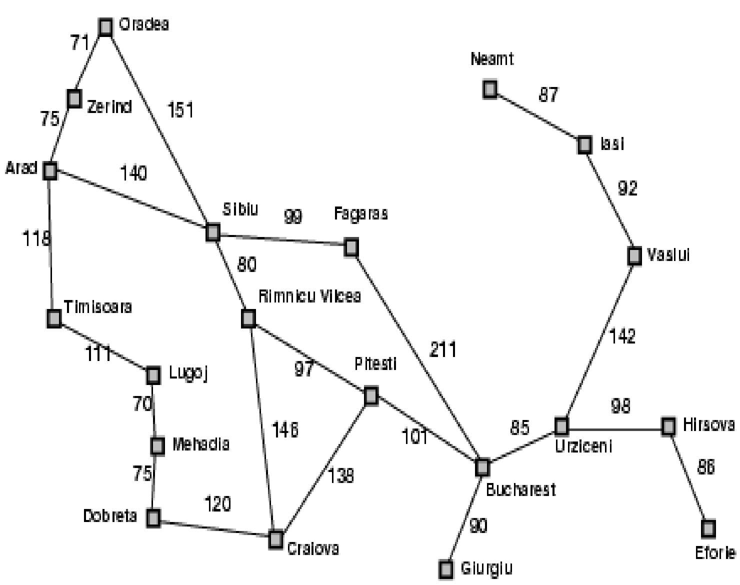
set C, the current node, to the highest-valued initial node;

Loop until success or no more children(fail)

select N, the highest-value child of C;

return C if its value is better than the value of N;

Hill Climbing search example



Straight–line distance to Bucharest

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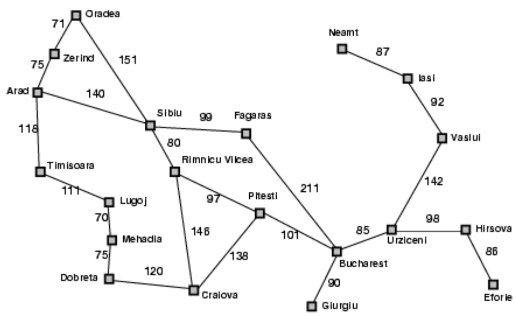

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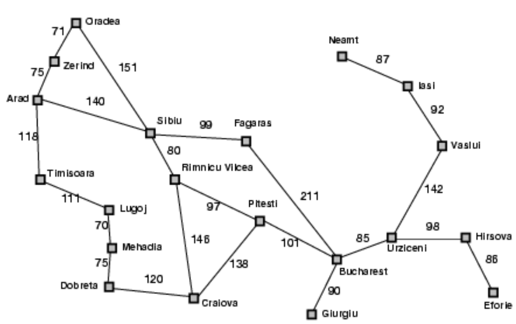

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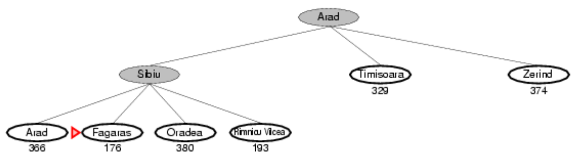

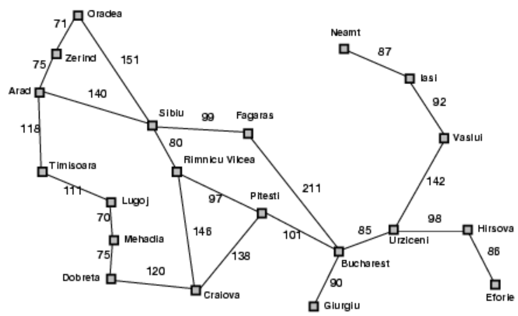

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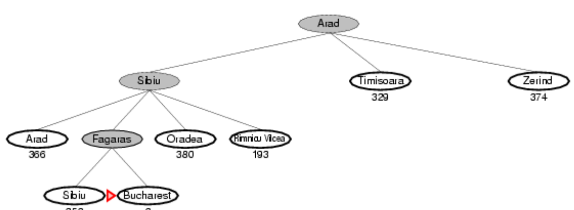

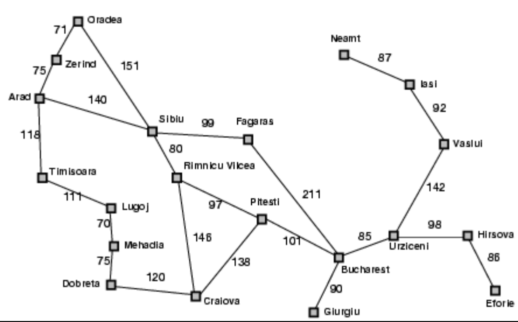

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Hill climbing

Complete:

No, Can get stuck in loop. Complete if loops are avoided.

