INT. J. ELECTRONICS, 1999, VOL. 86, NO. 12, 1441–1451

CMOS current feedback op amp-based chaos generators using novel active nonlinear voltage controlled resistors with odd symmetrical characteristics

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Novel grounded and floating CMOS active nonlinear resistors with odd symmetrical characteristics are designed. The nonlinear resistors are then incorporated into two chaotic oscillator circuits based on a CMOS current feedback op amp (CFOA). The slopes of both the negative and positive segments of the nonlinear characteristics are voltage controlled, allowing for a wide range of dynamic behaviour to be observed and easily tuned in a period doubling route to chaos. Nonlinear current–voltage characteristics are derived in a piecewise-linear form and shown possibly to be modelled using a cubic polynomial approximation. PSPICE simulations using a standard 2.0 μm technology file and numerical simulations of the derived chaotic mathematical models are included.

1. Introduction

An increasing interest in studying the behaviour of nonlinear electronic circuits has recently developed. Of specific interest is the study of the bounded steady-state behaviour in low-dimensional deterministic dynamical systems, which is not an equilibrium point, not periodic and not quasi-periodic; such a behaviour is termed chaos (Parker and Chua 1989). It has been shown that chaotic signals can be useful in applications such as signal encryption and secure communication systems (Hayes *et al.* 1993, Itoh and Murakami 1995, Pinkney *et al.* 1995, Parlitz *et al.* 1996, Grebogi *et al.* 1997). Such applications usually imply a considerable amount of signal processing, and hence chaotic oscillators that are suitable for VLSI integration are advantageous. Several attempts to introduce digital as well as analogue chaotic CMOS oscillators have been made. Digital CMOS chaos generators are usually based on iterating a form of the logistic equation (Vazquez *et al.* 1985, Restituto *et al.* 1992, Pham *et al.* 1995) while analogue CMOS chaotic oscillators have been based generally on the well-known Chua's circuit (Cruz and Chua 1992, 1993, Rodriguez-Vazquez and Delgado-Restituto 1993). Some realizations of chaotic neurons have also been introduced (Kanou *et al.* 1994). Two basic problems arise when designing a monolithic form of Chua's circuit; the first is to integrate the inductor. In Cruz and Chua (1993) and Rodriguez-Vazquez and Delgado-Restituto (1993), active RC emulation of the inductor using OTAs was used to overcome this problem. The second

Received 2 February 1998. Accepted 23 February 1999.

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International Journal of Electronics ISSN 0020–7217 print/ISSN 1362–3060 online
http://www.tandf.co.uk/JNLS/etn.htm
http://www.taylorandfrancis.com/JNLS/etn.htm

problem arises from the fact that the nonlinear resistor in Chua's circuit is the only active element. Therefore, the design procedure for this nonlinear resistor explained in Kennedy (1992), has been followed in Cruz and Chua (1993) and Rodriguez-Vazquez and Delgado-Restituto (1993). Recently, some RC chaotic oscillators that are not based on inductor emulation and that rely on simple nonlinear resistors were introduced (Namajunas and Tamasevicius 1996, Nakagawa and Saito 1996, Elwakil and Soliman 1997d, 1999a). Those current-mode oscillators that are based on the second generation current conveyor (CCII) (Elwakil and Soliman 1997d, 1999a) are suitable for direct VLSI integration using CMOS CCII configurations, such as the ones reported by Elwan and Soliman (1996, 1997).

