Two-Port Oscillators Based on Three Impedance Structure

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***Abstract*—This paper investigates the general analysis of the three impedance common B oscillators based on two port network. The concept is applied for 12 different impedance structures to obtain a second order oscillator where the condition and the frequency of oscillation are studied for each case. Then three special cases of two-port networks whose transmission matrices contain two non-zero elements are studied which represent MOS, BJT and gyrator circuits where six cases only can be adapted to have oscillation using gyrators. The effect on non-idealities of the current conveyor used to build gyrator on the condition and the frequency of oscillation is also studied. Finally three different cases are validated using the circuit simulations which match the theoretical study.**

***Keywords*—two port network; oscillators; gyrator.**

I. INTRODUCTION

The two-port network concept is old valuable technique in circuit theory [1]. Although it is not commonly used; it is worthy to use it in representation of any device as input and output port regardless of the nature of the device itself. The concept helps in deriving mathematical models and theories first then select the device with the suitable transmission matrix as this paper introduces. The two port network structure is shown in Fig.1 from which the transmission matrix is described by the following equation.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad (1)$$

Where V_1 and I_1 are the input port voltage and current while V_2 and I_2 are the output port voltage and current respectively. The concept of two port networks is unique as it allows making the theory first then selecting any active device; so it conquers limitations and opens lots of possibilities. The concept can be employed in any circuit structure such as amplifiers, filters and oscillators [2-6]. Two-port network transmission parameters were used in [2] to derive exact expressions for the voltage/current gains and the input/output impedances of common amplifier topologies. The derived expressions are valid both for BJT and MOS-based amplifiers. With the use of two-port network transmission parameters; the general characteristic equation of a cross-coupled circuit topology was derived in [3] which involves two active devices and four or six impedances to demonstrate non-balanced non-matched cross coupled oscillators.

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Another oscillator circuit was presented in [4], where the general characteristic equations for two classes of the two-stage Colpitts oscillator were derived as a function of the two-port network transmission parameters. All different valid possibilities that yield a second-order oscillator with a maximum of three capacitors, or two capacitors and one inductor were found. The general characteristic equations for a two port network with three and two impedances were derived in [5]. It classifies the oscillators into three categories common A, B and C according to which terminal is grounded [6].

Oscillators are widely used in many applications for examples, instrumentation, measurement systems and wireless communication systems. Several oscillators have been introduced in the literature using different active elements such as transistors [7], op-amps [8], current conveyors [9] and current feedback operational amplifiers [10-11].

In this paper, a general study for three impedance common B oscillators based on the employment of two port network concept is presented. The three impedances are considered to be two storage elements (capacitors or inductors or mixed) and one resistor to have a second-order oscillator. An investigation of three different two-port network devices whose transmission matrices contain two elements only is presented. These devices are MOS, BJT and gyrator circuits where only six cases can achieve oscillations using gyrators and no cases with the use of MOS or BJT. The oscillation frequency and conditions are studied for all the valid cases. The gyrator implementation used in this paper based on current conveyor (CCII) which led to the presented study of the non-idealities effect of the CCII on the derived oscillation frequency and condition for the valid cases. To validate the theoretical study, three different examples based on gyrator implementation are simulated using spice.

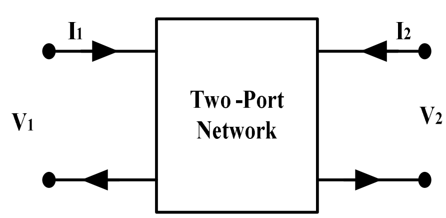


Fig. 1. Two port network

This paper is organized as follows; section II presents the general configuration of the common-B two-port network with all possible cases to have second order oscillator. Section III discusses three practical examples; section IV discusses the non-idealities effect on the behavior of the oscillation parameters, and section V illustrates the simulation results for different practical cases and finally the conclusion.

