## Game Playing

A (pure) strategy:

a complete set of advance instructions that specifies a definite choice for every conceivable situation in which the player may be required to act.

In a two-player game, a strategy allows the player to have a response to every move of the opponent.

Game-playing programs implement a strategy as a software mechanism that supplies the right move on request.

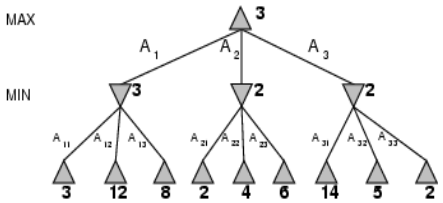
## Two-Person Perfect Information Deterministic Game

- Two players take turns making moves
- Call one Min and the other Max
- Deterministic moves: Board state fully known,
- One player wins by defeating the other (or else there is a tie)
- Want a strategy to win, assuming the other person plays rationally

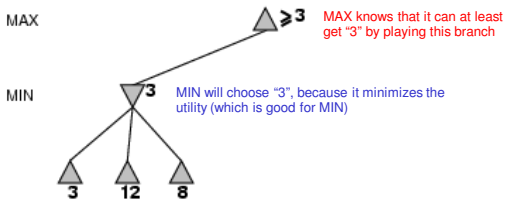
## Pruning the Minimax Tree

- Minimax works best for large trees, but it can be useful even in mini-games such as tic-tac-toe.
- Since we have limited time available, we want to avoid unnecessary computation in the minimax tree.
- Pruning: ways of determining that certain branches will not be useful. Then cut of these branches

## pruning




## α pruning

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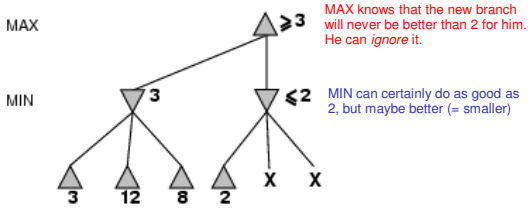

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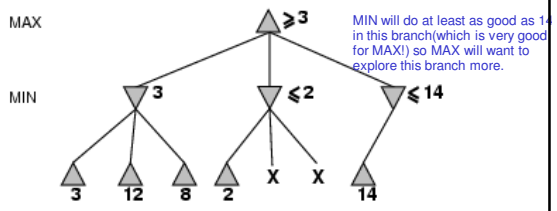
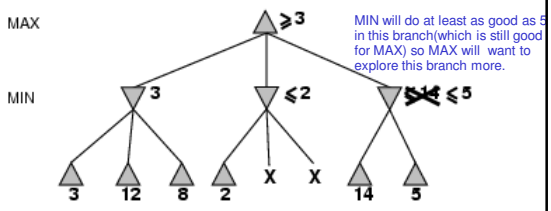


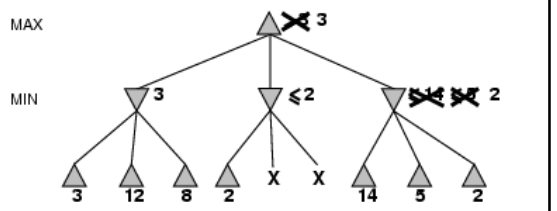

MIN will do at least as good as 14 in this branch(which is very good for MAX!) so MAX will want to explore this branch more.

## α pruning




MIN will do at least as good as 5 in this branch(which is still good for MAX) so MAX will want to explore this branch more.

## α pruning

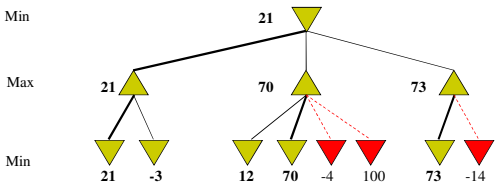



MIN will be able to play this last branch and get 2. This is worse than 3, so MAX will play 3.

## β pruning

- Similar idea to α pruning, but the other way around
- If the current minimum is less than the successor's max value, don't look down that max tree any more

## β pruning example



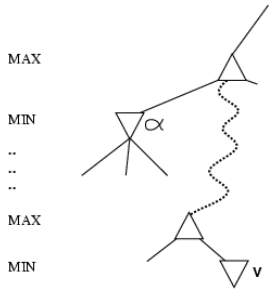

- Some subtrees at second level already have values > min from previous, so we can stop evaluating them.

## Why is it called α-β?

- α is the value of the best (i.e., highest-value) choice found so far at any choice point along the path for *max*
- If *v* is worse than α, *max* will avoid v

→ prune that branch

- Define β similarly for *min*



## α-β Pruning properties

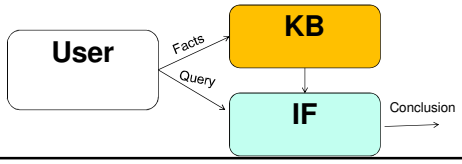
- Pruning by these cuts does not affect final result
  - May allow you to go much deeper in tree
- Properties:
  - Evaluating "best" branch first yields better likelihood of pruning later branches
  - Perfect ordering reduces time to $b^{m/2}$

## Properties of minimax

- Complete? Yes (if tree is finite)
- Optimal? Yes (against an rational opponent)
- Time complexity? $O(b^m)$
- Space complexity? O(bm) (depth-first exploration)
- For chess, b ≈ 35, m ≈100 for "reasonable" games
  → exact solution completely infeasible

## Representation and Reasoning

- In order to determine appropriate actions to take, an intelligent system needs to represent information about the world and draw conclusions based on general world knowledge and specific facts.
- Knowledge is represented by sentences in some language stored in a **knowledge base (KB).**
- A system draws conclusions from the KB to answer questions, take actions using **Inference Engine (IF)**.




## Knowledge Representation

- Logics are formal languages for representing information such that conclusions can be drawn
- **Syntax:** defines the sentences in the language
- **Semantics:** define the "meaning" of sentences: i.e., define truth of a sentence in a world
- E.g., the language of arithmetic
  - x+2 ≥ y is a sentence; x2+y > {} is not a sentence syntax
  - x+2 ≥ y is true in a world where x = 7, y = 1
  - x+2 ≥ y is false in a world where x = 0, y = 6

  } semantics

## Inference

- Logical Inference (deduction) derives new sentences in the language from existing ones,.

  Socrates is a man.

  All men are mortal.

  Socrates is mortal.
- Proper inference should only derive sound conclusions

## Logics

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## Examples of Types of Logics

| Language | What exist | Degree of belief of an Agent |
|---|---|---|
| Propositional Logic | Facts | {o,1} T or F |
| First Order Logic | Facts, Objects, Relations | {o,1} T or F |
| Temporal Logic | Facts, Objects, Relations, Time | {o,1} T or F |
| Probability Theory | Facts | Chances of belief [0,1] |
| Fuzzy Logic | Degree of truth about Facts | Degree of belief [0,1] |

## Propositional calculus & First-order logic

- Propositional logic assumes world contains facts.
- First-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, …
  - Relations: red, round, prime,…
  - Functions: fatherof, friend, in,…
- Propositional calculus
  $A \wedge B \Rightarrow C$
- First-order predicate calculus
  $(\forall x)(\exists y)$ Mother(y,x)

## Syntax of PC

- Connectives: $\neg, \wedge, \vee, \Rightarrow$
- Propositional symbols, e.g., P, Q, R, …
  - *True*, *False*
  - **Syntax of PC**
- sentence → atomic sentence | complex sentence
- atomic sentence → Propositional symbol, *True*, *False*
- Complex sentence → ¬sentence
  | (sentence ∧ sentence)
  | (sentence ∨ sentence)
  | (sentence ⇒ sentence)
  - **Rules of Inference:**
  - Ex: Modus ponens

## Semantics of PC

| A | B | ¬ A | A ∧ B | A ∨ B | A ⇒ B |
|---|---|---|---|---|---|
| *True* | *True* | *False* | *True* | *True* | *True* |
| *True* | *False* | *False* | *False* | *True* | *False* |
| *False* | *False* | *True* | *False* | *False* | *True* |
| *False* | *True* | *True* | *False* | *True* | *True* |

## Validity and Inference

- An Interpretation is an assignment of a truth value (True or False) to each atomic proposition
- A sentence that is true under all interpretation is valid (i.e. tautology)
- Validity can be checked by the truth table
- Inference can be done by checking the validity of each sentence. (may be applying truth table)
- An alternative to checking all rows of a truth table, one can use rules of inference to draw conclusions.

## Rules of Inference

- A sequence of inference rule applications that leads to a desired conclusion is called a **logical proof.**
- A |- B , denotes that B can be derived by some inference procedure from the set of sentences A.
- Inference rules can be verified by the truth-table
- Then used to construct **sound proofs.**
- Finding a proof is simply a **search problem** with the inference rules as operators and the conclusion as the goal

## Rules of Inference

- **Modus Ponens:** $\{\alpha \Rightarrow \beta, \alpha\} \vdash \beta$
- **And Elimination:** $\{\alpha \wedge \beta\} \vdash \alpha$; $\{\alpha \wedge \beta\} \vdash \beta$
- **Double negation Elimination:** $\{\neg\neg\alpha\} \vdash \alpha$
- **Implication Elimination** $\{\alpha \Rightarrow \beta\} \vdash \neg\alpha \vee \beta$
- **Unit resolution:** $\{\alpha \vee \beta, \neg\beta\} \vdash \alpha$
- **Resolution:** $\{\alpha \vee \beta, \neg\beta \vee \gamma\} \vdash \alpha \vee \gamma$

## Models and Entailment

- A model is any interpretation in which a statement is true.
- A sentence A **entails** B (A $\models$ B) if every model of A is Also a model of B. i.e. if A is true then B must be true
- A statement B is entailed from some KB if there is a logical inference to deduce B

  KB $\models$ B if KB→B

## Satisfiability

- A sentence is **satisfiable** if it is true under some interpretation (i.e. it has a model), otherwise the sentence is **unsatisfiable.**
- A sentence is **valid** if and only if its negation is unsatisfiable.
- •Therefore, algorithms for either validity or satisfiability
- checking are useful for logical inference.
- If there are *n propositional symbols in a sentence, then we* must check $2^n$ *rows for validity*
- **Satisfiability is** NP-complete, i.e. there is no polynomial-time algorithm to solve.
- Yet, many problems can be solved very quickly.

## Pros and cons of propositional logic

- ✓ Propositional logic is declarative:

  pieces of syntax correspond to facts
- ✓ Propositional logic is compositional:

  meaning of A ^ B is derived from meaning of A and B
- ✓ Meaning in propositional logic is context-independent
- (unlike natural language, where meaning depends on context)
- ➢ Propositional logic has very limited expressive power
- (unlike natural language)

## Propositional logic is a weak language

- Hard to identify "individuals." Ex. Mary, 3
- Can't directly talk about properties of individuals or relations between individuals. Ex. "Bill is tall"
- Generalizations, patterns, regularities can't easily be represented. Ex. all triangles have 3 sides
- **First-Order Logic** (abbreviated FOL or FOPC) is expressive enough to concisely represent this kind of situation.
  - FOL adds relations, variables, and quantifiers, e.g.,
    - "Every elephant is gray": $\forall$ x (elephant(x) → gray(x))
    - "There is a white elephant": $\exists$ x (elephant(x) ^ white(x))

## First-order logic

- First-order logic (FOL) models the world in terms of
  - Objects, which are things with individual identities
  - Properties of objects that distinguish them from other objects
  - Relations that hold among sets of objects
  - Functions, which are a subset of relations where there is only one "value" for any given "input"

Ex:Objects: Students, lectures, companies, cars ...

- Relations: Brother-of, bigger-than, outside, part-of, has-color, occurs-after, owns, visits, precedes, ...
- Properties: blue, oval, even, large, ...
- Functions: father-of, best-friend, second-half, one-more-than ...

## FOL Syntax

- **Variable symbols**
  - E.g., x, y, John
- **Connectives**: ¬, ∧, ∨, ⇒
  - **Quantifiers**
  - Universal ∀x
  - Existential ∃x

## FOL Syntax

*Sentence* → *Atomicsentence*
| ( *Sentence Connective Sentence* )
| *Quantifier Variable,. . . Sentence*
| *Sentence*
*AtomicSentence* → *Predicate(Term, . . .)*
| ( *Term* = *Term*
*Term* → *Function(Term, . . .)*
I *Constant*
| *Variable*
*Connective* → ¬, ∧, ∨, ⇒
*Quantifier* → ∀, ∃
*Constant* → *A* ( XI ( *John* 1 . . .
*Variable* → *a* | *x* | s | . . .
*Predicate* → *Before…*
*Function* → *Mother* | …

## Nested Quantifiers

- Combinations of universal and existential quantification are possible:

$\forall x \forall y\ Father(x,y) \equiv \forall y \forall x\ Father(x,y)$
$\exists x \exists y\ Father(x,y) \equiv \exists y \exists x\ Father(x,y)$
$\forall x \exists y\ Father(x,y) \neq \exists y \forall x\ Father(x,y)$
$\exists x \forall y\ Father(x,y) \neq \forall y \exists x\ Father(x,y)$
$x,y \in \{All\ people\}$