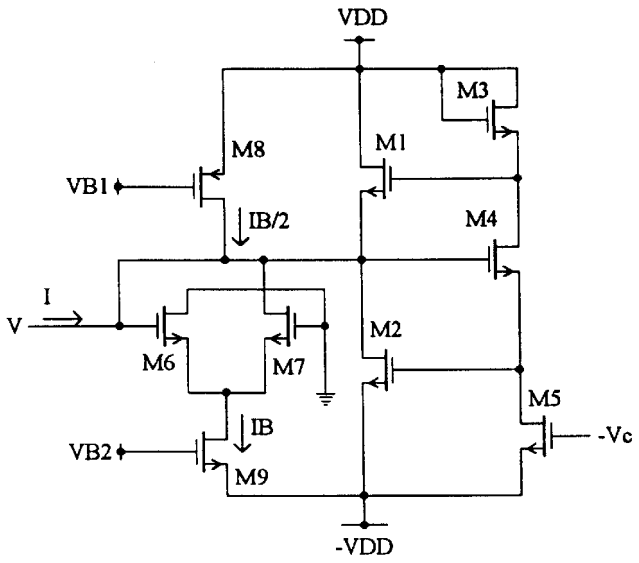
In this work, grounded and floating CMOS nonlinear resistors are designed. Unlike the procedure explained in Kennedy (1992), which requires two parallel op amp-based negative impedance converters with one of the op amps operating in its nonlinear region, the proposed nonlinear resistors are derived from linear voltage-controlled resistors that are modified using the nonlinearity of MOS differential pairs. Such a procedure benefits from the large number of already existing CMOS linear resistors and avoids unnecessary knowledge of nonlinear dynamics. Furthermore, design is done on the transistor level instead of the circuit level which results in a significant reduction in the number of transistors required, as compared with Cruz and Chua (1992). It also allows low-voltage and low-power optimization to be performed. Both resistors have negative slopes around the origin and positive slopes in the outer segments which are voltage controlled. Two chaotic oscillators based on the CMOS current feedback op amp (CFOA) of Mahmoud and Soliman (1998) are then constructed using the proposed nonlinear resistors. Chaotic behaviour is demonstrated with PSPICE simulations using the MOSIS 2.0 μm technology file. Mathematical models describing circuit dynamics are then derived and numerically integrated to confirm the chaotic behaviours.

2. Grounded nonlinear resistor with application to a minimum component chaotic oscillator

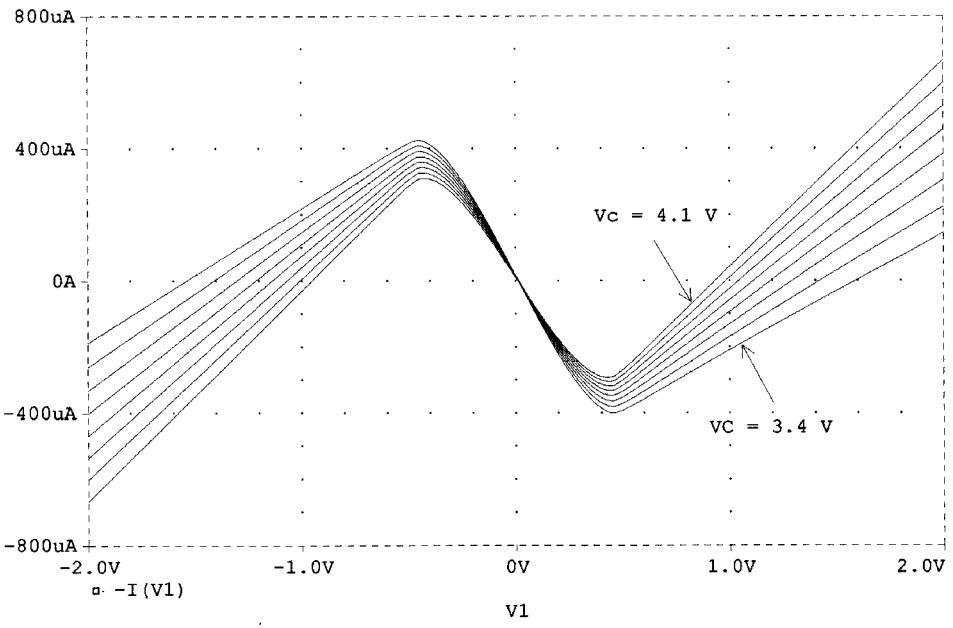
Figure 1(*a*) represents the proposed grounded nonlinear resistor. Two basic parts can be distinguished: a linear voltage controlled NMOS grounded resistor (transistors M1–M5) (Mahmoud *et al.* 1996) and a nonlinear differential pair transconductor (transistors M6 and M7) biased with two current sources (transistors M8 and M9). The I–V characteristics of the nonlinear resistor are shown in figure 1(*b*) for different values of the negative control voltage V_C. The slope of the inner segment can be varied by altering the bias current I_B of the differential pair through the bias voltages V_{B1} and V_{B2}.

The current–voltage characteristics shown are modelled by

$$I = \begin{cases} 2K_a(2V_C - V_{DD})V + \dfrac{I_B}{2} & V \le -\sqrt{\dfrac{2I_B}{K_b}} \\ \left[2K_a(2V_C - V_{DD}) - \sqrt{K_bI_B\left(1 - \dfrac{K_bV^2}{4I_B}\right)}\right]V & |V| \le \sqrt{\dfrac{2I_B}{K_b}} \\ 2K_a(2V_C - V_{DD})V - \dfrac{I_B}{2} & V \ge \sqrt{\dfrac{2I_B}{K_b}} \end{cases} \quad (1)$$



(*a*)

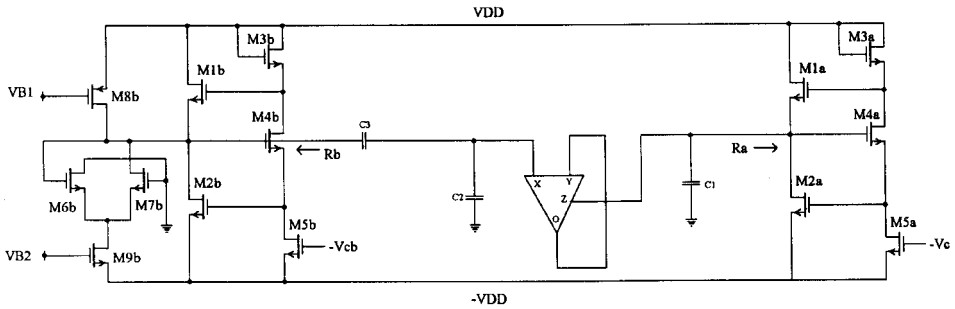


(*b*)

Figure 1. The proposed grounded nonlinear resistor and its *I–V* characteristics.

where K_a, K_b and I_B were taken in simulations as $110\,\mu A\,V^{-2}$, $13.75\,mA\,V^{-2}$ and $800\,\mu A$ respectively. The aspect ratios of the M1–M9 transistors were set to: 8/4, 8/4, 4/4, 4/4, 16/4, 500/2, 500/2, 200/2 and 200/4 respectively.

One of the minimum component chaotic oscillators of Elwakil and Soliman (1997b) is selected to investigate its chaotic behaviour using the proposed nonlinear



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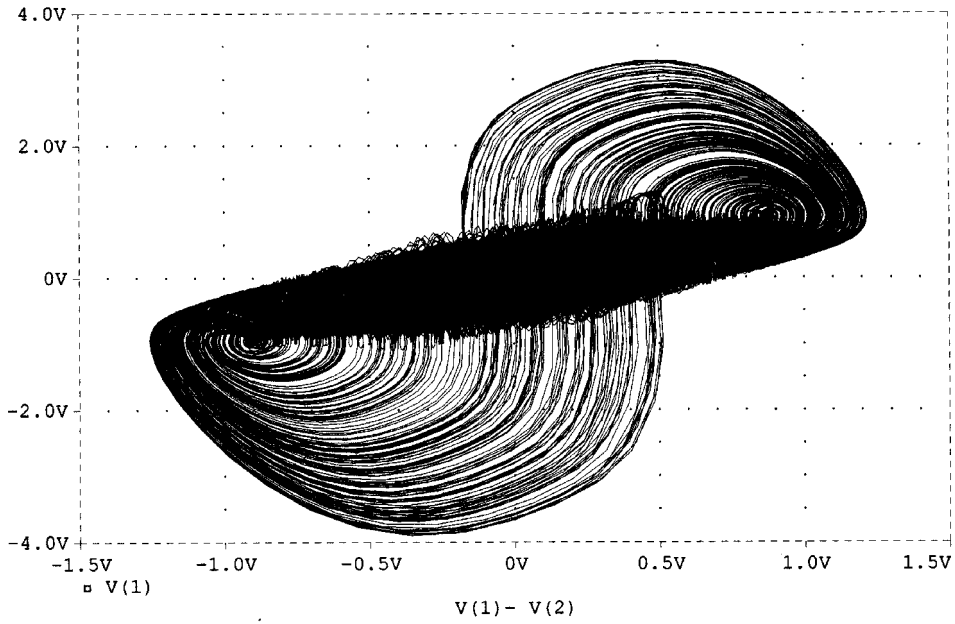


(*b*)

Figure 2. A minimum component chaotic oscillator using the proposed grounded nonlinear resistor and the corresponding PSPICE V_{C2}-V_{C3} trajectory.