Time complexity?

O(b^m), but with some good heuristic, it could give better results

Space complexity?

O(b^m), keeps all nodes in memory

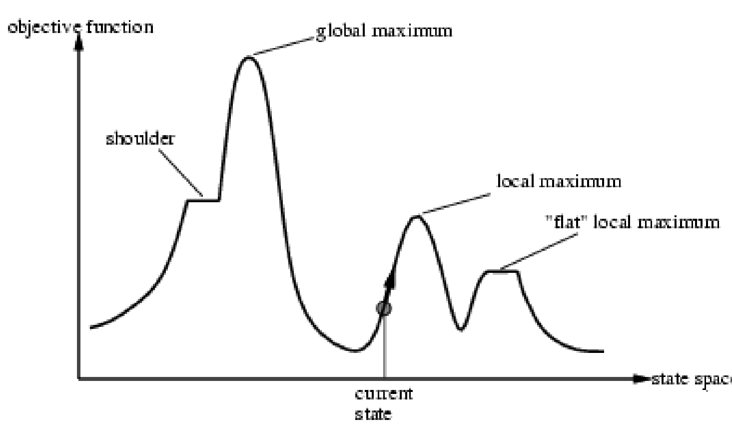
Optimality?

No

e.g. Arad→Sibiu→Rimnicu Virea→Pitesti→Bucharest is shorter!

Hill-climbing search

- Problem: depending on initial state, can get stuck in local maxima,…etc



Problems with hill climbing

1. Local maximum problem: there is a peak, but it is lower than the highest peak in the whole space.
2. The plateau problem: all local moves are equally unpromising, and all peaks seem far away.
3. The ridge problem: almost every move takes us down.

Solution:

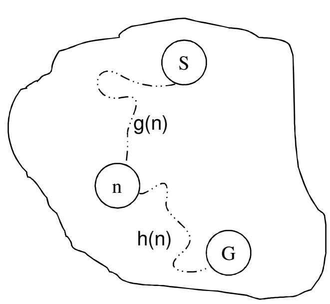
Random-restart hill climbing is a series of hill-climbing searches with a randomly selected start node whenever the current search gets stuck.

Algorithm A*

- One of the most important advances in AI search algs.
- Idea: avoid expanding paths that are already expensive

$$f(n) = g(n) + h(n)$$

- $g(n)$ = least cost path to n from S found so far
- $h(n)$ = estimated cost to goal from n
- $f(n)$ = estimated total cost of path through n to goal



The A* procedure

Hill-climbing (and its improved versions) may miss an optimal solution. Here is a search method that ensures **optimality** of the solution.

The algorithm

keep a list of partial paths (initially root to root, length 0);

repeat

succeed if the first path P reaches the goal node;

otherwise remove path P from the list;

extend P in all possible ways, add new paths to the list;

sort the list by the sum of two values: the real cost of P till now, and an estimate of the remaining distance;

prune the list by leaving only the shortest path for each node reached so far;

until

success or the list of paths becomes empty;

The A* procedure

A heuristic that never overestimates is also called **optimistic** or **admissible**.

We consider three functions with values ≥ 0:

- g(n) is the actual cost of reaching node n,
- h(n) is the actual *unknown* remaining cost,
- h*(n) is the optimistic estimate of h(n).