## Logical equivalence in PC

- Two sentences are logically equivalent iff true in same models: α ≡ ß iff α ⊨ β and β ⊨ α
- EXamples:

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$ commutativity of ∧
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$ commutativity of ∨
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$ associativity of ∧
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$ associativity of ∨
$\neg(\neg\alpha) \equiv \alpha$ double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$ contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$ implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$ biconditional elimination
$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$ de Morgan
$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$ de Morgan
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$ distributivity of ∧ over ∨
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$ distributivity of ∨ over ∧

## A common mistake to avoid

- Typically, ⇒ is the main connective with ∀
- Common mistake: using ∧ as the main connective with ∀:
- Ex:

∀x At(x,CU) ∧ Smart(x)
means "Everyone is at CU and everyone is smart"

Yet to say Everyone at CU is smart
∀x At(x,CU) ⇒ Smart(x)

## Another common mistake to avoid

- Typically, ∧ is the main connective with ∃
- Common mistake: using ⇒ as the main connective with ∃:

$\exists x$ At(x,CU) ⇒ Smart(x)

is true if there is anyone who is smart not at CU.

Yet to say: there exists someone in CU that is smart

$\exists x$ At(x,CU) ∧ Smart(x)

## Properties of quantifiers

∀x ∀y is the same as ∀y ∀x
∃x ∃y is the same as ∃y ∃x

∃x ∀y is not the same as ∀y ∃x
∃x ∀y Loves(x,y)
- "There is a person who loves everyone in the world"

∀y ∃x Loves(x,y)
- "Everyone in the world is loved by at least one person"

- Quantifier duality: each can be expressed using the other

| Exp. | Negation |
|---|---|
| ∀x Likes(x,IceCream) | ∃x ¬Likes(x,IceCream) |
| ∃x Likes(x,Broccoli) | ∀x ¬Likes(x,Broccoli) |

## Equality

Equality:

$term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object

FOPC can include equality as a primitive predicate or require it to be as identity relation

Equal(x,y) or x=y

Examples:

to say "that Mary is taking two courses", you need to insure that x,y are different

∃x ∃y ( takes(Mary,x) ^ takes (Mary,y) ^ ~ (x=y))

To say "Everyone has exactly one father"

∀x ∃y father(y,x) ^ ∀z father(z,x) ➔ y=z

## Higher Order Logic

- FOPC is called first order because it allows quantifiers to rang only over objects (terms).

  ∀x, ∀y [*x=y or x>y or y>x*]

- **Second-Order Logic** allows quantifiers to range over predicates and functions as well

  ∀f, ∀g [*f=g* <=> (∀x f(x)=g(x))]

- **Third-Order Logic** allows quantifiers to range over predicates of predicates,.. etc

## Examples of FOPC

- Brothers are siblings

  ∀x, ∀y *Brother(x,y)* => *Sibling(x,y)*

- One's mother is one's female parent

  ∀m, ∀c *Mother(c)* = m ⇔ *(Female(m)* ∧ *Parent(m,c))*

- "Sibling" is symmetric

  ∀x, ∀y *Sibling(x,y)* ⇔ *Sibling(y,x)*

## Translating English to FOL

- Every gardener likes the sun.

```
(∀x) gardener(x) => likes(x,Sun)
```

## Translating English to FOL

- Every gardener likes the sun.

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(∀x) gardener(x) => likes(x,Sun)
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- You can fool some of the people all of the time.

```
(∃x) person(x) ^ ((∀t) time(t)) => can-fool(x,t))
```

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  `(∀x) person(x) => ((∃t) time(t) ^ can-fool(x,t))`
- All purple mushrooms are poisonous.
  `(∀x) (mushroom(x) ^ purple(x)) => poisonous(x)`

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- No purple mushroom is poisonous.
  `~(∃x) purple(x) ^ mushroom(x) ^ poisonous(x)`
  or, equivalently,
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  or, equivalently,
  `(∀x) (mushroom(x) ^ purple(x)) => ~poisonous(x)`
- There are exactly two purple mushrooms.
  `(∃x)(∃y) mushroom(x) ^ purple(x) ^ mushroom(y) ^ purple(y) ^ ~(x=y) ^ (∀z) (mushroom(z) ^ purple(z)) => ((x=z) v (y=z))`