

NULLATOR-NORATOR MODELS OF CHUA'S SECOND TYPE L-C MUTATOR

Ahmed M. Soliman
Assistant Professor of Engineering
The College of Steubenville

Abstract - Equivalent realizations of type 2 L-C mutator using nullators and norators are given here. Two different approaches are used. The first is based on similar ideas to that used by Daniels in the realization of fundamental gyrators. The second is based on the factorization of the transfer matrix of the mutator.

INTRODUCTION

Two types of L-C mutators were defined by Chua [1]. The equivalent realizations of type 1 L-C mutator (gyrator) using nullators and norators was given by Mitra [2] and Daniels [3,4]. The present paper gives equivalent realizations of type 2 L-C mutator using the singular elements.

DEFINITIONS

1) Singular elements:

a) The nullator is a bilateral and lossless one port network defined by

$$V = I = 0.$$

b) The norator is a one port network which passes an arbitrary current and sustains an arbitrary independent voltage [5]. The symbols of the nullator and norator are given in Figure 1(a) and (b) respectively.

The equivalent circuit [6] of an ideal transistor ($h_{11}=h_{22}=h_{12}=0, h_{21}=-1$)

using these pathological elements is given in Figure 1(c).

2) Type 2 L-C mutator, is characterized in the time domain by,

$$\begin{aligned} v_1 &= \pm T_1 \frac{dv_2}{dt} \\ i_1 &= \mp \frac{1}{T_2} \int i_2 dt \end{aligned} \quad (1)$$

The transmission matrix of type 2 L-C mutator is given by:

$$T_{LC_2}(s) = \begin{bmatrix} \pm T_1 s & 0 \\ 0 & \mp \frac{1}{T_2 s} \end{bmatrix} \quad (2)$$

It can easily be seen when port 2 of the mutator is terminated in a capacitance C , the impedance seen at port 1 will be inductive, and the inductance is given by $\frac{T_1 T_2}{C}$.

Considering equation (2) with both signs positive, the transmission matrix can be factored to:

$$T_{LC_2}(s) = \begin{bmatrix} T_1 s & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{T_2 s} \end{bmatrix} \quad (3-a)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{T_2 s} \end{bmatrix} \begin{bmatrix} T_1 s & 0 \\ 0 & 1 \end{bmatrix} \quad (3-b)$$

The above equations represent a cascade connection of type 1 L-R mutator and type 1 R-C mutator [1], as shown in Figure 2.

SIMULATION OF THE MUTATOR

Daniels [3] introduced a synthesis procedure and used it in realizing a four port network, known as a fundamental gyrator, which includes type 1 L-C mutator as a special case. The applicability of the above synthesis procedure in realizing type 2 L-C mutator will be given next.

Consider a six port network, satisfying the following conditions,

$$V_1 = + \left(\frac{-V_3}{I_3} \right) \left(\frac{-I_4}{V_4} \right) V_2 \quad (4-a)$$

$$I_1 = - \left(\frac{-V_6}{I_6} \right) \left(\frac{-I_5}{V_5} \right) I_2 \quad (4-b)$$

It is seen that the above equations realizes type 2 L-C mutator by terminating ports 3 and 5 in resistances R_1 and R_2 and ports 4 and 6 in capacitances C_1 and C_2 respectively, and in this case $T_i = C_i R_i$ ($i = 1, 2$). One possible solution of the above equations is:

$$V_1 = V_3, V_5 = V_6, V_2 = V_4 \quad (5-a)$$

$$I_1 = -I_5, I_2 = I_6, I_3 = I_4 \quad (5-b)$$

Nullators will be used first to satisfy the voltage equalities, and then norators will be added to generate the current relations. One possible realization is shown in Figure 3(a). Replacing each nullator-norator pair sharing a common node by the equivalent transistor, one gets a 5 transistor model for the mutator, as shown in Figure 3(b). Other idealized five transistors models are given in Figures 3(c) and (d).

Next, consider the transmission matrix,

$$T(s) = \begin{bmatrix} -\frac{Z_3}{Z_4} & 0 \\ 0 & -\frac{Z_6}{Z_5} \end{bmatrix} \quad (6)$$

By using resistances as Z_3 and Z_5 and capacitances as Z_4 and Z_6 , the above matrix realizes the type 2 L-C mutator. It is possible to factor the transmission matrix into two different ways, leading to two different realizations.