Admissible heuristics

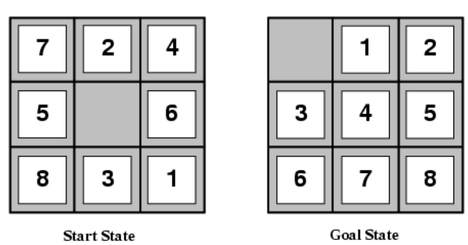
- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Theorem: If $h(n)$ is admissible, A^* using is optimal

Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



- $h_1(S) = ?$
- $h_2(S) = ?$

Admissible heuristics

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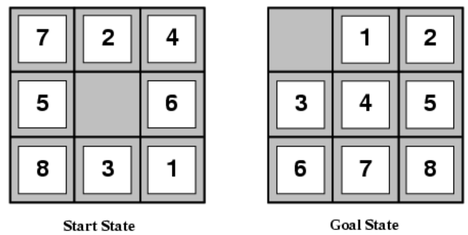


- $h_1(S) = ?$ 8
- $h_2(S) = ?$ 3+1+2+2+2+3+3+2 = 18

Admissible heuristics

- If $h_2(n) >= h_1(n)$ for all n, both are admissible
- Then h_2 dominates h_1 and is usually better for search

Typical Costs

- d = 14 IDS = 3,473,941 nodes
  - A*(h_1) = 539 nodes
  - A*(h_2) = 113 nodes
- d = 24 IDS ~ 54,000,000,000 nodes
  - A(h_1) = 39,135 nodes
  - A(h_2) = 1,641 nodes

Remark: Given h_1 and h_2 any two admissible functions

then $h(n) = \max\{h_1(n), h_2(n)\}$ is also admissible

Admissible heuristics and relaxed problems

- Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere,
- then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution
- Remark:
  - the optimal solution cost of a relaxed problem is less than the optimal solution cost of the real problem

A* Algorithm- Properties

- **Admissibility**: An algorithm is called admissible if it always terminates and terminates in optimal path
- **Theorem**: A* is admissible.
- **Lemma**: Any time before A* terminates there exists on *OL* a node *n* such that $f(n) <= f^*(s)$
- **Observation**: For optimal path $s \rightarrow n_1 \rightarrow n_2 \rightarrow \ldots \rightarrow g$,
  1. $h^*(g) = 0, g^*(s)=0$ and
  2. $f^*(s) = f^*(n_1) = f^*(n_2) = f^*(n_3)\ldots = f^*(g)$

Algorithm A*

- $f^*(n) = g^*(n) + h^*(n)$, where,
- $g^*(n)$ = actual cost of the optimal path (s, n)
- $h^*(n)$ = actual cost of optimal path (n, g)
- $g(n) \leq g^*(n)$
- By definition, $h(n) \leq h^*(n)$
- *h(n) <= h*(n) where h*(n) is the actual cost of optimal path to G(node to be found) from n*

Lemma

Any time before A* terminates there exists in the open list a node n' such that $f(n') <= f^*(S)$

For any node n_i on optimal path,

$f(n_i) = g(n_i) + h(n_i)$

$<= g^*(n_i) + h^*(n_i)$

Also $f^*(n_i) = f^*(S)$

Let n' be the fist node in the optimal path that is in OL. Since all parents of n' have gone to CL,

$g(n') = g^*(n')$ and $h(n') <= h^*(n')$

$=> f(n') <= f^*(S)$

A* always terminates

Proof

If A* does not terminate

Let e be the least cost of all arcs in the search graph.

Then $g(n) >= e.l(n)$ where $l(n)$ = # of arcs in the path from S to n found so far. If A* does not terminate, $g(n)$ and hence $f(n) = g(n) + h(n)$ $[h(n) >= 0]$ will become unbounded.

This is not consistent with the lemma. So A* has to terminate.

Admissibility of A*

The path formed by A* is optimal when it has terminated

Proof

Suppose the path formed is not optimal

Let G be expanded in a non-optimal path.