resistor. The whole chaotic oscillator is implemented in CMOS, as shown in figure 2(*a*). The CFOA used is that of Mahmoud and Soliman (1998) with the input resistance of its inverting input terminal (R_X) adjusted to 65 Ω. The linear grounded resistor R_a formed by transistors M1a–M5a is implemented as in Mahmoud *et al.* (1996) while R_b is the nonlinear resistor of figure 1(*a*). As can be seen, parasitic effects are greatly suppressed since all elements, except for C_3, are grounded. Moreover, the CFOA connection forces the voltages at its four terminals to be approximately equal. However, the small voltage drop on R_X is responsible for stimulating chaos by allowing a third rather than a second-order system to exist (Elwakil and Soliman 1999b). The circuit is thus described by the

following equation set

$$\left.\begin{aligned} R_a C_1 \dot{V}_{C1} &= R_a C_3 \dot{V}_{C3} + R_a C_2 \dot{V}_{C2} - V_{C1} \\ R_X C_2 \dot{V}_{C2} &= V_{C1} - V_{C2} - R_X C_3 \dot{V}_{C3} \\ C_3 \dot{V}_{C3} &= I(V) \end{aligned}\right\} \quad (2)$$

where, $I(V)$ is as given by (1) with $V = V_{C2} - V_{C3}$. Figure 2(*b*) demonstrates the observed $V_{C2} - V_{C3}$ phase space trajectory using PSPICE with $C_1 = C_2 = C_3 = C = 1\text{nF}$, $R_a = 1.75\,\text{k}\Omega$ ($V_C = -3.8\,\text{V}$) and using $\pm 5\,\text{V}$ supplies.

Although the nonlinear resistor can be modelled using three-segment piecewise-linear characteristics, an approximation using a continuous cubic polynomial displays similar results. Thus, setting

$$X = \frac{V_{C1}}{V_{Tn}}, \quad Y = \frac{V_{C2}}{V_{Tn}}, \quad Z = \frac{V_{C3}}{V_{Tn}}, \quad t_n = \frac{t}{R_a C}, \quad \alpha = \frac{R_a}{R_X}$$

where V_{Tn} is the NMOS threshold voltage (1 V), the dimensionless form of equation set (2) becomes

$$\left.\begin{aligned} \dot{X} &= (\alpha - 1)X - \alpha Y \\ \dot{Y} &= \alpha(X - Y) - \dot{Z} \\ \dot{Z} &= \beta_1(Y - Z) + \beta_2(Y - Z)^3 \end{aligned}\right\} \quad (3)$$

Numerical integration of equation set (3) was carried out using a Runge–Kutta fourth-order algorithm with a 0.001 step size taking $\alpha = 16$, $\beta_1 = -0.65$ and $\beta_2 = 0.2$. The attractor obtained is plotted in figure 3. The Jacobian of the linearized

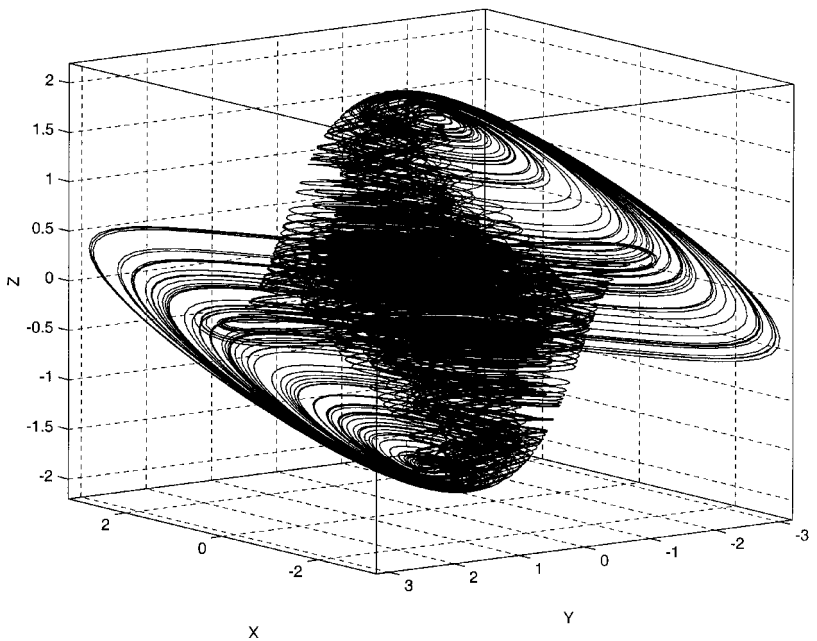


Figure 3. Chaotic attractor in the *X*–*Y*–*Z* phase space obtained via numerical integration of equation (3).

