Empirical Discrete Distribution

- Find the random variate for the following discrete distribution.
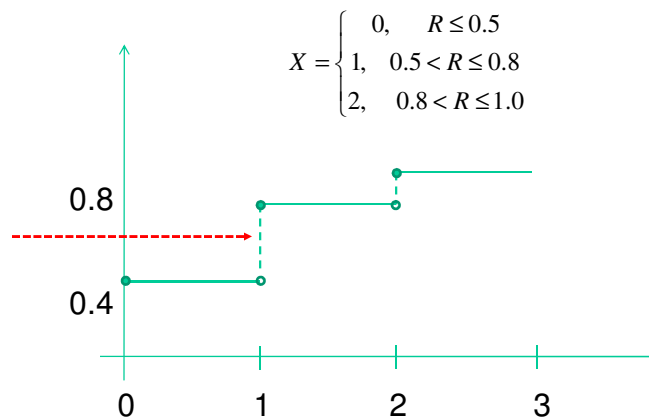
x	p(x)
0	0.50
1	0.30
2	0.20

- At first find the F
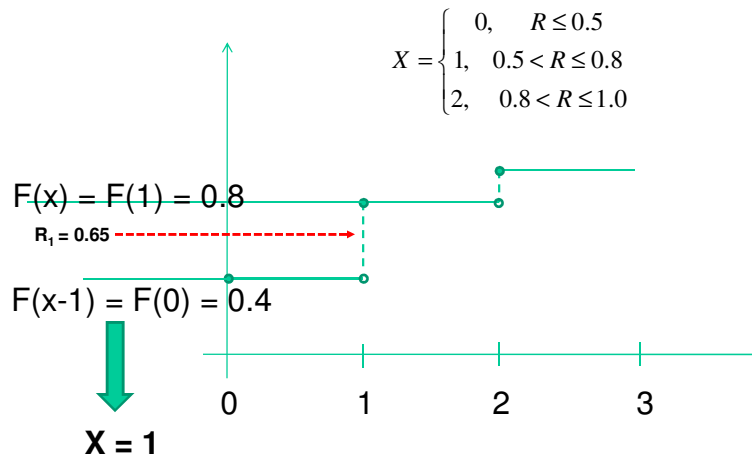
x	p(x)	F(x)
0	0.50	0.50
1	0.30	0.80
2	0.20	1.00

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.5, & 0 \le x < 1 \\ 0.8, & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$

Empirical Discrete Distribution

$$X = \begin{cases} 0, & R \le 0.5 \\ 1, & 0.5 < R \le 0.8 \\ 2, & 0.8 < R \le 1.0 \end{cases}$$

Empirical Discrete Distribution

$$X = \begin{cases} 0, & R \le 0.5 \\ 1, & 0.5 < R \le 0.8 \\ 2, & 0.8 < R \le 1.0 \end{cases}$$

F(x) = F(1) = 0.8

$R_1 = 0.65$

F(x-1) = F(0) = 0.4

0   1   2   3

X = 1

Example of Empirical Distribution

- There are four pumps(i-1 to 4) at a petrol station. And on average 1/3 of the customers used pump 1, 1/6 of them used pump 2, another 1/3 used pump 3 and the remaining 1/6 used pump 4. Outline a procedure for selection of pumps by various customers.
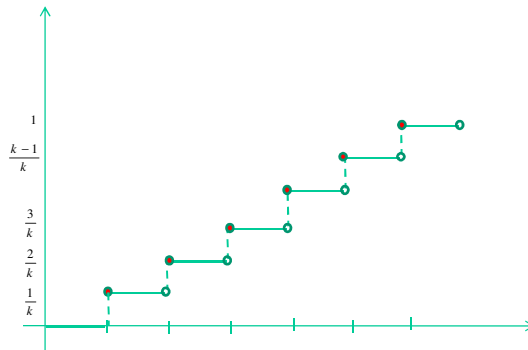
i	1	2	3	4
P(i)	1/3	1/6	1/3	1/6
F(i)	1/3	½	5/6	1

- The procedure:
- Generate r ~U(0,1)

     If $0 \le r < 1/3$ select pump 1

     If $1/3 \le r < 1/2$ select pump 2

     If $1/2 \le r < 5/6$ select pump 3

     If $5/6 \le r < 1$ select pump 4

Discrete Uniform Distribution
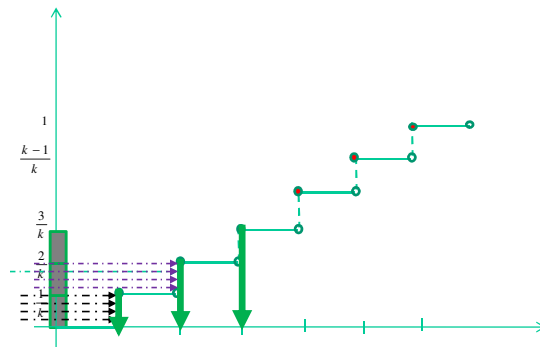
- Consider $p(x) = \dfrac{1}{k}, \; x = 1, 2, 3 \ldots k$

$$F(x) = \begin{cases} 0, & x < 1 \\ \dfrac{1}{k}, & 1 \le x < 2 \\ \dfrac{2}{k}, & 2 \le x < 3 \\ \vdots & \vdots \\ \dfrac{k-1}{k}, & k-1 \le x < k \\ 1, & k \le x \end{cases}$$

Discrete Uniform Distribution

- Generate R~U(0,1)

$$0 \le R \le \frac{1}{k}$$

$$\frac{1}{k} < R \le \frac{2}{k}$$

$$\frac{2}{k} < R \le \frac{3}{k}$$

$$\frac{3}{k} < R \le \frac{4}{k}$$

$$\frac{i-1}{k} < R \le \frac{i}{k}$$

Discrete Uniform Distribution

$$0 \le R \le \frac{1}{k}$$

$$\frac{1}{k} < R \le \frac{2}{k}$$

$$\frac{2}{k} < R \le \frac{3}{k}$$

$$\frac{3}{k} < R \le \frac{4}{k}$$

$$\frac{i-1}{k} < R \le \frac{i}{k}$$

$$\frac{i-1}{k} < R \le \frac{i}{k}$$

$$\Rightarrow i - 1 < Rk \le i$$

$$\Rightarrow i - 1 < Rk \le i$$

$$\Rightarrow i < Rk + 1 \text{ and } Rk \le i$$

$$\Rightarrow Rk \le i < Rk + 1$$

$$\Rightarrow i = \lfloor RK \rfloor = \text{output } X$$

$$\therefore X = \lfloor RK \rfloor$$

Discrete Uniform Distribution

- Algorithm to generate random variate for p(x)=1/k where x = 1, 2, 3, … k

  – Generate R~U(0,1) uniform random number
  – Return $\lfloor RK \rfloor$