First, writing equation (6) as,

$$T(s) = \begin{bmatrix} -\frac{Z_3}{Z_4} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -\frac{Z_6}{Z_5} \end{bmatrix} \quad (7-a)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -\frac{Z_6}{Z_5} \end{bmatrix} \begin{bmatrix} -\frac{Z_3}{Z_4} & 0 \\ 0 & 1 \end{bmatrix} \quad (7-b)$$

The above equations indicate that the mutator can be realized as a cascade of a VNIC and a CNIC in an arbitrary order. Using Braun's [7] realizations of the NIC, it is seen that there are 18 possible models for the mutator. Figure 4 includes some of the possible models. A four transistors equivalent circuits can be directly obtained from these models, except in Figures 4(a) and (b) one must add two series connections of nullator and norator (equivalent to open circuit), such that all nullators and norators will be in

pairs, each pair shares a common node.

Next, equation (6) can be written as,

$$T(s) = \begin{bmatrix} 0 & +Z_3 \\ -\frac{1}{Z_5} & 0 \end{bmatrix} \begin{bmatrix} 0 & +Z_6 \\ -\frac{1}{Z_4} & 0 \end{bmatrix} \quad (8)$$

Hence the mutator can be realized as a cascade of two negative-impedance inverters. The nullator-norator models shown in Figure 5, are obtainable directly from equation 8. The final realization is in the form of the Carlin-Youla nullor lattice [5,8]. The above realization can also be obtained using Daniels technique [3].

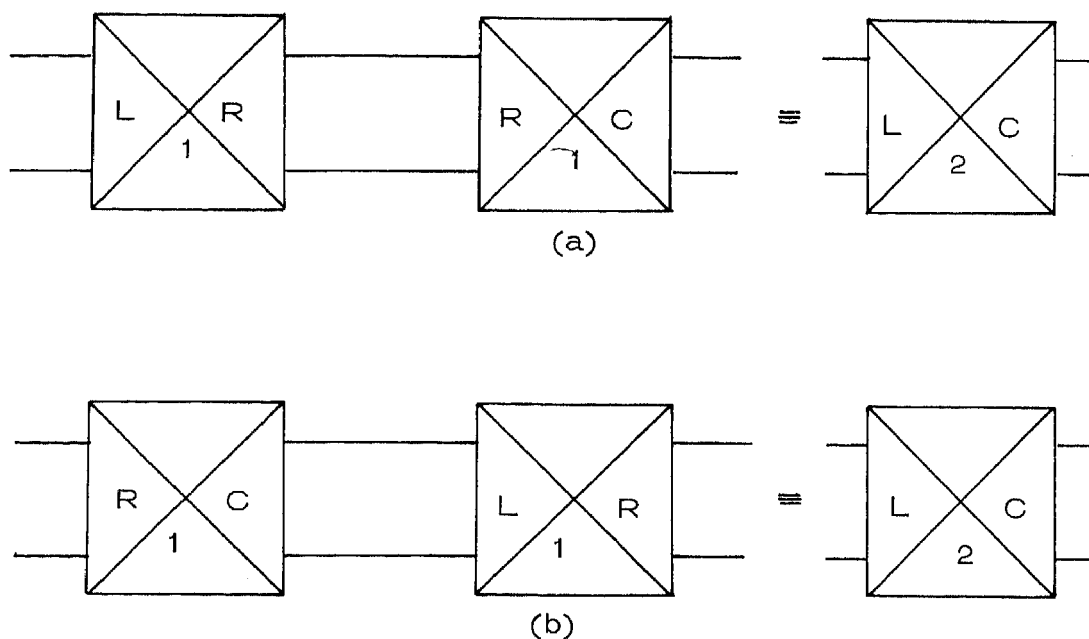
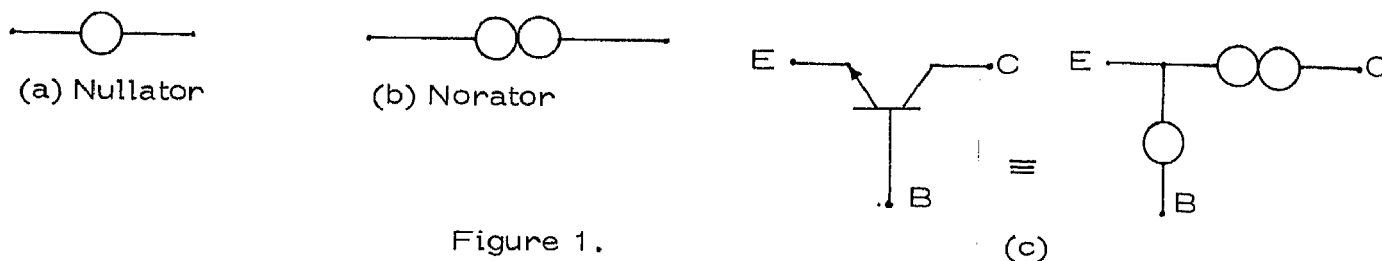
CONCLUSIONS