system is given by

$$J = \begin{bmatrix} (\alpha - 1) & -\alpha & 0 \\ \alpha & -(\alpha + \Delta) & \Delta \\ 0 & \Delta & -\Delta \end{bmatrix} \tag{4}$$

where, in the inner region of the nonlinear resistor (around the origin), $\Delta_{min} = \beta_1 = -0.65$ while, in the outer regions, simulations indicate a $\Delta_{max} \cong 0.95$. Hence, the following eigenvalues are calculated

$$\begin{cases} 0.41357, & -0.05678 \pm j5.0144 & \text{for } \Delta_{min} \\ -3.4145, & 0.25676 \pm j2.0945 & \text{for } \Delta_{max} \end{cases}$$

It is worth noting that the use of a cubic nonlinearity in Chua's circuit has been reported in by Zhong (1994), where it was shown that the double-scroll chaotic dynamics were preserved. However, the implementation of this nonlinearity required three commercial analogue multipliers, which indeed have complicated internal circuitry. It should thus be possible to realize a monolithic Chua's circuit with the proposed nonlinear resistor of figure 1(*a*) with much fewer overheads.

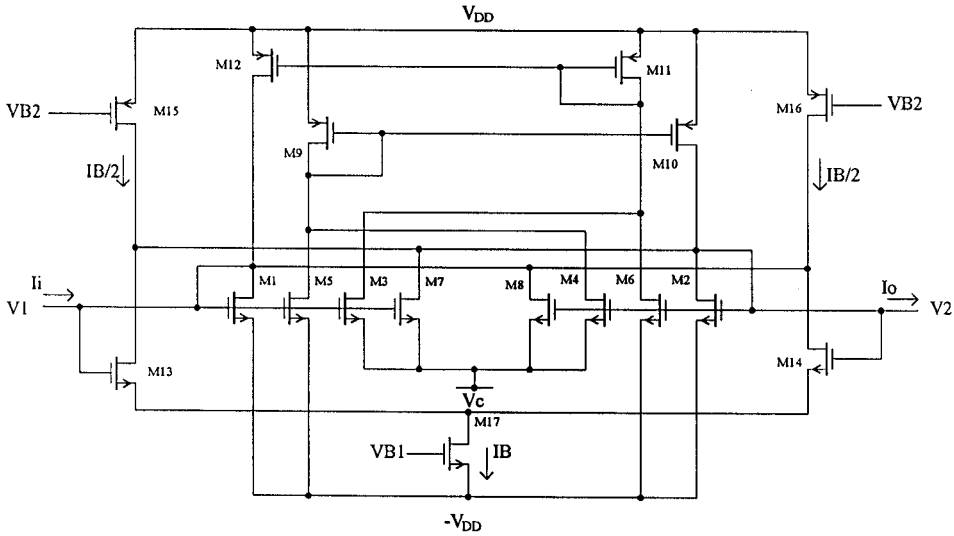
3. Floating nonlinear resistor with application to a Wien-bridge chaotic oscillator

Figure 4(*a*) represents the proposed floating nonlinear resistor. Two basic parts can be distinguished: a linear voltage-controlled CMOS floating resistor (transistors M1–M12) (Elwan *et al.* 1996) and a nonlinear differential pair transconductor (transistors M13 and M14) biased with two current sources (transistors M15–M17). The *I–V* characteristics of the nonlinear resistor are shown in figure 4(*b*) with the negative control voltage V_C scanned from -3 V to -1 V in 0.5 V steps. The negative slope around the origin can also be altered by changing the bias current I_B of the differential pair through the bias voltages V_{B1} and V_{B2}. The relation between the current flowing in the resistor ($I = I_i = I_o$) and the voltage between its two terminals ($V = V_2 - V_1$) is given by

$$I = \begin{cases} K_a(V_C + V_{DD})V + \dfrac{I_B}{2} & V \le -\sqrt{\dfrac{2I_B}{K_b}} \\ \left[K_a(V_C + V_{DD}) - \sqrt{K_b I_B\left(1 - \dfrac{K_b V^2}{4I_B}\right)}\right]V & |V| \le \sqrt{\dfrac{2I_B}{K_b}} \\ K_a(V_C + V_{DD})V - \dfrac{I_B}{2} & V \ge \sqrt{\dfrac{2I_B}{K_b}} \end{cases} \tag{5}$$

where K_a, K_b and I_B were taken in simulations as 206.25 μA V^{-2}, 5.5 mA V^{-2} and 800 μA respectively. The transistors' aspect ratios are as given in table 1.

