Fractional Order Four-Phase Oscillator Based on Double Integrator Topology

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***Abstract*— This paper presents a generalization of Soliman's four-phase oscillator into the fractional-order domain. The extra degrees of freedom provided by the fractional-order parameters α and β add more flexibility to the design of the circuit. The design procedure and equations of the proposed oscillator are presented and verified using Matlab and PSPICE. Also, the stability analysis for fractional order systems is studied for different cases of α and β.**

***Keywords*—Caputo; Constant phase element; Fractional calculus; Oscillator;**

I. INTRODUCTION

The concept of fractional order calculus was proposed by l'Hôpital in 1695 when he claimed that following the fractional solution approach will lead to useful consequences in the future [1].

One of the general definitions for the fractional order derivative is the Caputo definition presented in the follwing equation:

$$D_{t_0}^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_{t_o}^{t} f^{(m)}(u)(t-u)^{m-\alpha-1}\, du\,, \qquad (1)$$

where m is an integer that bounds the value of the fractional order α as follows: $(m-1) < \alpha < m$. Figure 1(a) shows the fractional-order derivative of order α for $f(x) = x$. The integer derivative is represented by a constant line equal to 1 is shown at the cure upper left edge.

The Laplace transform for (1) with initial conditions set to zero is:

$$\mathcal{L}\{D_{t_0}^{\alpha} f(t)\} = S^{\alpha}\mathcal{L}\{f(t)\}. \qquad (2)$$

Recently the Fractional order calculus was used in the analysis and design of electronic circuits for different applications. In [2] the authors used the fractional calculus to get the actual value of super-capacitors. Also, in [3] a fractional order filter was designed. In [4], the fractioanl order analysis was used in implemeting a fractional PI controller. In all of the afore-mentioned works, applying fractional order analysis lead to improved results that were not achievable by using conventional calculus.

The concept of fractional element or the so called Constant

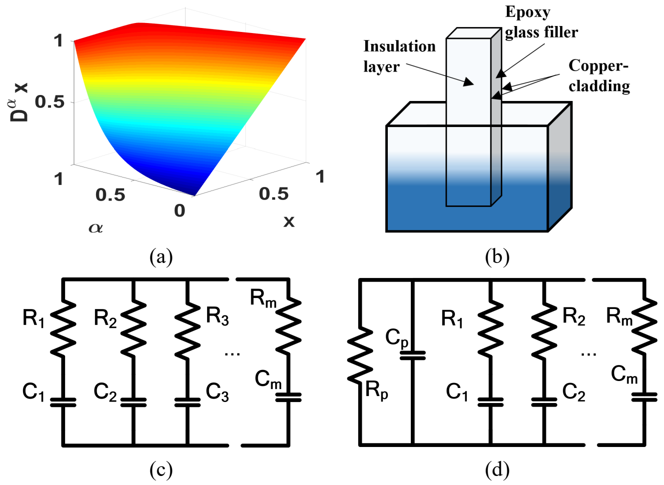

Fig.1: (a) Fractional order derivative of x with order α, (b) Chemical realization of CPE (c) RC ladder realization of fractional order CPE, and (d) Optimized CPE network

Phase Element (CPE) is deduced from (2). Its impedance can be defined by $Z = Qs^{\alpha}$, where Q is the constant value and α is the fractional order, which decides the type of this general element. For standard elements the value of α is limited to the following set: $\{0, 1, -1\}$, which represent resistor, inductor, and capacitor, respectively. Using the fractional calculus, the value of α can be any value between -1 and 1.

The practical realizations of fractional order elements on circuit level made it considered as a perspective alternative for designing electronic circuits. There are many practical realizations of fractional order capacitors proposed in literature. In [5], the authors realized a CPE using chemical approach, shown in Fig.1(b). Furthermore, A design for CPE using field-effect transistors was proposed in [6]. Moreover, an RC ladder structure shown in Fig.1(c), was presented in [7]. Figure 1(d) shows realization of CPE based on optimized RC network [8].