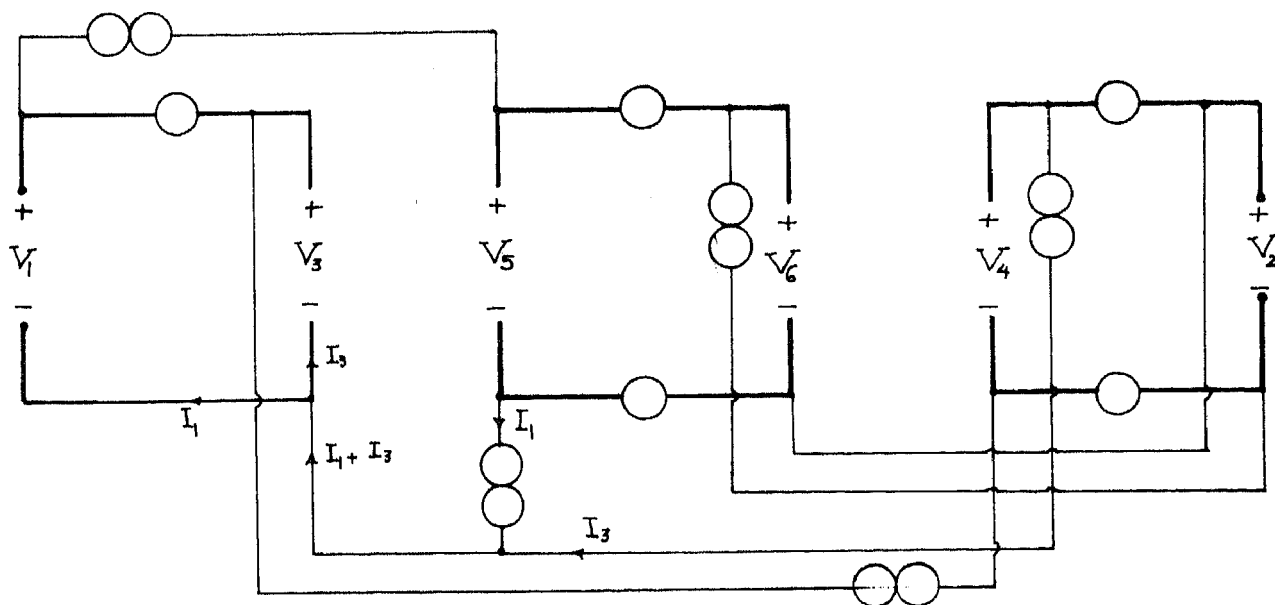
Equivalent realizations of type 2 L-C mutator were given. The realizations shown in Figure 2 includes five transistors. The realizations given in Figure 4 are three terminal networks, requiring four transistors, and consisting of a cascade connection of a VNIC and a CNIC, whereas those in Figure 5 are four terminals and are a cascade of two negative-impedance inverters.

REFERENCES

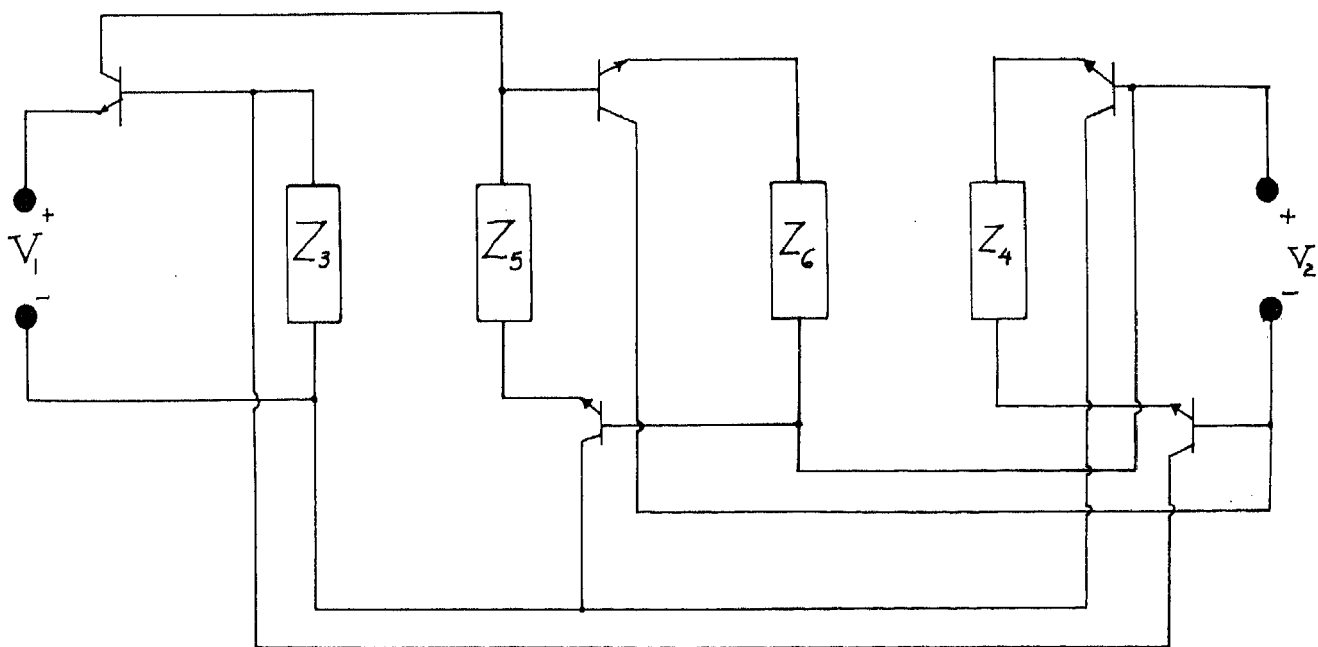
- [1] L. O. Chua, "Synthesis of nonlinear network elements", Proceedings IEEE, Vol. 56, No. 8, August 1968, pp. 1325-1340.
- [2] S. K. Mitra, "Equivalent circuits of gyrators", Electronics Letters, 3, July 1967, pp. 333-334.
- [3] R. W. Daniels, "The synthesis of fundamental gyrators", IEEE Trans. on Circuit Theory, Vol. CT-16, No. 4, November 1969, pp. 543-544.

- [4] R. W. Daniels, "A synthesis procedure for gyrators and related circuits", Ph.D. dissertation, Northeastern University, Boston, Mass., February 1969.
- [5] H. J. Carlin and D. C. Youla, "Network synthesis with negative resistors", Proc. IRE, Vol. 49, May 1961, pp. 907-920.
- [6] B. R. Myers, "Nullor model of the transistor", Proceedings IEEE, Vol. 53, July 1965, pp. 758-759.
- [7] J. Braun, "Equivalent NIC networks with nullators and norators", IEEE Trans. on Circuit Theory, Vol. CT-12 No. 3, September 1965, pp. 441-442.
- [8] L. T. Bruton, "Nonideal performance of a class of positive immittance converters", IEEE Trans. on Circuit Theory, Vol. CT-16, No. 4, November 1969, pp. 572-573.