Owing to their popularity and ease of design, Wien-type oscillators have been focused upon to be modified for chaos (Morgul 1995, Namajunas and Tamasevicius 1995, Elwakil and Soliman 1997a). The Wien oscillator in Elwakil and Soliman (1997c), which employs a CFOA, is selected to investigate its chaotic behaviour with the proposed nonlinear resistor. The whole circuit is implemented in CMOS



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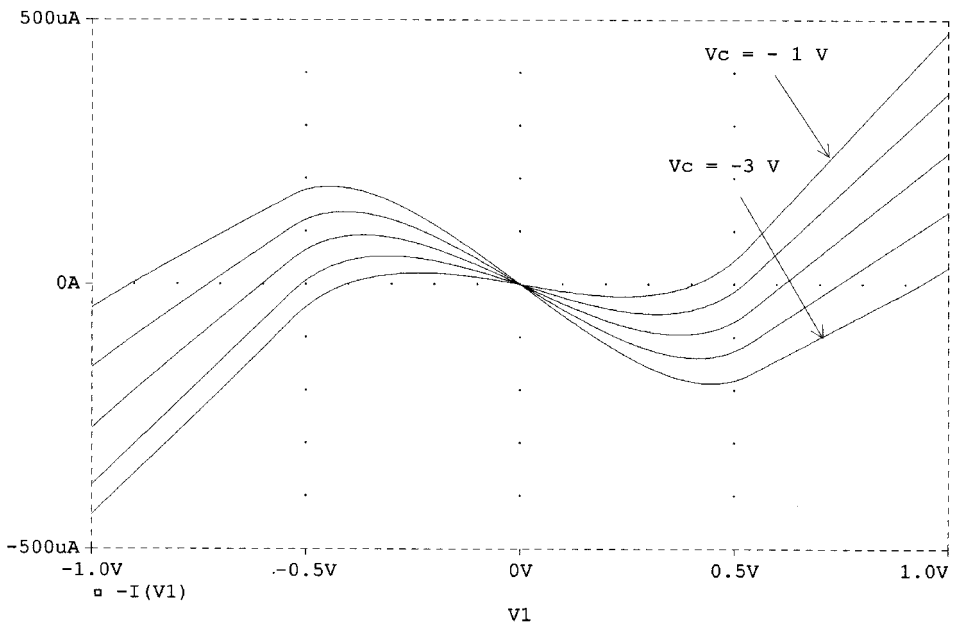

(*b*)

Figure 4. The proposed floating nonlinear resistor and its *I–V* characteristics.

as shown in figure 5(*a*) where the linear grounded resistors R_1, R_2 and R_3 are implemented as shown in figure 5(*b*), NR is the nonlinear resistor of figure 4(*a*) and the CFOA is that of Mahmoud and Soliman (1998). It was found that it is necessary to include the very small CFOA output resistance (R_o) in the analysis to model the observed chaotic behaviour (Elwakil and Soliman 1999b). This R_o

Transistors	Aspect ratio
M1–M8	15/4
M9–M12, M15, M16	80/4
M13–M14	400/4
M17	160/4

Table 1. Aspect ratios of the transistors of the proposed nonlinear resistor.