II. COMMON B TOPOLOGY

The general structure of the common B topology based on two port network with three impedances [5] is shown in Fig.2. As deduced in [5]; the general characteristic equation can be written as:

$$\mathrm{a}_{11}\mathrm{Z}_2(\mathrm{Z}_3+\mathrm{Z}_1)+\mathrm{a}_{12}(\mathrm{Z}_1+\mathrm{Z}_2+\mathrm{Z}_3)+\mathrm{a}_{21}\mathrm{Z}_1\mathrm{Z}_2\mathrm{Z}_3 +\mathrm{a}_{22}\mathrm{Z}_1(\mathrm{Z}_2+\mathrm{Z}_3)-(1+\mathrm{A})\mathrm{Z}_1\mathrm{Z}_2=0 \qquad (2)$$

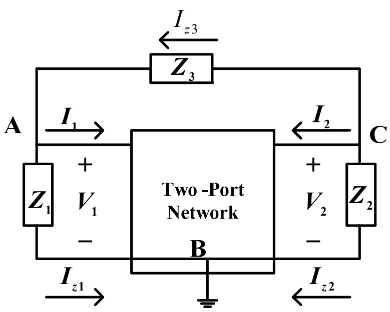


Fig. 2. Common- B topolog

TABLE I. ALL POSSIBLE IMPEDANCE COMBINATIONS

#	Z_1	Z_2	Z_3	OSCILLATION CONDITION	OSCILLATION FREQUENCY ω_o
1	$\frac{1}{sC_1}$	R	$\frac{1}{sC_2}$	$(a_{12}+a_{11}R) = \frac{-(a_{22}-(1+A))(C_2R)}{(C_1+C_2)}$	$\sqrt{\frac{R\,a_{21}+a_{22}}{C_1C_2Ra_{12}}}$
2	$\frac{1}{sC_1}$	sL	R	$a_{12}RC_1 = L(-a_{11}-a_{22}-a_{21}R + (1+A))$	$\sqrt{\frac{Ra_{22}+a_{12}}{C_1L(a_{12}+a_{11}R)}}$
3	$\frac{1}{sC_1}$	R	sL	$a_{12}RC_1 = -L(a_{22}+a_{21}R)$	$\sqrt{\frac{(a_{22}+a_{11})R+a_{12}-R(1+A)}{C_1L(a_{12}+a_{11}R)}}$
4	R	$\frac{1}{sC_1}$	$\frac{1}{sC_2}$	$(a_{12}+a_{22}R) = \frac{-a_{11}(C_2R)+C_2R(1+A)}{(C_1+C_2)}$	$\sqrt{\frac{a_{11}+R\,a_{21}}{C_1C_2Ra_{12}}}$
5	R	$\frac{1}{sC_1}$	sL	$a_{12}RC_1 = -L(a_{11}+a_{21}R)$	$\sqrt{\frac{(a_{22}+a_{11})R+a_{12}-R(1+A)}{C_1L(a_{12}+a_{22}R)}}$
6	R	sL	$\frac{1}{sC_1}$	$a_{12}RC_1 = -L(a_{11}+a_{21}R)$	$\sqrt{\frac{Ra_{22}+a_{12}}{C_1L(a_{12}+(a_{22}+a_{11})R-R(1+A))}}$
7	sL	$\frac{1}{sC_1}$	R	$a_{12}RC_1 = L(1+A-(a_{11}+a_{22} + a_{21}R))$	$\sqrt{\frac{a_{12}+a_{11}R}{C_1L(a_{12}+a_{22}R)}}$
8	sL	R	$\frac{1}{sC_1}$	$a_{12}RC_1 = -L(a_{22}+a_{21}R)$	$\sqrt{\frac{(a_{12}+a_{11}R)}{C_1L(a_{12}+(a_{22}+a_{11})R-R(1+A)}}$
9	R	sL_1	sL_2	$(a_{12}+a_{22}R) = \frac{L_1R(1+A-a_{11})}{(L_1+L_2)}$	$\sqrt{\frac{Ra_{12}}{L_1L_2(a_{11}+a_{21}R)}}$
10	sL_1	R	sL_2	$(a_{12}+a_{11}R) = \frac{L_1R(1+A-a_{22})}{(L_1+L_2)}$	$\sqrt{\frac{Ra_{12}}{L_1L_2(a_{22}+a_{21}R)}}$
11	sL_1	sL_2	R	$a_{12} = \frac{-a_{11}L_2R-a_{22}RL_1}{(L_1+L_2)}$	$\sqrt{\frac{Ra_{12}}{L_1L_2(a_{11}+a_{22}+a_{21}R-(1+A))}}$
12	$\frac{1}{\mathrm{sC}_1}$	$\frac{1}{\mathrm{sC}_2}$	R	$a_{12} = \frac{-a_{11}C_1R-a_{22}C_2R}{(C_1+C_2)}$	$\sqrt{\frac{Ra_{21}+a_{22}+a_{11}-(1+A)}{C_1C_2Ra_{12}}}$

Where $\mathrm{A}=\mathrm{a}_{11}\mathrm{a}_{22}-\mathrm{a}_{12}\mathrm{a}_{21}$. In common B the characteristic equation depends on the four elements of the transmission matrix. By the substitution in the previous characteristic equations with the three impedances; all possible combinations (12 cases) for the three impedances are shown in Table I with the general oscillation frequency and condition. They are deduced in terms of a transmission matrix parameter of an active element regardless of the active element itself. It could be transistor, current conveyor, current feedback operational amplifier, op-amp, gyrator or any other active device.

III. MOS, BJT AND GYRATOR PRACTICAL EXAMPLES

In this section, the transmission matrices for some practical two port networks are studied as special cases to create 2nd order oscillators. These three networks have only two transmission parameters of the four parameters.

A. MOS and BJT

The transmission matrices of MOS and BJT as derived in [5-6] using small signal model with KCL and KVL analysis are shown in (3)

$$\begin{bmatrix}\mathrm{a}_{11} & \mathrm{a}_{12}\\ \mathrm{a}_{21} & \mathrm{a}_{22}\end{bmatrix}=\begin{bmatrix}\frac{-1}{\mathrm{g_m r_{ds}}} & -\left(\mathrm{r_s}+\frac{1}{\mathrm{g_m}}\right)\\ 0 & 0\end{bmatrix} \qquad (3a)$$

$$\begin{bmatrix}\mathrm{a}_{11} & \mathrm{a}_{12}\\ \mathrm{a}_{21} & \mathrm{a}_{22}\end{bmatrix}=\begin{bmatrix}0 & -\left(\mathrm{r_e}+\frac{1}{\mathrm{g_m}}\right)\\ 0 & -\frac{1}{\beta}\end{bmatrix} \qquad (3b)$$

By substitution with these two matrices in the general characteristic equation of common B; equations (4a) and (4b) represent the general characteristic equation using MOS and BJT respectively.

$$\frac{-\mathrm{Z}_2(\mathrm{Z}_3+\mathrm{Z}_1)}{\mathrm{g_m r_{ds}}}-\left(\mathrm{r_s}+\frac{1}{\mathrm{g_m}}\right)(\mathrm{Z}_3+\mathrm{Z}_1+\mathrm{Z}_2)-\mathrm{Z}_1\mathrm{Z}_2=0 \quad (4a)$$

$$-\left(\mathrm{r_e}+\frac{1}{\mathrm{g_m}}\right)(\mathrm{Z}_3+\mathrm{Z}_1+\mathrm{Z}_2)\frac{-\mathrm{Z}_1(\mathrm{Z}_2+\mathrm{Z}_3)}{\beta}-\mathrm{Z}_1\mathrm{Z}_2 =0 \qquad (4b)$$

It is clear from (4) that the condition of oscillation will always need negative component to be achieved regardless of the impedance connection; hence 2nd order oscillators cannot be obtained from common B using three impedances with MOS and BJT transistors as a two port network.