The first fractional order oscillator was proposed in [9], where the authors designed a generalized Wien bridge oscillator. In [10], the authors presented a methodology for designing fractional order sinusoidal oscillators. A complete study of all two-port network fractional order oscillators was presented in [11].

In this work, a generalized analysis for Soliman's four-phase oscillator is presented, where the conventional integer

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order capacitors are replaced by fractional ones. This paper is organized as follows; a generalized analysis for the Soliman's four-phase oscillator is presented in Section II. In Section III, the simulation results of proposed fractional order analysis are presented and verified by PSPICE for different cases of α and β. Finaly, the conclusion is discussed in Section IV.

II. Generalized Fractional Order Four Phase Oscillator

The schematic of Soliman's generalized oscillator is shown in Fig.2. The oscillator is implemented based on double integrator topology. This oscillator, has independent control for both oscillation frequency and the oscillation condition, which offers flexibility in controlling the oscillation condition without affecting the desired oscillation frequency, however this is valid only for the integer order case. The fractional order capacitors were realized in PSPICE based on the procedure presented in [4]. Each stage is responsible for generating a signal and its inverted replica. The phase shift between the two stages outputs in conventional operation is 90° [12–14]. R_1, R_2, C_1 and C_2 control the oscillation frequency (f), as shown in (3).

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{R_1 R_2 C_1 C_2}} \tag{3}$$

The state matrix and the characteristic equation of the presented oscillator are given by:

$$\begin{bmatrix} D^{\alpha} V_1 \\ D^{\beta} V_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C_1 R_1} \\ -\frac{1}{C_2 R_2} & \frac{1}{C_2}\left[\frac{1}{R_4} - \frac{1}{R_3}\right] \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \tag{4a}$$

$$s^{\alpha+\beta} - \frac{1}{C_2}\left[\frac{1}{R_4} - \frac{1}{R_3}\right] s^{\alpha} + \frac{1}{C_1 C_2 R_1 R_2} = 0, \tag{4b}$$

To sustain oscillations, there must be a solution for ω satisfies the following equations [10]:

$$\omega^{\alpha+\beta}\cos\left(\frac{(\alpha+\beta)\pi}{2}\right) - \frac{1}{C_2}\left[\frac{1}{R_4} - \frac{1}{R_3}\right]\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right) + \frac{1}{C_1 C_2 R_1 R_2} = 0, \tag{5a}$$

$$\omega^{\beta}\sin\left(\frac{(\alpha+\beta)\pi}{2}\right) - \frac{1}{C_2}\left[\frac{1}{R_4} - \frac{1}{R_3}\right]\sin\left(\frac{\alpha\pi}{2}\right) = 0. \tag{5b}$$

Selecting R_4 to be the oscillation condition, the outputs waveforms can be written as follows:

$$R_4 = \frac{\frac{1}{C_2}\sin\left(\frac{\alpha\pi}{2}\right)}{\omega^{\beta}\sin\left(\alpha + \frac{\beta\pi}{2}\right) + \frac{1}{C_2 R_3}\cos\left(\frac{\alpha\pi}{2}\right)} \tag{6a}$$

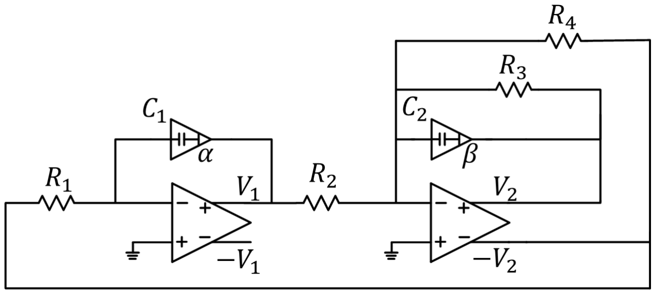


Fig.2: Soliman's four phase oscillator

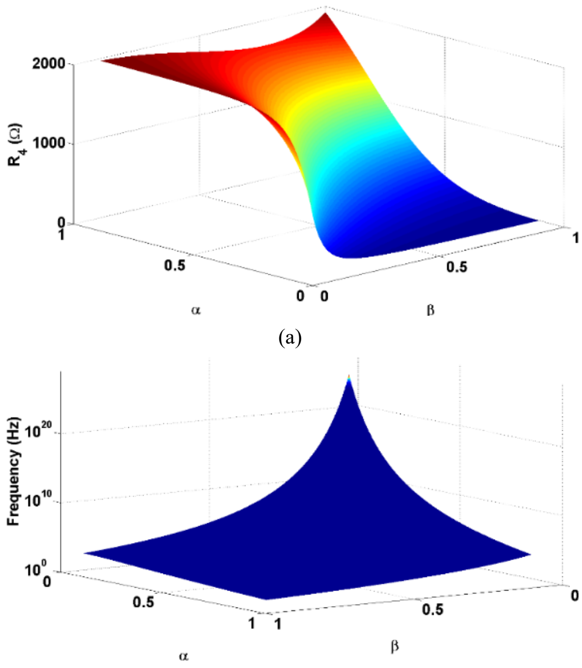
$$R_4 = \frac{\frac{1}{C_2}\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)}{\omega^{\alpha+\beta}\cos\left(\alpha + \frac{\beta\pi}{2}\right) + \frac{\omega^{\alpha}}{C_2 R_3}\cos\left(\frac{\alpha\pi}{2}\right) + \frac{1}{C_1 C_2 R_1 R_2}} \tag{6b}$$

By eliminating R_4 from (6), the general expression for the oscillation frequency of the output waveforms is as follows:

$$f = \frac{1}{2\pi}\left(\frac{\sin\left(\frac{\alpha\pi}{2}\right)}{C_1 C_2 R_1 R_2 \sin\left(\frac{\beta\pi}{2}\right)}\right)^{\frac{1}{\alpha+\beta}} \tag{7}$$

As a validation of the fractional order solution, it should give the conventional response when the orders are set to the integer values. By substituting the values of $\alpha = 1$ and $\beta = 1$ in (7), it will be as same as the relation in (3), which confirms that the integer order solution is just a special case of the generalized solution.

Figures 3(a) and 3(b) show the oscillation condition and frequency surfaces versus $\alpha - \beta$ plane, respectively. Both figu-



(a)

(b)

Fig.3: The effect of values of α and β on (a) R_4 and (b) output frequency.

res were plotted using the following parameters: $R_1 = R_2 = R_3 = 2\text{k}\Omega$ and $C_1 = C_2 = 1.2\text{x}10^{-6}$, while the required values of R_4, and the corresponding frequency are calculated for different values of both α and β from 0.1 to 1. It is obvious that the range of R_4 resistance encounters a small varying range over the full range of α and β. On the other hand, the output frequency shows an interesting response over the range of α and β, where the frequency increases dramatically as the value of the fractional order element reaches zero, and it is at the lowest value is near the conventional integer order case at $\alpha = \beta = 1$. This represent the importance of analyzing circuits in generalized fractional order solution.

The system response for different values of R_1 is presented in Fig.4, using the same values of the other components as in Fig.3. From the figure, it is obvious that the value of the output frequency is inversely proportional to the value of R_4 for all the values of α. It is shown that the output frequency is at its maximum value at small values of α, while using a very small resistance for R_4. Referring to Figs.(3-5), it is clear that the fractional order of the system has the dominant control over the oscillation frequency.