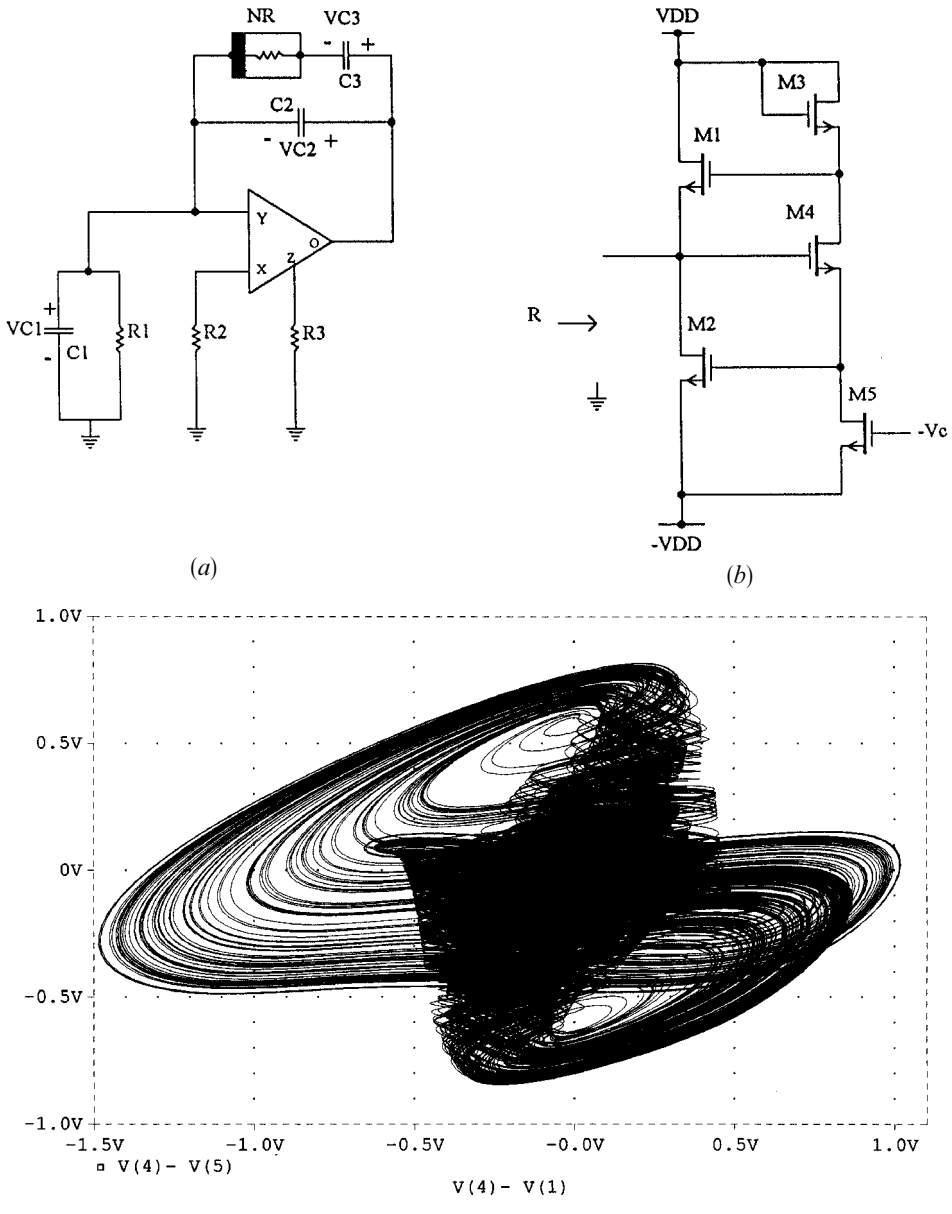

Figure 5. Wien bridge chaotic oscillator using the proposed floating nonlinear resistor and the corresponding PSPICE V_{C2}-V_{C3} trajectory.

was measured to be 15Ω. Thus, the circuit can be described by the following equation set

$$\left.\begin{aligned} R_o C_1 \dot{V}_{C1} &= \left(K - 1 - \frac{R_o}{R_1}\right) V_{C1} - V_{C2} \\ R_1 C_2 \dot{V}_{C2} &= R_1 C_1 \dot{V}_{C1} - R_1 C_3 \dot{V}_{C3} + V_{C1} \\ C_3 \dot{V}_{C3} &= I(V) \end{aligned}\right\} \tag{6}$$

where $I(V)$ is as given by (5) with $V = V_{C2} - V_{C3}$. Figure 5(*c*) shows the PSPICE $V_{C2} - V_{C3}$ trajectory with $C_1 = 2\,\text{nF}$, $C_2 = C_3 = 1\,\text{nF}$, $R_1 = 1.32\,\text{k}\Omega$, $R_2 = 10\,\text{k}\Omega$, $R_3 = 30.1\,\text{k}\Omega$, the nonlinear characteristics of figure 4(*b*) at $V_C = -2\,\text{V}$ and using $\pm 5\,\text{V}$ supplies.