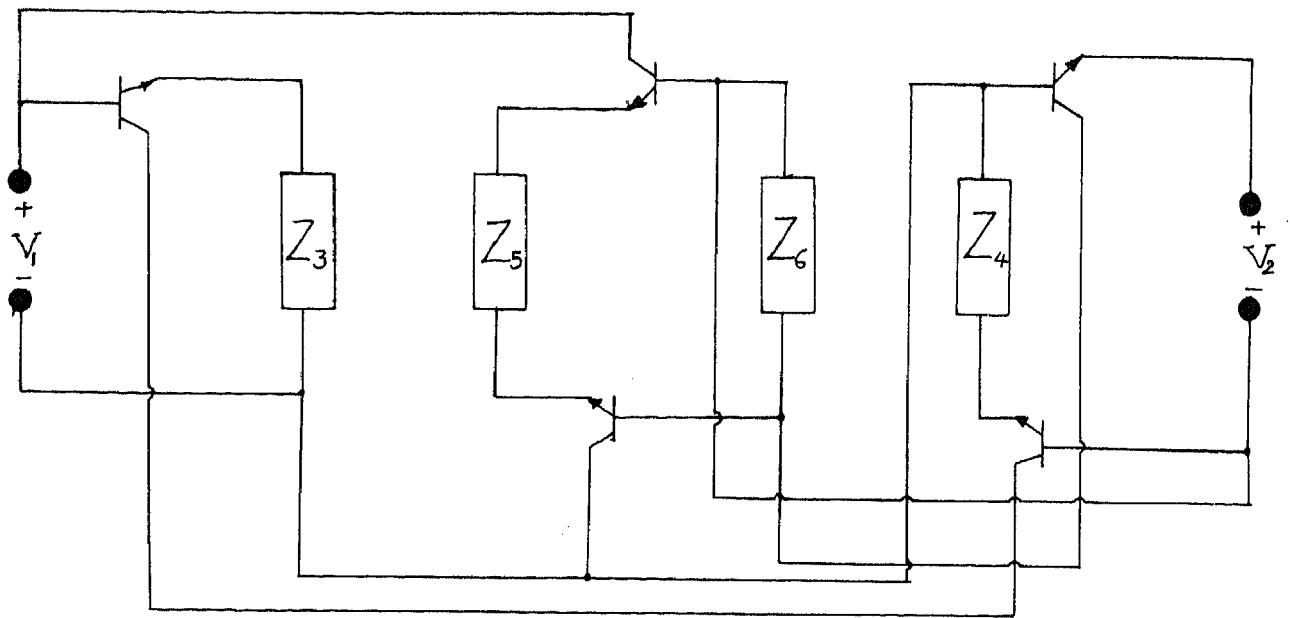


(a)

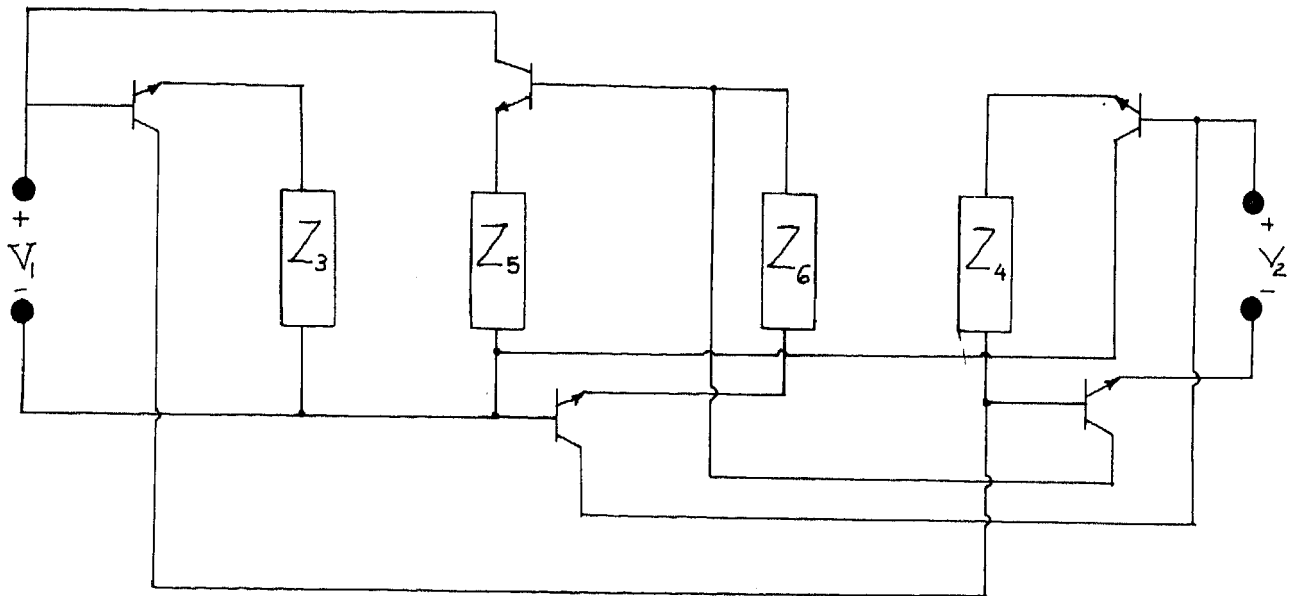


(b)

Figure 3

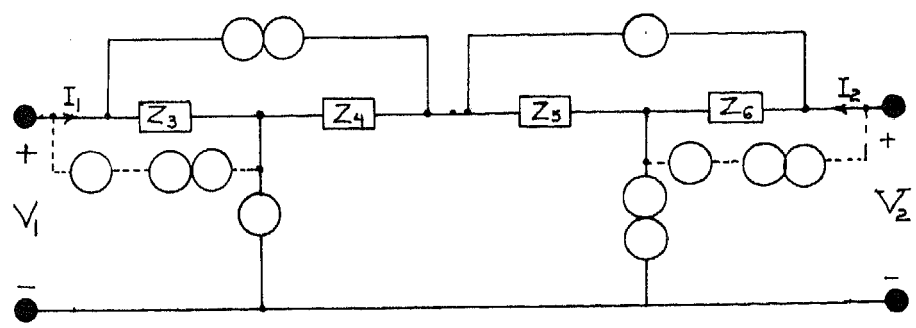


(c)

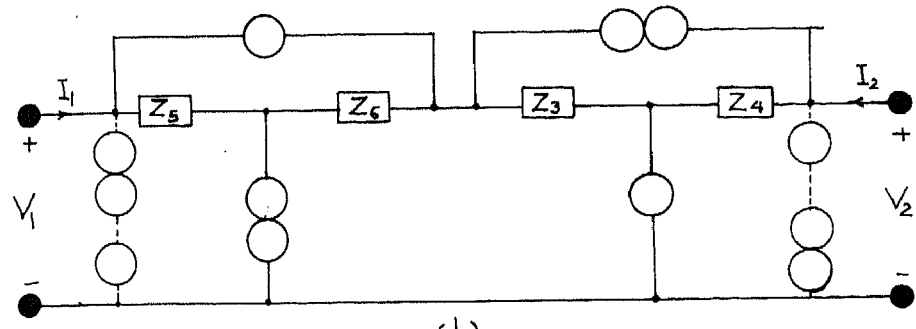


(d)

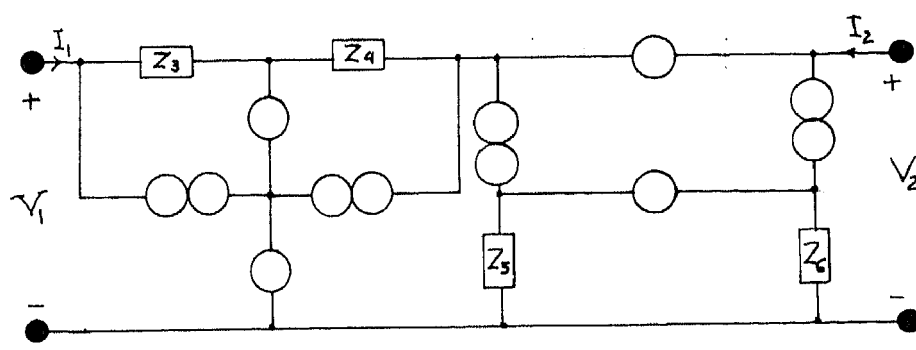
Figure 3 (Cont'd)



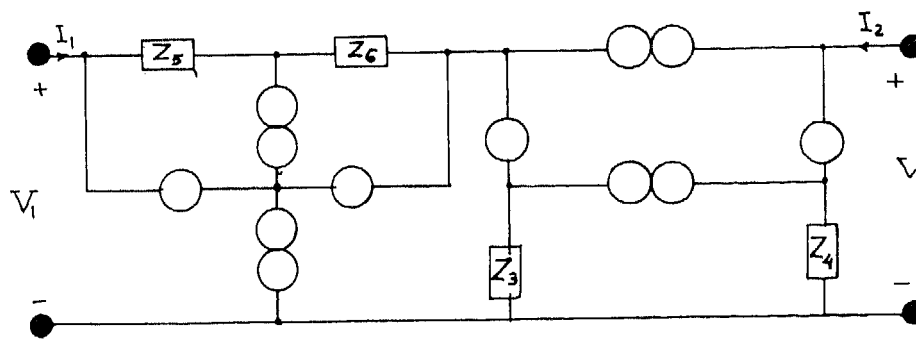
(a)