At the point of expansion of G,

$f(G) = g(G) + h(G)$

$= g(G) + 0$

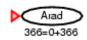
$> g^*(G) = g^*(S) + h^*(S)$

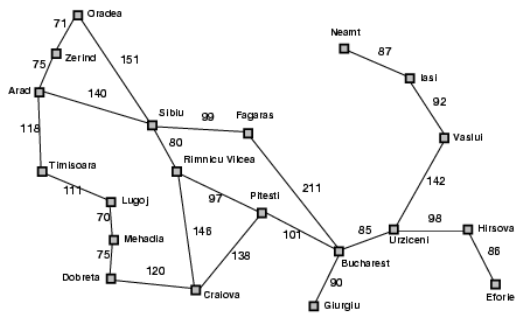
$= f^*(S)$ [$f^*(S)$ = cost of optimal path]

This is a contradiction

So path should be optimal

A* search example





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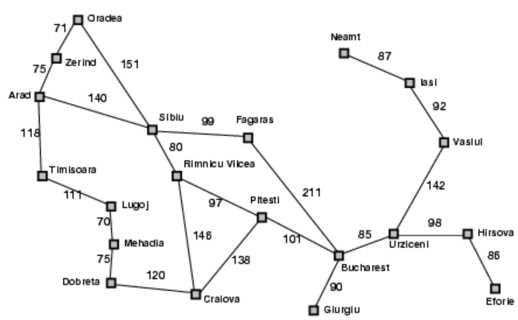
A* search example



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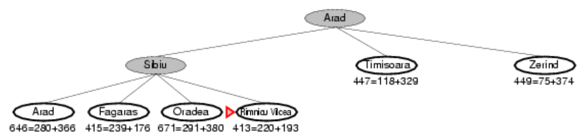

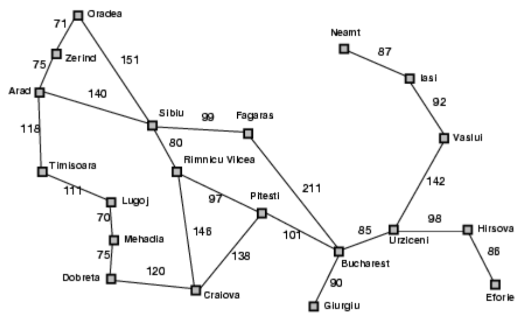

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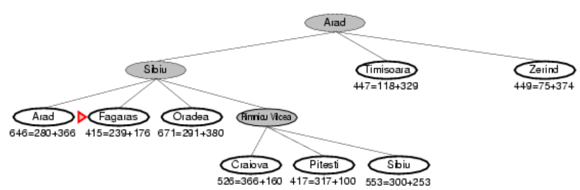

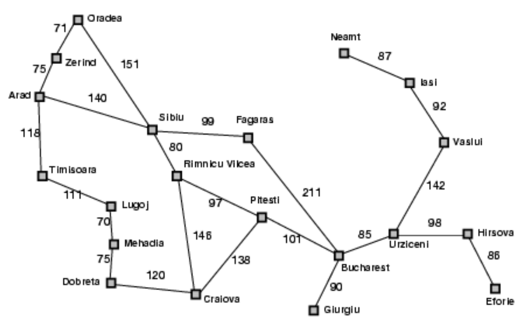

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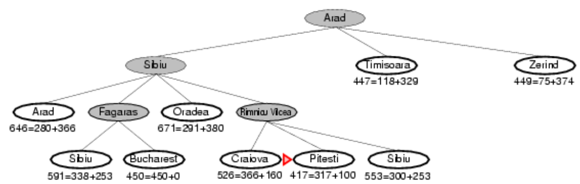

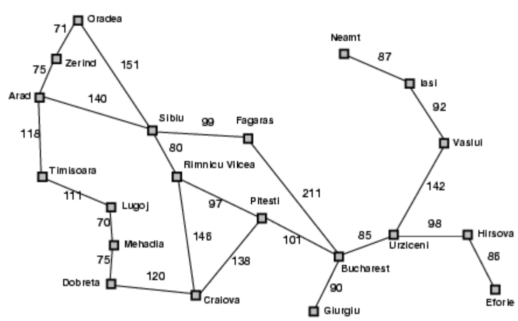

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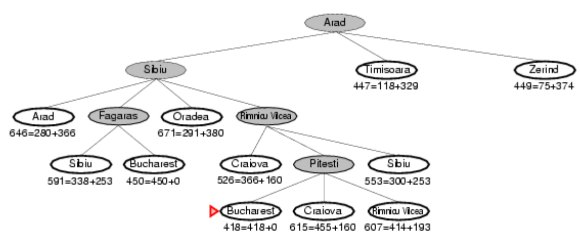

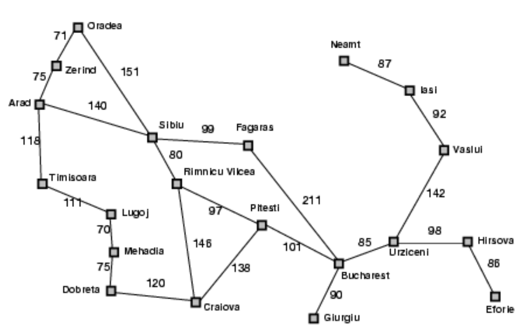

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Properties of A*

- Complete?