The nonlinear resistor characteristics can be approximated using a continuous cubic polynomial. Thus, setting

$$X = \frac{V_{C1}}{V_{Tn}}, \quad Y = \frac{V_{C2}}{V_{Tn}}, \quad Z = \frac{V_{C3}}{V_{Tn}}, \quad t_n = \frac{t}{R_1 C_1}, \quad \alpha = \frac{R_1}{R_o}$$

and for the choice of $C_1 = 2C$, $C_2 = C_3 = C$, the dimensionless form of equation set (6) becomes

$$\left.\begin{aligned} \dot{X} &= \alpha\left[\left(K - 1 - \frac{1}{\alpha}\right)X - Y\right] \\ \dot{Y} &= 2(X + \dot{X}) - \dot{Z} \\ \dot{Z} &= \beta_1(Y - Z) + \beta_2(Y - Z)^3 \end{aligned}\right\} \tag{7}$$

Numerical integration of (7) was carried out using a Runge–Kutta fourth-order algorithm with a 0.001 step size, taking $K = 2.968$, $\alpha = 25$, $\beta_1 = -0.53$ and $\beta_2 = 0.3$. The observed *Y–Z* trajectory is plotted in figure 6. The Jacobian of the linearized system is given by

$$J = \begin{bmatrix} \alpha\left(K - 1 - \dfrac{1}{\alpha}\right) & -\alpha & 0 \\ 2\left[1 + \alpha\left(K - 1 - \dfrac{1}{\alpha}\right)\right] & -(2\alpha + \Delta) & \Delta \\ 0 & \Delta & -\Delta \end{bmatrix} \tag{8}$$

where, around the origin, $\Delta_{\min} = \beta_1 = -0.53$ while, in the outer regions, simulations indicate a $\Delta_{\max} \cong 0.85$. Hence, the following eigenvalues are calculated

$$\begin{cases} 0.35344, & -0.54672 \pm j8.6417 \quad \text{for } \Delta_{\min} \\ -3.7266, & 0.11329 \pm j3.3752 \quad \text{for } \Delta_{\max} \end{cases} \tag{9}$$

4. Conclusion

Novel grounded and floating voltage-controlled nonlinear resistors with odd symmetrical characteristics were introduced. Application to chaotic oscillator design was then demonstrated on two selected chaotic oscillators based on a CMOS CFOA. It should be stressed that the proposed nonlinear resistors were derived from linear resistors modified by the inherent nonlinearity of MOS differential pairs. More

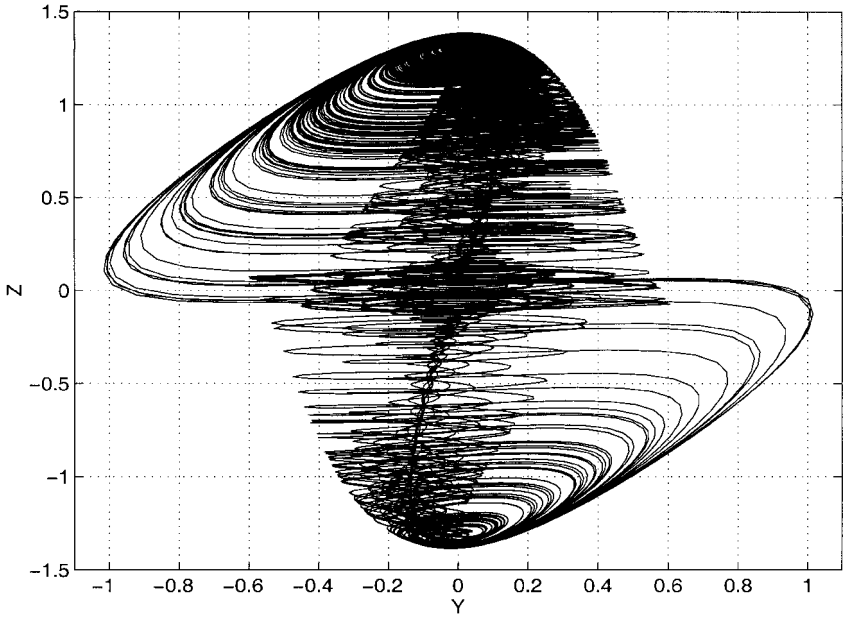

Figure 6. Chaotic Y–Z phase portrait obtained via numerical integration of equation (7).

nonlinear resistors can be derived in a similar manner. The chaotic attractors observed here support the results reported by Elwakil and Soliman (1998), who studied the relationship between chaotic attractors observed using a simple two-segment passive nonlinear resistor and those observed using active resistors with odd symmetrical characteristics.

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