B. Gyrator circuit

A gyrator is a two-port nonreciprocal device with the property that the input impedance is proportional to the reciprocal of the load impedance. It was first introduced in [12]. Large number of papers has been written concerning the implementation and application of the gyrator. Gyrators have been realized utilizing vacuum tubes, transistors, operational amplifiers and current conveyors to obtain their nonreciprocal property [13-19]. The transmission matrix of a non-ideal gyrator circuit shown in Fig.3 can be written as follows

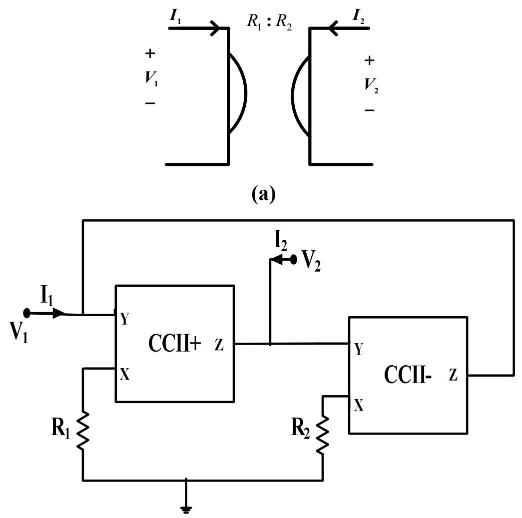


Fig.3. Gyrator (a) symbol and (b) implementation using CCII

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & R_1 \\ \frac{1}{R_2} & 0 \end{bmatrix} = T \tag{5}$$

By substituting in Table I with (5); common B can have 2nd order oscillations using gyrator described by the T matrix as an active element as shown in Table II where six cases out of the twelve cases of common B can realize second order oscillations; the condition and the frequency of oscillation for each case can be found in Table II with the use of gyrator with T matrix as an active element. It is clear from this table that some cases have similarities such as the ones between cases 1, 2 and 9 and cases 4, 7, and 10 respectively. These cases have exactly the same condition and frequency of oscillation. For cases 11 and 12; it is not practical to achieve the condition of oscillations using the gyrator structure. The other cases may oscillate if negative resistance is used.

IV. NUMERICAL DISCUSSION

The circuit shown in Fig.3b represents gyrator implementation which was introduced in [19]. The non- ideal gyrator can be achieved by having different two grounded resistances; three cases were chosen to be discussed and simulated which are cases 1, 2 and 9 of common B topology with the use of the gyrator as the two port network as shown in Fig. 3b.The characteristic equation of the cases 1, 2, and 9 are shown in Table III which are similar to cases 4, 7, and 10 respectively. Fig.4 shows the oscillation conditions C_1/C_2, C_1RR_1/L, and L_2/L_1 for the above cases as function of the parameters R_1/R_2 and R/R_1 based on Table II where there is a region where oscillation can occur and other is not possible. Taking into consideration the non-idealities of CCII the new transmission matrix can be written as

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & \frac{R_1}{\alpha\beta} \\ \frac{\alpha\beta}{R_2} & 0 \end{bmatrix} = T \tag{6}$$