(b)

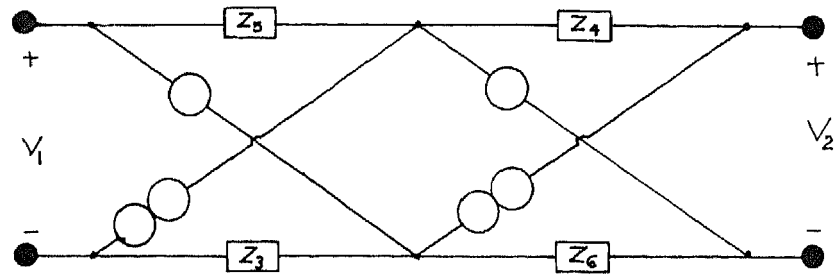


(c)

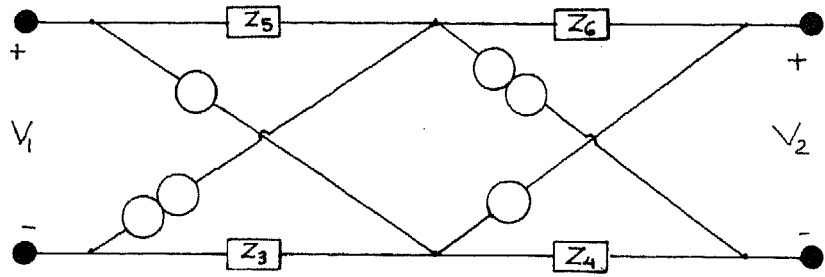


(d)

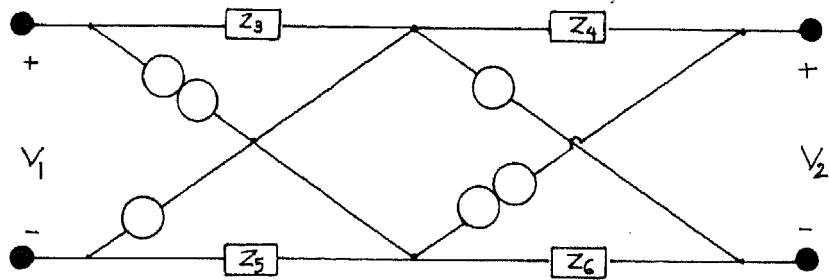
Figure 4.



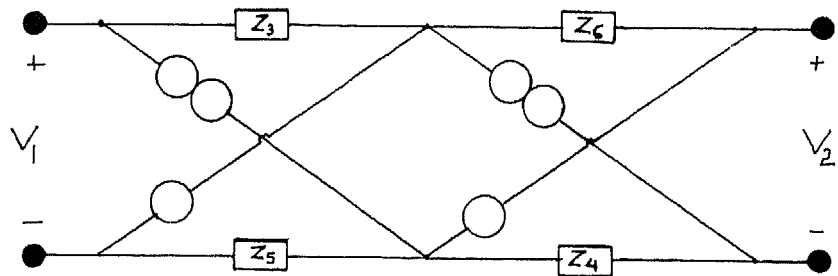
(a)



(b)



(c)



(d)

Figure 5.

Midwest Symposium on Circuits and Systems



3 1858 019 050 636

PROCEEDINGS
OF THE
FIFTEENTH
Midwest Symposium
on
Circuit Theory

MAY 4-5, 1972

Presented by
The Electrical Engineering Department
University of Missouri - Rolla
in Cooperation With
The Circuit Theory Group and the
Control Systems Society
Institute of Electrical and Electronic Engineers

Sponsored by
DEPARTMENT OF ELECTRICAL ENGINEERING
EXTENSION DIVISION
UNIVERSITY OF MISSOURI - ROLLA
ROLLA, MISSOURI 65401