11.4 Competitive Learning

11.4.2 A Kohonen Network for Learning Prototypes

Kohonen Learning & Classification of Data

- **Winner-Take-All Classification methed.**
- Kohonen learning is **unsupervised**, with a set of prototypes randomly created and then refined until they come to explicitly represent the clusters of data.
- It builds a set of evolving and explicit prototypes to represent the data clusters.
- As the algorithm continues, the learning constant is progressively reduced so that each new input vector will cause less perturbation in the prototypes.

Data Points & Prototypes

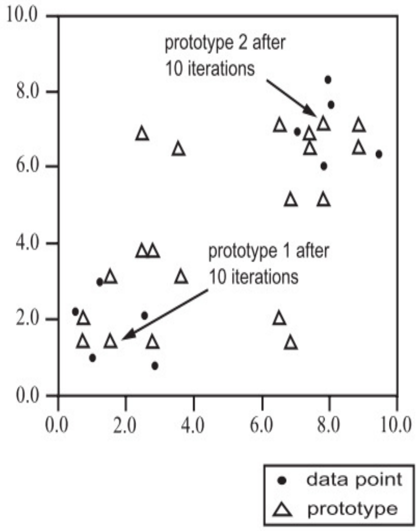

Figure 11.14 The use of a Kohonen layer, unsupervised, to generate a sequence of prototypes to represent the classes of Table 11.3.

- Figure 11.14 presents again the data points of Table 11.3. Superimposed on these points are a series of prototypes created during network training.

Data Points & Prototypes

- The perceptron training algorithm converged after a number of iterations, resulting in a network weight configuration defining a **linear separation between the two classes.**

x_1	x_2	Output
1.0	1.0	1
9.4	6.4	−1
2.5	2.1	1
8.0	7.7	−1
0.5	2.2	1
7.9	8.4	−1
7.0	7.0	−1
2.8	0.8	1
1.2	3.0	1
7.8	6.1	−1

Table 11.3 A data set for perceptron classification.

Kohonen Learning

- Kohonen learning has a strong inductive bias in that **the number of desired prototypes is explicitly identified at the beginning of the algorithm and then continuously refined**. This allows the net algorithm designer to identify a specific number of prototypes to represent the clusters of data.

A Kohonen Learning Network For Classification Of The Data Of Table 11.3

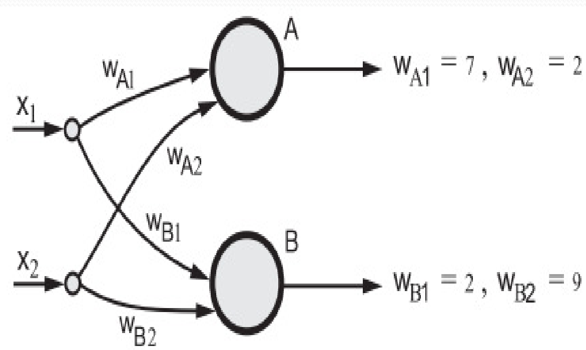

Figure 11.15 The architecture of the Kohonen based learning network for the data of Table 11.3 and classification of Figure 11.14.

A Kohonen Learning Network

- The data are represented in Cartesian two dimensional space, so prototypes to represent the data clusters will also be ordered pairs.
- We select two prototypes, one to represent each data cluster. We have randomly initialized node A to (7, 2) and node B to (2, 9).
- Random initialization only works in simple problems such as ours; an alternative is to set the weight vectors equal to representatives of each of the clusters.

Kohonen Learning Algorithm

- **The winning node will have a weight vector closest to that of the input vector. This weight vector for the winning node will be rewarded by being moved even closer to the input data, while the weights on the losing nodes are left unchanged.**
- Kohonen learning is **unsupervised**, in that a simple measure of the distance between each prototype and the data point allows selection of the winner.
- Kohonen learning selects data points for analysis in random order.

Kohonen Learning Algorithm

- For point (1, 1), we measure the distance from each prototype:

$$||(1, 1) - (7, 2)|| = (1 - 7)^2 + (1 - 2)^2 = 37, \text{ and}$$
$$||(1, 1) - (2, 9)|| = (1 - 2)^2 + (1 - 9)^2 = 65.$$

- **Node A (7, 2) is the winner since it is closest to (1, 1).** ||(1, 1) − (7, 2)|| represents the distance between these two points; we do not need to apply the square root function in the Euclidean distance measure because the relation of magnitudes is invariant.

Kohonen Learning Algorithm

- We now reward the winning node, using the learning constant c set to 0.5. For the second iteration:

$$\begin{aligned} W^2 &= W^1 + c(X^1 - W^1) \\ &= (7, 2) + .5((1, 1) - (7, 2)) = (7, 2) + .5((1 - 7), (1 - 2)) \\ &= (7, 2) + (-3, -.5) = (4, 1.5) \end{aligned}$$

- At the second iteration of the learning algorithm we have, for data point (9.4, 6.4):

$$||(9.4, 6.4) - (4, 1.5)|| = (9.4 - 4)^2 + (6.4 - 1.5)^2 = 53.17 \text{ and}$$
$$||(9.4, 6.4) - (2, 9)|| = (9.4 - 2)^2 + (6.4 - 9)^2 = 60.15$$

Kohonen Learning Algorithm

- Again, node A is the winner. The weight for the third iteration is:

$$\begin{aligned} W^3 &= W^2 + c(X^2 - W^2) \\ &= (4, 1.5) + .5((9.4, 6.4) - (4, 1.5)) \\ &= (4, 1.5) + (2.7, 2.5) = (6.7, 4) \end{aligned}$$

- At the third iteration we have, for data point (2.5, 2.1):

$$\|(2.5, 2.1) - (6.7, 4)\| = (2.5 - 6.7)^2 + (2.1 - 4)^2 = 21.25, \text{ and}$$
$$\|(2.5, 2.1) - (2, 9)\| = (2.5 - 2)^2 + (2.1 - 9)^2 = 47.86.$$

Kohonen Learning Algorithm

- Node A wins again and we go on to calculate its new weight vector. Figure 11.14 shows the evolution of the prototype after 10 iterations.
- The algorithm used to generate the data of Figure 11.14 selected data randomly from Table 11.3, so the prototypes shown will differ from those just created.
- **The progressive improvement of the prototypes can be seen moving toward the centers of the data clusters.**