TABLE II. IMPEDANCE CASES USING GYRATOR

CASE #	OSCILLATION CONDITION	OSCILLATION FREQ. ω_0	
1	$\frac{R_1}{R} = \frac{C_2(1-\frac{R_1}{R_2})}{(C_1+C_2)}$	$\sqrt{\frac{1}{C_1C_2R_1R_2}}$	√
2	$R_1RC_1 = L(1-\frac{R}{R_2}-\frac{R_1}{R_2})$	$\sqrt{\frac{1}{C_1L}}$	√
3	$R_1RC_1 = -L\frac{R}{R_2}$	$\sqrt{\frac{\left(R_1-R\left(1-\frac{R_1}{R_2}\right)\right)}{C_1LR_1}}$	×
4	$\frac{R_1}{R} = \frac{C_2(1-\frac{R_1}{R_2})}{(C_1+C_2)}$	$\sqrt{\frac{1}{C_1C_2R_1R_2}}$	√
5	$R_1RC_1 = -L\frac{R}{R_2}$	$\sqrt{\frac{\left(R_1-R\left(1-\frac{R_1}{R_2}\right)\right)}{C_1LR_1}}$	×
6	$R_1RC_1 = -L\frac{R}{R_2}$	$\sqrt{\frac{R_1}{C_1L\left(R_1-R\left(1-\frac{R_1}{R_2}\right)\right)}}$	×
7	$R_1RC_1 = L(1-\frac{R_1}{R_2}-\frac{R}{R_2})$	$\sqrt{\frac{1}{C_1L}}$	√
8	$R_1RC_1 = -L\frac{R}{R_2}$	$\sqrt{\frac{R_1}{C_1L\left(R_1-R\left(1-\frac{R_1}{R_2}\right)\right)}}$	×
9	$\frac{R_1}{R} = \frac{L_1(1-\frac{R_1}{R_2})}{(L_1+L_2)}$	$\sqrt{\frac{R_1R_2}{L_1L_2}}$	√
10	$\frac{R_1}{R} = \frac{L_1(1-\frac{R_1}{R_2})}{(L_1+L_2)}$	$\sqrt{\frac{R_1R_2}{L_1L_2}}$	√
11	$R_1 = 0$	$\sqrt{\frac{RR_1}{L_1L_2\left(\frac{R}{R_2}-\left(1-\frac{R_1}{R_2}\right)\right)}}$	×
12	$R_1 = 0$	$\sqrt{\frac{\frac{R}{R_2}-\left(1-\frac{R_1}{R_2}\right)}{C_1C_2RR_1}}$	×

Where α and β is tolerance in the conveying current and voltage respectively. Fig.5 shows the oscillation condition C_1/C_2, C_1RR_1/L and L_2/L_1 for the above cases taking the effect of α and β at specific ratios of $R_1/R_2 = 1/4$ and $R/R_1 = 2$. It shows how the oscillation condition responds to the deviation of α and β from one.

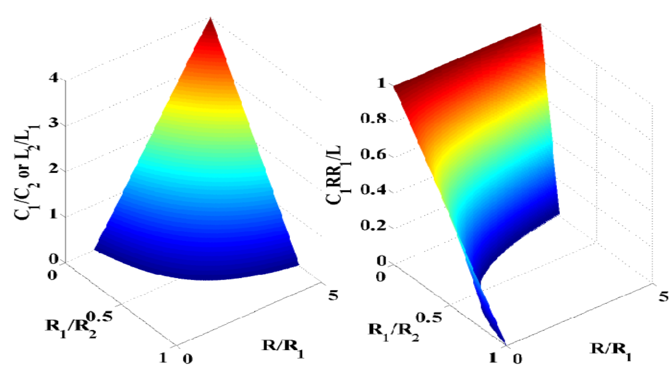
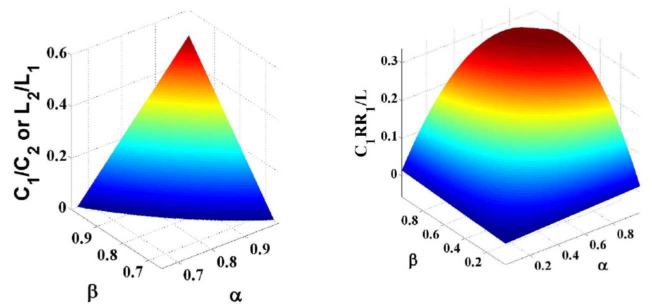


Fig.4.Condition of oscillation versus R_1/R_2 and R/R_1 for (a) case 1, 4, 9, 10 and (b) case 2 and 7.