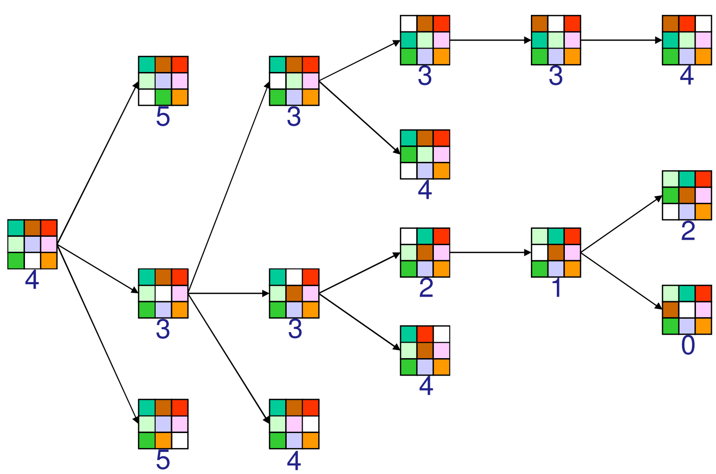
Yes (unless there are infinitely many)

- Time/Space?
- Exponential mostly
- Optimal?

Yes

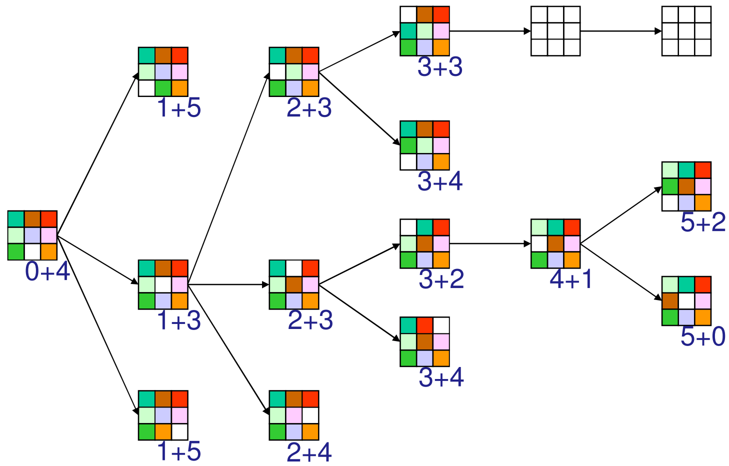
8-Puzzle

f(N) = h(N) = number of misplaced tiles



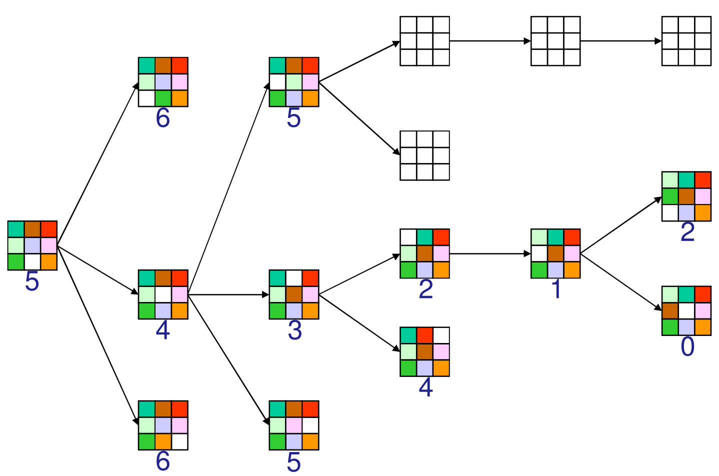
8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles



8-Puzzle

f(N) = h(N) = Σ distances of tiles to goal



Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution