System Frequency Tuning for Heaving Buoy Wave Energy Converters

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Abstract—In this paper, we propose a control technique that maximizes power absorption in point absorber wave energy converter systems. This technique involves the tuning of the Eigen-frequency of the converter system to meet the incident waves energy frequency, providing near-resonance operation. The system was modeled, where the effects of radiation forces due to oscillator-generated waves were considered. The radiation forces depend on the hydrodynamic coefficients, the added mass, and the hydrodynamic damping, which are calculated through solving the hydrodynamic boundary value problem for heaving bodies. The power absorbed through the power take-off system corresponds to the external damping of the system. The frequency tuning is performed by changing the effective stiffness of the system through the introduction of an external stiffness that is connected in parallel to the buoy stiffness. We propose changing the effect of this external stiffness by means of a stepless V-belt variable speed drive, which changes the speed ratio continuously. The effect of changing the drive speed ratio on the system Eigen-frequency was investigated. The effects of the power take-off system equivalent damping on both the absorbed power and the absorption efficiency for a frequency-tuned system are presented. Numerical results showed promising power absorption and efficiency values, compared with other control techniques.

I. INTRODUCTION

Heaving buoy point absorbers are floating oscillators which oscillate over waves due to their buoyancy. They are advantaged over other types of Wave Energy Converters (WEC) in that they are omni-directional and able to collect wave energy equally from any direction [1], [2]. They are commonly modeled as single-degree-of-freedom vibrating systems. The main parameters affecting the oscillatory motion of these devices are the buoy’s mass, system stiffness, and the damping. The stiffness for floating devices is due to the buoyancy force which mainly depends on the floater size and shape. The damping is caused by the power take-off systems (PTO) connected to these oscillators, which damp the oscillations through absorbing its energy by means of electrical generators.

In addition to the restoring force and the PTO damping force, oscillating floaters are subjected to radiation forces, generated due to the added mass of fluid, which move with the oscillator during its oscillating motion, in addition to the hydrodynamic damping, which is the resistance of the motion of the fluid itself. The added mass and the hydrodynamic damping are called the hydrodynamic coefficients and can be calculated by the solution of the hydrodynamic boundary value problem (HBVP) [3].

Heaving buoy point absorbers achieve maximum energy capture when tuned to the energy frequency or the energy Eigen period of the wave [1]. As wave frequencies change according to the sea state, these devices must be controlled. Control techniques are categorized according to the time taken to adjust or tune the system; namely, long term control, short term control, and wave-by-wave control [1]. Long and short term control techniques include tuning the system by adjusting its natural frequency. Wave-by-wave control techniques include tuning the system by means of phase control such as latching control and declutching control. They can lead to significant power absorption increase; however, they often require perfect foreknowledge of the incident wave, in addition to a more complex PTO mechanism as compared with long and short control techniques. Flocard and Finnigan showed an example of long term control by inertia adjustment while, Lopes et al. showed a phase control strategy with latching control [4], [5], [6].

For most power extraction, damping must be adjusted to achieve maximum energy conversion efficiency. For very high damping, the resulting motion is low, which causes reduced generated power. For very small damping, little power is taken off. Therefore, the correct selection for the PTO system damping is of great importance [6].

In this research, we propose a control technique that involves the tuning of a heaving buoy WEC system Eigen-period to meet the incident waves energy period, providing near-resonance operation; hence, maximum power absorption. The tuning of the Eigen period of the system, is performed by changing the effective stiffness of the system through the introduction of a variable external stiffness that is connected parallel to the buoy stiffness. We change the effect of this external stiffness by means of a stepless V-belt variable-speed drive, which introduces a continuous variable speed ratio to be used for different sea states. The effects of the PTO damping on the system and the absorbed power are investigated.
II. MATHEMATICAL MODELING

We consider a cylindrical buoy of diameter $D_b$ and mass $m_b$. This buoy is subjected to hydrostatic and restoring forces that correspond to a system equivalent stiffness $k_{r,eff}$. The system damping $b$ is the system PTO damping. A schematic of the heaving buoy WEC system is shown in Fig. 1. Initially, the stiffness $k_{r,eff}$ is the static restoring force of the buoy $k$:

$$k = \rho g A,$$

where $\rho$ is the water density, $g$ is the gravitational acceleration, and $A$ is the cross-sectional area of the cylindrical buoy. The buoy is subjected to radiation force $f_r$ which is due to waves generated by the oscillations of the buoy. The force $f_r$ depends on the hydrodynamic coefficients, the added mass $m_r$ and the hydrodynamic damping $b_r$, and the frequency of oscillations of the buoy $\omega$.

The system was modeled, where the effects of radiation forces $f_r$ on the buoy heave displacement $z$ were considered, as presented by (2).

$$(m_b + m_r(\infty))\ddot{z}(t) + b \dot{z}(t) + k_{r,eff}z(t) + f_r(t) = f_e(t),$$

where

$$f_r(t) = \int_{-\infty}^{t} h_r(t - \tau) \dot{z}(\tau) d\tau,$$

(3)

$f_e$ is the excitation force, $m_r(\infty)$ is the high frequency limit of the added mass, and $h_r$ is the inverse Fourier transform of the radiation impedance $\zeta_r(\omega) = b_r(\omega) + i\omega m_r(\omega)$.

We calculate the hydrodynamic coefficients from the solution of the hydrodynamic boundary value problem [7, 8]:

$$m_r + i \frac{b_r}{\omega} = \left(\frac{1}{3} + \frac{1}{8} \left(\frac{r}{h - d}\right)^2 + \sum_{j=0}^{N_r} \gamma_{0j} L_{0s_j} \right) + \frac{4}{\pi} \left(\frac{h - d}{r}\right) \sum_{n=1}^{N_n} \frac{(n) I_1(k_n r)}{n I_0(k_n r)} \left\{ \sum_{j=0}^{N_p} \gamma_{0j} L_{ns_j} - \left(\frac{-1}{n^2 \pi^2}\right) S, \right.$$  

(4)

where $r$ is the radius of the buoy, $h$ is the water depth, $d$ is the draft, and $S = \pi r^2 (h - d) \rho$. The functions and variables $\gamma_{0j}$, $L_{ns_j}$, $I_1(k_n r)$, $I_0(k_n r)$, $N_p$ are defined in the Appendix.

State-space modeling is used to approximate the convolution integral in (3), calculating the radiation force $f_r$. We use the companion-form state-space description to approximate this integral by a linear sub-system as follows [9]:

$$f_r(t) = C_p X_p(t)$$

(5)

where

$$\dot{X}_p(t) = A_p X_p(t) + B_p u_p(t),$$

(6)

where $f_r(t)$ is radiation force which is the sub-system output, $u_p(t)$ is the sub-system input which is the heaving velocity of the buoy and, $X_p$ is the state vector of the sub-system. The companion-form realization is used here with matrices $A_p$, $B_p$ and, $C_p$ as follows [9]:

$$C_p = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$A_p = \begin{bmatrix} 0 & 0 & 0 & 0 & -a_1 \\ 1 & 0 & 0 & 0 & -a_2 \\ 0 & 1 & 0 & 0 & -a_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & -a_{n-1} \\ 0 & 0 & 0 & 0 & -a_n \end{bmatrix}$$

$$B_p = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix}^T$$

where $a_n$ and $b_n$ are unknown parameters that are calculated such that the following target function $Q$ is minimized:

$$Q = \sum_{k=1}^{m} G(t_k) [h_r(t_k) - C_p e^{A_p t_k} B_p]^2,$$

(7)

where $G(t_k)$ is a weight function to be chosen, and $h_r(t_k)$ is the value of the impulse response function at chosen instants $t_k$. A reasonably good approximation is often obtained even if $n$ is a rather small integer [9]. We used the pattern-search algorithm [10] for minimization of the target function $Q$ in (7) with $G(t_k) = 1$, $n = 3$, $\Delta t_k = 0.1sec$, and $m = 51$. State matrices coefficients ($a_n$’s and $b_n$’s) are calculated by minimization of the target function $Q$.

By solving the system differential equation (2) along with (7), the buoy displacement $z(t)$, and velocity $\dot{z}(t)$ can be calculated. The average absorbed power is:

$$P_{av} = 0.5 b v^2,$$

(8)

where $v$ is the buoy velocity amplitude.

As the wave energy period continuously changes according to the sea state, tuning of the system natural frequency must provide the suitable value in order to achieve near-resonance operation. The wave energy period $T_w$, or frequency $\omega_w$, can be calculated from the Pierson-Moskowitz spectrum which is an empirical relationship that defines the distribution of energy versus frequency, where the spectrum input is the wind speed. The spectrum formula is as follows [11, 12, 13]:

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \exp(-\beta \left(\frac{g}{U \omega}\right)^4),$$

(9)
where $S(\omega)$ is the power spectral density, $\omega$ is the frequency, $U$ is the wind speed measured at 19.5 m above the sea surface, and $\alpha$ and $\beta$ take the values of 0.0081 and 0.74, respectively. Spectrum calculations provide the significant wave height $H_s$, the energy period $T_e$, and the available power per unit width of incident waves $P_{avt}$ (power flux, kW/m) as follows:

$$H_s = 4\sqrt{M_0}$$

$$T_e = 0.84123B^{-0.25}$$

$$P_{avt} = 0.5H_s^2T_e, \quad (12)$$

where $B$ is represented as:

$$B = 0.7401\left(\frac{g}{2\pi U}\right)^4, \quad (13)$$

$M_0$ is the spectrum zeroth moment ($M_0 = \alpha g^2/4B(2\pi)^4$).

Fig. 2 shows the change of energy frequency with wind speed calculated by (11) and (13).

III. PROPOSED CONTROL TECHNIQUE

Since the wind speed is not constant, energy frequency of incident waves changes as well. To maintain near-resonance operation and maximize power absorption, system natural frequency must be controlled.

Here, we propose connecting an external stiffness $k_e$ in parallel to the buoy stiffness $k$ such that the effective stiffness $k_{eff}$ is changed. The buoy stiffness $k$ must be less than the lowest effective stiffness $k_{eff}$, corresponding to the lowest natural frequency required for the system. We change the effect of the external stiffness $k_e$ through the use of a stepless V-belt variable-speed drive with a controllable speed ratio $\Delta$ (see Fig. 3). When connecting $k_e$, the effective stiffness $k_{eff}$ becomes:

$$k_{eff} = k + \frac{k_e}{\Delta^2}, \quad (14)$$

where $\Delta$ is:

$$\Delta = \frac{R_s}{R_b}, \quad (15)$$

and $R_s$ and $R_b$ are the running radii for both pulleys connected to spring and buoy respectively.

The pulley running radius $R_\alpha$ is controlled through the control of its corresponding movable sheave axial position $S_\alpha$, shown in Fig. 4, which is calculated as:

$$S_\alpha = 2\tan(\phi)(R_\alpha - R_{\alpha,\text{min}}), \quad (16)$$

where $\phi$ is the taper angle of the sheaves, $R_\alpha$ is the running radius of the $\alpha$ pulley, and $R_{\alpha,\text{min}}$ is the minimum value for the running radius. The movable sheave axial positions $S_\alpha$ are controlled by linear actuators.

To achieve near-resonance operation, $k_e$ must be changed according to the energy frequency $\omega_e$ or the wind speed $U$. The effect of $k_e$ is changed by the control of the sheave axial positions $S_s$ and $S_b$, which changes the speed ratio $\Delta$. We measure the energy frequency by means of an incremental encoder with a separate smaller buoy. The measured displacements are processed to generate the power spectrum density of the waves, hence, $\omega_e$ is calculated.

We calculated $\Delta$ for a selected energy frequency range of 0.8-2.4 rad/sec. This frequency range corresponds to a wind speed range of 4.5-13.5 m/sec, a normal range to operate WECs (see Fig. 2). The sheave axial positions $S_s$ and $S_b$ were calculated from (14) and (16) and are presented in Fig. 5. For this frequency range, we calculated speed ratio $\Delta$, shown in Fig. 6. All calculations used a selected buoy...
IV. RESULTS AND DISCUSSION

Energy absorption of WECs is evaluated based on the value of its average absorbed power $P_{av}$ as compared with the available power per its physical width. This comparison leads to the calculation of the optimum value for power take-off (PTO) system damping at which power absorption is maximum. The absorption efficiency of WECs is calculated as follows:

$$\eta = \frac{P_{av}}{P_{avl} \times \text{floater width}}.$$  \hspace{1cm} (17)

The absorption efficiency can also be represented by the capture width $\lambda$, which is the width of the wave front containing the same available power as the absorbed power by the device [2]:

$$\lambda = \frac{P_{av}}{P_{avl}}.$$  \hspace{1cm} (18)

If $\lambda = D_b$, the absorption efficiency $\eta$ equals 100%. The absorption efficiency can exceed 100% ($\lambda > D_b$) since the device interacts with the entire surrounding wave field.

Numerical results were performed for our selected buoy, (see Section III), where the effects of the PTO damping on both the buoy oscillations displacement $z$ and velocity $\dot{z}$ were investigated at wind speeds of 7, 10, and 12 m/s. Results are presented in Figs. 7 and 8 for various values of PTO damping. Fig. 7 shows the ratio between $z$ and the significant wave amplitude $0.5H_s$. Results confirmed that the displacement $z$ and velocity $\dot{z}$ increased as the damping decreased. This increase is most significant at a PTO damping of approximately $b=2$ kN.s/m.

The effects of the PTO damping on the average absorbed power changes, and on the efficiency and capture width were also studied and presented in Fig. 9 and Fig. 10.

Damping must be selected so that it is neither too high

\[ \text{Buoy-side sheave, } S_b \quad \text{Spring-side sheave, } S_s \]

Fig. 5: Required sheaves axial positions $S_s$ and $S_b$ versus energy frequency $\omega_c$.

Fig. 6: Required speed ratio $\Delta$ versus energy frequency $\omega_c$.

Fig. 7: Buoy displacement amplitude $z / \text{significant wave amplitude } 0.5H_s$ versus PTO damping $b$ at different wind speeds.

Fig. 8: Buoy velocity amplitude $\dot{z}$ versus PTO damping $b$ at different wind speeds.
nor too low. It affects the absorption efficiency $\eta$ or the capture width $\lambda$. Fig. 9 show that the optimum value for damping, hence, maximum power absorption is achieved at approximately $b=0.4 \text{kN.s/m}$ at which the efficiency exceeds 100%. This is just a theoretical optimum since the buoy velocities are too high due to this low damping value, as shown in Fig. 8.

Since the optimum damping value depends on the incident wave energy frequency, or wind speed, the system should be pre-tuned to suit for the range of energy frequencies where it is deployed or tuned. Here, we recommend a PTO damping of $1 \text{kN.s/m}$, which gives acceptable velocity amplitude at a typical wind speed range, as shown in Figs. 8 and 9. At this damping, our proposed tuning method shows a high absorption efficiency, which exceeds 80% at acceptable oscillation velocity amplitude (for wind speeds less than 10 m/s - see Figs. 8 and Fig. 10). The achieved efficiencies are higher than those with latching control, which are normally in the range of 60%\(^{s}\) [5]. This system has also the advantage of longer time operations with reduced demands from the PTO as compared to the latching and declutching control techniques.

V. CONCLUSION AND FUTURE WORK

Heaving buoy WEC oscillations were modeled with the effect of radiation forces taken into consideration. We proposed a method to increase the power absorption by tuning the system natural frequency to the energy frequency of the incident waves. We tune the system by introducing an external stiffness, whose effect on the buoy is controlled using a stepless V-belt variable speed drive; thus, tune the natural frequency of the system. The damping of the power take-off system is selected such as to achieve highest possible power absorption with suitable oscillation velocity amplitudes. Simulation results showed promising power absorption and efficiency values, when compared with other control techniques.

The optimum PTO damping, which corresponds to suitable velocity amplitudes, changes as the energy frequency changes. Future work involves the tuning of the damping to achieve maximum efficiency at different wind speeds, while keeping suitable buoy velocities. In our future work, we are working on experimentally validating our numerical model.

REFERENCES


**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>Frequency</td>
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<tr>
<td>$\omega_e$</td>
<td>Incident waves energy frequency</td>
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<tr>
<td>$\omega_n$</td>
<td>System natural frequency</td>
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<tr>
<td>$\phi$</td>
<td>Taper angle for the conical sheave</td>
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<td>$\lambda$</td>
<td>Capture width</td>
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<tr>
<td>$\eta$</td>
<td>Absorption efficiency</td>
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<tr>
<td>$\zeta_r$</td>
<td>Radiation impedance</td>
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<tr>
<td>$b$</td>
<td>Power take-off system damping</td>
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<tr>
<td>$b_r$</td>
<td>Hydrodynamic damping</td>
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<tr>
<td>$\delta$</td>
<td>Draft (submerged height)</td>
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<tr>
<td>$D_b$</td>
<td>Oscillator (buoy) diameter</td>
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<tr>
<td>$f_r$</td>
<td>Radiation force</td>
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<tr>
<td>$h$</td>
<td>Water depth</td>
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<tr>
<td>$h_r$</td>
<td>Inverse Fourier transform of the radiation impedance</td>
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<td>$H_0^{(1)}$</td>
<td>Hankel function of first kind of order zero</td>
</tr>
<tr>
<td>$H_s$</td>
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<tr>
<td>$k$</td>
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<td>External stiffness</td>
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<td>$k_{eff}$</td>
<td>Effective stiffness (restoring)</td>
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</tr>
<tr>
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<td>Oscillator (buoy) mass</td>
</tr>
<tr>
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<td>Added mass</td>
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<tr>
<td>$M_n$</td>
<td>$n^{th}$ spectral moment</td>
</tr>
<tr>
<td>$P_{av}$</td>
<td>Average absorbed power</td>
</tr>
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<td>Available power per unit width of wave front</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Speed ratio</td>
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<td>Running radius for $\alpha$ pulley</td>
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<tr>
<td>$R_{\alpha,min}$</td>
<td>Minimum running radius for $\alpha$ pulley</td>
</tr>
<tr>
<td>$S_\alpha$</td>
<td>Axial movable sheave position for $\alpha$ pulley</td>
</tr>
<tr>
<td>$S(\omega)$</td>
<td>Power spectral density</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Incident waves energy period</td>
</tr>
<tr>
<td>$U$</td>
<td>Wind speed</td>
</tr>
<tr>
<td>$X_p$</td>
<td>State vector of radiation force calculation sub-system</td>
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<tr>
<td>$z(t)$</td>
<td>Oscillator (buoy) displacement</td>
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$N_{s_0}$ and $N_{s_j}$ are calculated as:

$$N_{s_0} = \frac{1}{2} \left(1 + \frac{\sinh(2s_0h)}{2s_0h}\right),$$

$$N_{s_j} = \frac{1}{2} \left(1 + \frac{\sin(2s_jh)}{2s_jh}\right).$$

$I_0(k_nr)$ and $I_1(k_nr)$ are modified Bessel functions of the first kind of order $0$ and $1$ respectively and $k_n = \frac{n\pi}{n\pi}$. $\gamma_{0j}$ can be calculated directly from:

$$\gamma_{0j} = \chi_{0t}$$

where $\varepsilon_{ij}$ ($l = 0, 1, \ldots, N_p$) is a square matrix of order $(N_p + 1)$ and $\chi_{0t}$ is a vector of length $(N_p + 1)$. The $\varepsilon_{ij}$ can be calculated by:

$$\varepsilon_{ij} = \frac{h}{h - d} g_{0j}s_{s_i} + 2 \sum_{n=1}^{N_p} G_{0n}L_{nsi}L_{nsj},$$

where $\delta_{s_i s_j}$ is the Kronecker delta function and,

$$g_{0j} = \frac{s_j r k_0(s_j r)}{k_0(s_j r)}$$

$$g_{00} = \frac{s_0 r H_0^{(1)}(s_0 r)}{H_0^{(1)}(s_0 r)}$$

$$G_{0n} = -\frac{k_n r I_0(k_n r)}{L_0(k_n r)}$$

where $K_0$ is modified Bessel function of the second kind of order zero and $H_0^{(1)}$ is the Hankel function of first kind of order zero [7].

The $\chi_{0t}$ can be calculated by [7]:

$$\chi_{0t} = -\frac{r^2}{2(h - d)^2} L_{0s} + \sum_{n=1}^{N_p} \frac{2(-1)^n}{n^2 \pi^2} G_{0n}L_{ns},$$

where $s_0$ and $s_j$ are calculated from:

$$\omega^2 = g s_0 \tanh(s_0 h),$$

and

$$\omega^2 = -g s_j \tan(s_j h),$$

where $j = 1, 2, \ldots,$