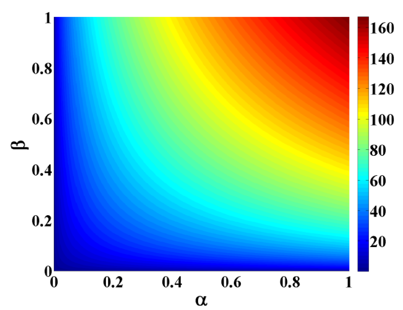
TABLE III. CHARACTERISTIC EQUATIONS OF THE SIMULATED CASES

CASE #	CHARACTERISTIC EQUATION
1	$s^2+\frac{R_1(C_1+C_2)-C_2R(1-\frac{R_1}{R_2})}{C_1C_2RR_1}s+\frac{1}{C_1C_2R_2R_1}=0$
2	$s^2+\frac{R_1RC_1-L(1-\frac{R}{R_2}-\frac{R_1}{R_2})}{C_1LR_1}s+\frac{1}{C_1L}=0$
9	$s^2+\frac{(R_1(L_1+L_2)-L_1R\left(1-\frac{R_1}{R_2}\right))R_2}{R\,L_1L_2}s+\frac{R_1R_2}{L_1L_2}=0$

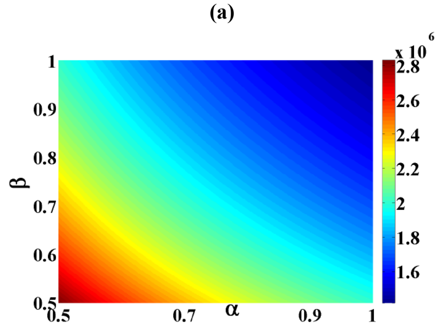


(a) (b)

Fig.5.Condition of oscillation versus α and β for (a) case 1, 4, 9, 10 and (b) case 2 and 7.



(a)



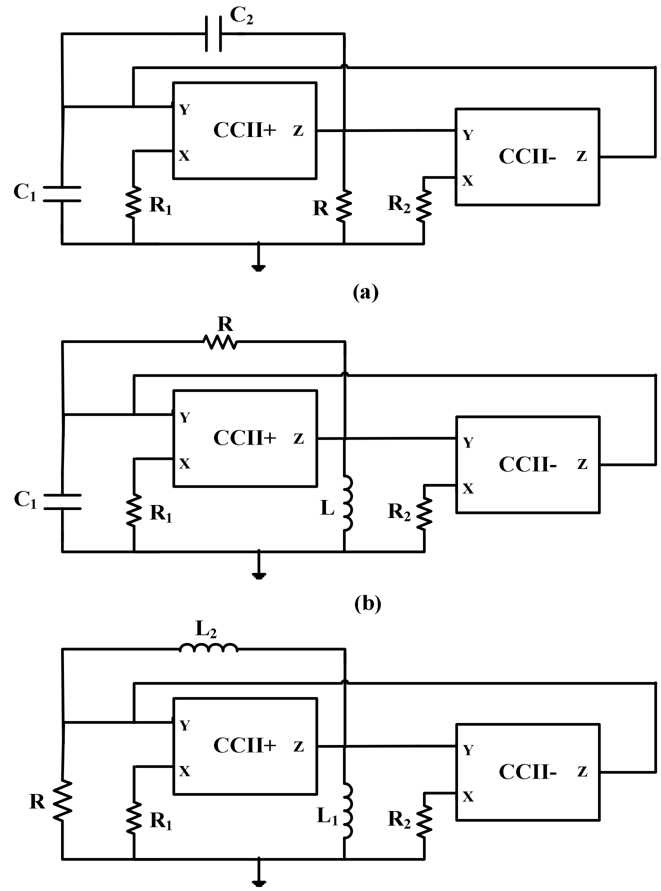
(b)

Fig.6.Frequency of oscillation contour map versus α and β for (a) case 1 and (b) case 9

Fig.6 shows a contour map describes how the frequency of oscillation responses with the non-idealities of CCII α and β for cases 1 and 9 while for case 2 the frequency of oscillations is unaffected with the non-idealities of CCII and it could be concluded from table I.

V. SIMULATION RESULTS

For case 1 (see Fig.7a), the simulation parameters are chosen to be $C_2 = 2C_1 = 2\mu F, R_2 = 4R_1 = 12k\Omega$ and $R = 2R_1 = 6k\Omega$; so $\omega_o = 0.167k$ rad/sec as shown in Fig.8a.