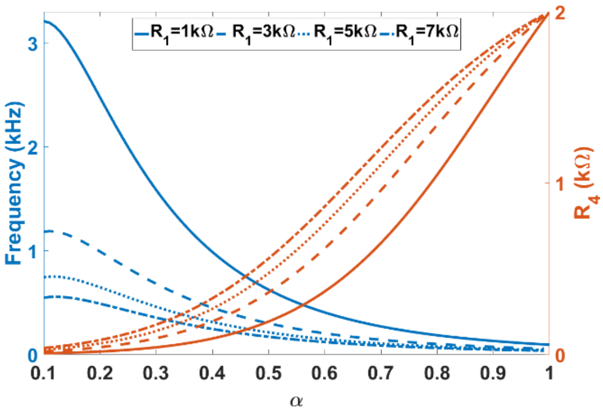


Fig.4: The effect of R_1 on R_4 for different values of α

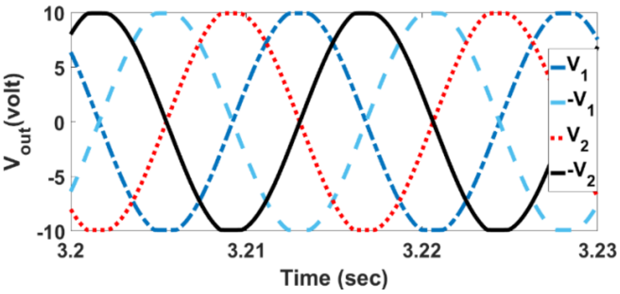
III. Simulation Results

In this section, the SPICE simulation of generalized Soliman's oscillator is presented. The Op-Amp used in this design is THS-4531 and biased by $V_{ss} = 10\text{V}$ and $V_{cc} = -10\text{V}$. The simulation result of the conventional integer order case is shown in Fig.5(a), for $R_1 = R_2 = R_3 = R_4 = 2\text{K}\Omega$, and $C_1 = C_2 = 1.2 \times 10^{-6}$. The output frequency is 66.3 Hz. The oscillation frequency for this case can be calculated using (3) or (7) while setting the order α and β to 1.

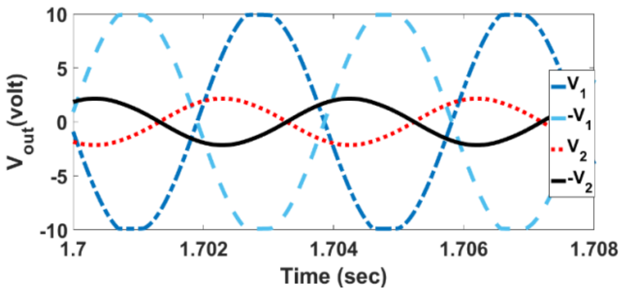
The design parameters of different values of α and β are presented in Table.1. The values of R_1, R_2 and R_3 are set to 2KΩ. The phases are uniformly spaced, and the amplitudes are equal only in the case of integer order, shown in Fig.5(a). For the fractional order cases shown in Figs. 5(b-d), the phases are no more uniformly spaced and the amplitudes are different among the integrator stages. The stage with the lower fractional order has rail-to-rail output signal, and the vice versa.

Table 1: Design Parameters for Fractional Order Oscillator

α	β	$R_4(\Omega)$	Frequency(Hz)
1.0	1.0	2000	66.3
0.6	1.0	516	262
0.6	0.8	633	3699
0.8	0.6	1443	987



(a)



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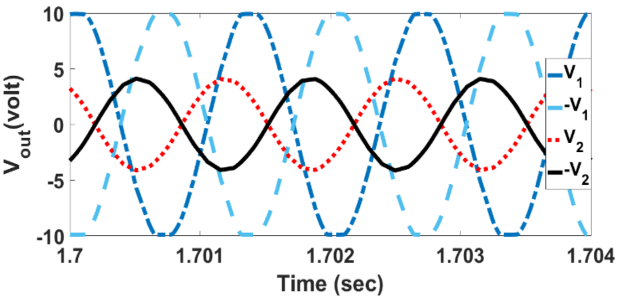

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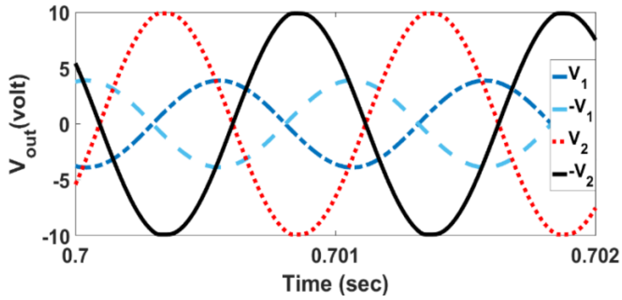

(d)

Fig.5: Output of generalized fractional order oscillator for, (a) $\alpha = 1$ and $\beta = 1$, (b) $\alpha = 0.6$ and $\beta = 1$, (c) $\alpha = 0.6$ and $\beta = 0.8$ and (d) $\alpha = 0.8$ and $\beta = 0.6$

The stability analysis is vital when designing an oscillatory system. To sustain the stable oscillation the roots of the characteristic equation should fall exactly on the imaginary axis of the S-plane. In fractional order systems, the solution is different from the case of the integer order solution, in which some the physical range of the solution is limited to specific area decided by the order and the number of the fractional order elements.

A study for the stability of fractional order systems was presented in [15], where the authors applied the stability analysis for different cases of α and β and proposed a new plane called W-plane. The W-plane is a translation for the conventional S-plane, which decides the limits of acceptable range of the transfer function roots

To check the stability of the designed oscillator the roots are plotted in W-plane as shown in Fig.6, where the red line presents the feasible area of the S-plane when a fractional order system is analyzed[15]. It is obvious that area of the feasible range is changed according to the value of α and β. For the integer case, when $\alpha = \beta = 1$, the roots falls on the imaginary axis.

IV. Conclusion

Analyzing the system based on the fractional order calculus leads to having a better understanding of the system behavior. The extra degrees of freedom added by generalizing the conventional integer orders to α and β, which in turn offers a powerful control over the system response and design flexibility. While the conventional methodologies solve only particular case, the generalized fractional order solution offers a comprehensive analysis for the system response.

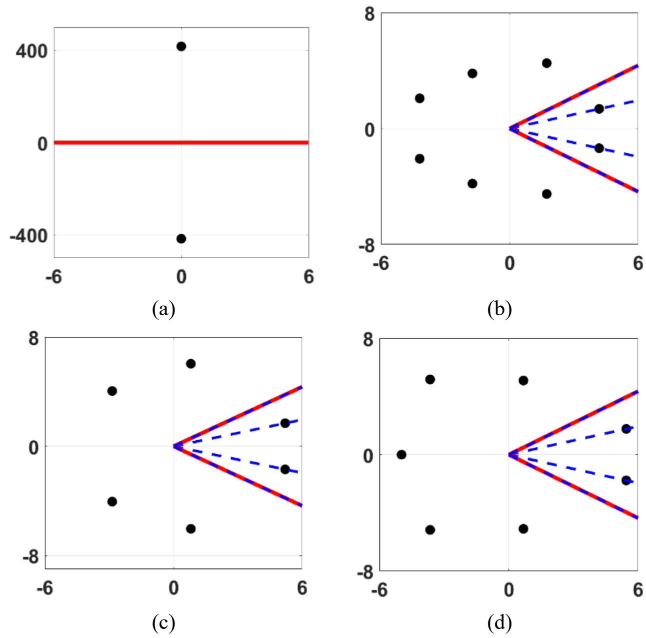

Fig.6: Stability regions of system defined in (6) for different orders, (a) $\alpha = 1$ and $\beta = 1$, (b) $\alpha = 0.6$ and $\beta = 1$, (c) $\alpha = 0.6$ and $\beta = 0.8$, and (d) $\alpha = 0.8$ and $\beta = 0.6$

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