

## Active RC high selectivity notch filter

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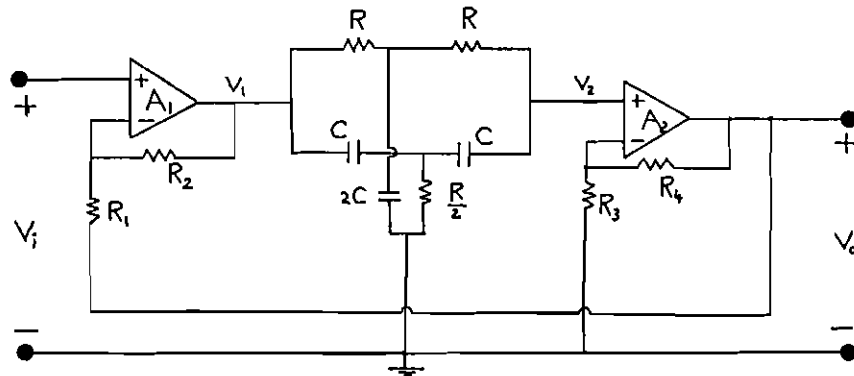
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A general configuration for realizing a high  $Q$  notch filter is given. The circuit has a very high input impedance and a very low output impedance. The frequency limitation equations based on the one-pole roll-off model of the operational amplifier are given.

### 1. Introduction

Active filters are being classified now based on the selectivity that can be achieved by the circuit (Moschytz 1970). The  $Q$  range is divided into three decades, low  $Q$  (0.5 to 5), medium  $Q$  (5 to 50) and high  $Q$  (50 to 500). Recently a general configuration for realizing a medium selectivity notch filter was given (Soliman 1972). The purpose of this paper is to modify the previous circuit to make it capable of realizing a high  $Q$  notch characteristics.

### 2. Basic configuration



The circuit is shown in the figure. The open circuit voltage transfer function is given by :

$$G(s) \equiv \frac{V_o}{V_i} = \frac{T(s)}{\left(\frac{1}{K_1} + \frac{1}{A_1}\right)\left(\frac{1}{K_2} + \frac{1}{A_2}\right) + \left(\frac{K_1 - 1}{K_1}\right) T(s)} \quad (1)$$

where

$$K_1 = 1 + \frac{R_2}{R_1}, \quad K_2 = 1 + \frac{R_4}{R_3} \quad (2)$$

and

$$T(s) = \frac{V_2}{V_1} = \frac{s^2 + \left(\frac{1}{CR}\right)^2}{s^2 + \left(\frac{4}{CR}\right)s + \left(\frac{1}{CR}\right)^2} \quad (3)$$

From (3) in (1), hence

$$G(s) = \frac{s^2 + \left(\frac{1}{CR}\right)^2}{\left[\left(\frac{1}{K_1} + \frac{1}{A_1}\right)\left(\frac{1}{K_2} + \frac{1}{A_2}\right) + \frac{K_1 - 1}{K_1}\right]} \times \left[ \frac{s^2 + \frac{4}{CR}}{1 + \frac{1}{K_1 \left(\frac{1}{K_1} + \frac{1}{A_2}\right)\left(\frac{1}{K_2} + \frac{1}{A_2}\right)}} \frac{K_1 - 1}{K_1 - 1} s + \left(\frac{1}{CR}\right)^2 \right] \quad (4)$$

For  $A_1$  and  $A_2$  approaching infinity, the above transfer function reduces to

$$G(s) \simeq \frac{K_1 K_2 \left[ s^2 + \left(\frac{1}{CR}\right)^2 \right]}{[K_2(K_1 - 1) + 1] \left[ s^2 + \frac{4}{CR(1 + K_2(K_1 - 1))} s + \left(\frac{1}{CR}\right)^2 \right]} \quad (5)$$

which realizes a notch characteristics having

$$\omega_0 = \frac{1}{CR} \quad (6)$$

$$Q = \frac{K_2(K_1 - 1) + 1}{4} \simeq \frac{K_1 K_2}{4} \quad \text{for } K_1, K_2 \gg 1 \quad (7)$$

$$H \equiv |G(0)| \equiv |G(\infty)| = \frac{K_1 K_2}{K_2(K_1 - 1) + 1} \simeq 1 \quad \text{for } K_1, K_2 \gg 1 \quad (8)$$

Equations (6) and (7) are the design equations of the filter. It is seen that  $C$  and  $R$  control  $\omega_0$  without affecting  $Q$  of the circuit, on the other hand the selectivity of the filter is adjusted by  $K_1$  and  $K_2$  without changing the notch frequency.

### 3. Active sensitivities

The effect of the finite gain of the operational amplifiers is discussed next. Assuming a real gain of the operational amplifiers, i.e.  $A_1 = A_2 = A_0$ , and for the case of interest namely  $K_1 = K_2$  and  $Q \gg 1$ , the actual values of  $\omega_0$  and  $Q$  are given by

$$\omega_{0a} = \omega_0 \quad (9)$$

$$Q_a \simeq \frac{Q}{\left(1 + \frac{2\sqrt{Q}}{A_0}\right)^2} \quad (10)$$

Thus

$$\frac{\omega_{ns}}{A_0} = 0 \quad (11)$$

$$\frac{Q_n}{A_0} \approx \frac{\frac{4\sqrt{Q}}{A_0}}{1 + \frac{2\sqrt{Q}}{A_0}} \approx \frac{4\sqrt{Q}}{A_0} \quad \text{for } Q \ll A_0 \quad (12)$$

which is proportional to the square root of  $Q$  rather than  $Q$  as in the medium selectivity circuit (Soliman 1972).

#### 4. Frequency limitation equations

The frequency limitation equations of this network are obtained from (4) by setting,

$$A_1 = A_2 = \frac{A_0}{1 + \frac{s}{\omega_1}} \approx \frac{GB}{s} \quad (13)$$

where

$A_0$  is the open loop d.c. gain of the operational amplifier,  
 $\omega_1$  is the open loop 3 dB bandwidth in radians per second,

and

$$GB = A_0\omega_1$$

and following Budak-Petrela analysis (1972), hence ;

$$\frac{\Delta\omega_0}{\omega_0} \approx -\frac{2\omega_0}{\sqrt{Q} \cdot GB} \quad (14)$$

$$\frac{\Delta Q}{Q} = \frac{2\omega_0}{\sqrt{Q} \cdot GB} \quad (15)$$

where it is assumed that  $Q \gg 1$  and  $K_1 = K_2$ .

As an example, for the  $\mu A 741$  operational amplifier having  $A_0 = 10^5$ ,  $f_1 = 10$  Hz, if the maximum allowable change in  $\omega_0$  and  $Q$  is not to exceed 2% and for  $Q = 100$ , the maximum notch frequency that can be realized using this circuit is 100 kHz.

#### 5. Conclusions

A general circuit for realizing a high  $Q$  notch characteristics is given. The circuit has a very high input impedance and a very low output impedance. The frequency limitation equations based on the one pole rolloff model of the operational amplifier are given.

#### REFERENCES

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