(a) (b) (c)

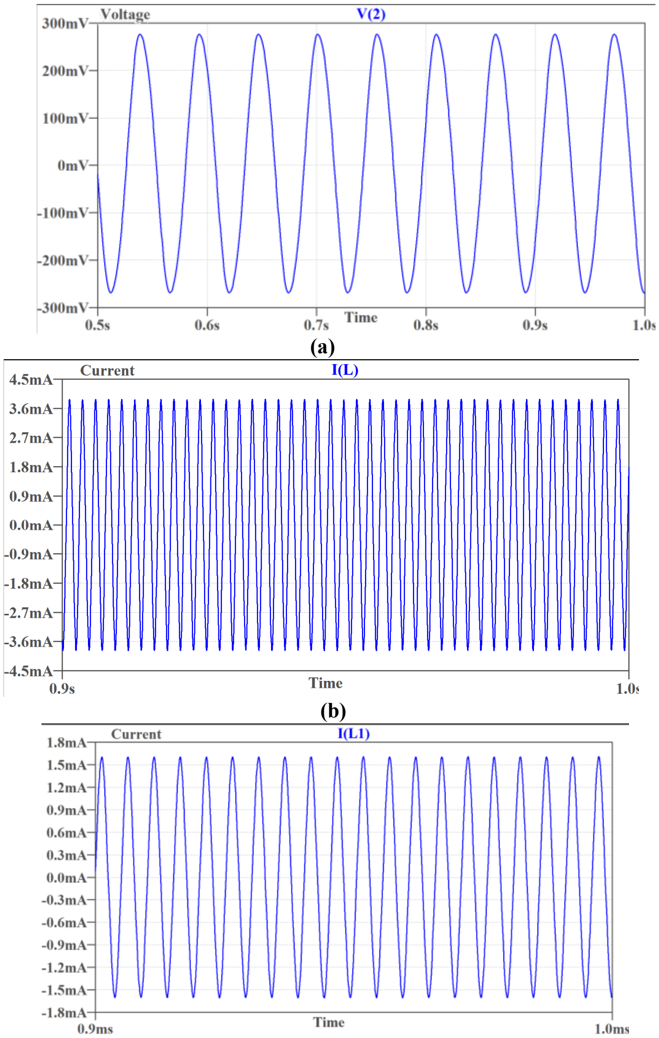
Fig.7.Common B Oscillators (a) case1 (b) case2 (c) case9

For cases 2 (see Fig.7b), the simulation parameters are chosen to be $C_1 = 0.19\mu F, L = 0.615H, R_2 = 4R_1 = 2.616k\Omega$ and $R = 2R_1 = 1.308k\Omega$ so $\omega_o = 3k$ rad/sec as in Fig. 8b. Similarly, the parameters of case 9 (see Fig. 7c) are chosen as $L_1 = 2L_2 = 6mH, R_2 = 4R_1 = 12k\Omega$ and $R = 2R_1 = 6k\Omega$, then the oscillation frequency is $\omega_o = 1.4M$ rad/sec as in Fig.8c. Note that all these simulations were done using spice with the use of commercial model AD844 to simulate CCII+.

VI. CONCLUSION

A study for common B oscillator topology based on the two port network concept with three impedance structure was proposed. It has been analyzed for all possible realizations using different impedance structures. The effect of each structure on the condition and the frequency of oscillation was summarized in different Tables.

Finally applications using MOS, BJT and non-ideal gyrator were proposed to verify the mathematical analysis to obtain a second order oscillation which proved that common B topology can't have a second order oscillator using MOS and BJT. However when the gyrator was used; 2nd order oscillator could be realizable. Six cases out of twelve general cases could be obtained. The symmetry of these six cases was clarified through the paper where three cases of them were discussed and simulated. Non-idealities effects of the CCII used in the gyrator on the condition and the frequency of oscillation were studied. The condition of oscillation was affected by these non-idealities but for the frequency of oscillation only two cases were affected. Simulation results for the three cases were presented to verify the theoretical concept.



(c)

Fig.8. SPICE simulation results for (a) case1 (b) case2, and (c) case 9

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