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Active RC low-pass filter suitable for integration

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A canonic active RC low-pass filter having both capacitors earthed is given. The ω_0 and the Q sensitivities with respect to all circuit components are derived. A unique feature of the circuit is that ω_0 is absolutely insensitive to the gain-bandwidth of the operational amplifiers.

1. Introduction

Many active *RC* low-pass filters are available (Mitra 1969, Soliman 1973). Most of these require floating capacitors. For integrated circuit fabrication, it is desirable to have all capacitors grounded (Newcomb 1968). In this paper a new active *RC* second-order low-pass filter which uses two earthed capacitors is described. The filter has a very high input impedance and a very low output impedance, hence cascading second-order sections is possible without using any buffer.

2. Basic configuration

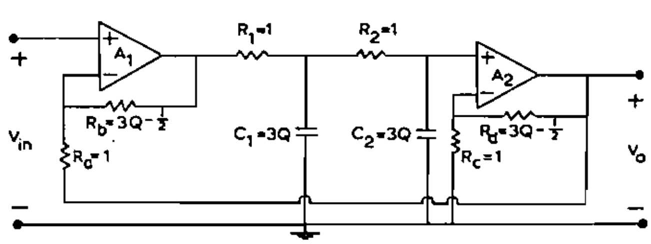
The circuit is shown in fig. 1. By direct analysis taking the effect of the finite gain of the operational amplifiers (OAs) into consideration, thus :

$$\frac{V_0}{V_{in}}=\frac{1}{\dfrac{K_1-1}{K_1}+\left(\dfrac{1}{K_1}+\dfrac{1}{A_1}\right)\left(\dfrac{1}{K_2}+\dfrac{1}{A_2}\right)\times[s^2C_1C_2R_1R_2+s(C_1R_1+C_2R_2+C_2R_1)+1]} \tag{1}$$

where

$$K_1=1+\frac{R_b}{R_a} \quad \text{and} \quad K_2=1+\frac{R_d}{R_c}$$

Fig. 1



A new active RC low-pass filter. Circuit components are given for $\omega_0=1$.

As A_1 and A_2 approach infinity, the transfer function becomes :

$$\frac{V_0}{V_{\text{in}}}=\frac{K_1K_2}{s^2C_1C_2R_1R_2+s(C_1R_1+C_2R_2+C_2R_1)+1+K_2(K_1-1)} \tag{2}$$

which realizes a low-pass filter characteristics having :

$$\omega_0=\sqrt{\frac{1+K_2(K_1-1)}{C_1C_2R_1R_2}} \tag{3}$$

$$Q=\frac{\sqrt{\{C_1C_2R_1R_2[1+K_2(K_1-1)]\}}}{C_1R_1+C_2R_2+C_2R_1} \tag{4}$$

From eqn. (2) it is seen that the decomposition of the denominator of the transfer function is similar to that obtained in the Sallen–Key negative feedback low-pass filter (1955) which belongs to Mitra's type A decomposition (1969).

3. Design formulas

Design equations can be choosen in a variety of ways. One possible solution for a normalized value of $\omega_0=1$ is :

$$\left.\begin{aligned} &R_1=R_2=1\\ &C_1=C_2=3Q\\ &K_1=K_2\simeq 3Q+\tfrac{1}{2} \quad \text{for} \quad Q>\tfrac{1}{2} \end{aligned}\right\} \tag{5}$$

4. Passive sensitivities

From (3), the ω_0 sensitivities with respect to all passive circuit components are given by :

$$\left.\begin{aligned} &S^{\omega_0}_{R_1}=S^{\omega_0}_{R_2}=S^{\omega_0}_{C_1}=S^{\omega_0}_{C_2}=-\tfrac{1}{2}\\ &S^{\omega_0}_{R_d}=-S^{\omega_0}_{R_c}<\tfrac{1}{2}\\ &S^{\omega_0}_{R_b}=-S^{\omega_0}_{R_a}<\tfrac{1}{2} \end{aligned}\right\} \tag{6}$$

From (4), the Q sensitivities with respect to all passive circuit components are given by :

$$\left.\begin{aligned} &S^{Q}_{R_d}=-S^{Q}_{R_c}<\tfrac{1}{2}\\ &S^{Q}_{R_b}=-S^{Q}_{R_a}<\tfrac{1}{2}\\ &S^{Q}_{R_1}=-S^{Q}_{R_2}=\tfrac{1}{2}\left(\frac{-R_1C_1-R_1C_2+R_2C_2}{R_1C_1+R_1C_2+R_2C_2}\right)=-\tfrac{1}{6}\\ &S^{Q}_{C_1}=-S^{Q}_{C_2}=\tfrac{1}{2}\left(\frac{-R_1C_1+R_1C_2+R_2C_2}{R_1C_1+R_1C_2+R_2C_2}\right)=\tfrac{1}{6} \end{aligned}\right\} \tag{7}$$

5. Active sensitivities

Assuming a real gain of the OAs, $A_1 = A_2 = A_0$ and for the case of interest namely $R_1 = R_2 = R$, $C_1 = C_2 = C$ and $K_1 = K_2 = K$, eqn. (1) reduces to

$$\frac{V_0}{V_{in}} = \frac{K^2}{\left(1+\frac{K}{A_0}\right)^2} \frac{1}{s^2C^2R^2 + 3sCR + \left[1 + \frac{K(K-1)}{\left(1+\frac{K}{A_0}\right)^2}\right]} \tag{8}$$

Thus the actual values of ω_0 and Q are given by :

$$\omega_{0a} = \frac{1}{CR}\sqrt{\left(1 + \frac{K(K-1)}{\left(1+\frac{K}{A_0}\right)^2}\right)} \tag{9}$$

$$Q_a = \tfrac{1}{3}\sqrt{\left(1 + \frac{K(K-1)}{\left(1+\frac{K}{A_0}\right)^2}\right)} \tag{10}$$

For

$$Q \gg \tfrac{1}{2}, \quad \frac{Q}{A_0} \ll 1$$

$$\omega_{0a} = \frac{\omega_0}{1+\frac{3Q}{A_0}} \tag{11}$$

$$Q_a \simeq \frac{Q}{1+\frac{3Q}{A_0}} \tag{12}$$

$$\overset{\omega_{0a}}{\underset{A_0}{S}} = \overset{Q_a}{\underset{A_0}{S}} = \frac{\frac{3Q}{A_0}}{1+\frac{3Q}{A_0}} \simeq \frac{3Q}{A_0} \quad \text{for} \quad \frac{Q}{A_0} \ll 1 \tag{13}$$

6. Frequency limitation equations

Here the effect of the roll-off of the OA gain is taken into account. From (1) and setting

$$A_1 = A_2 = \frac{A_0}{1+\frac{s}{\omega_1}} \simeq \frac{GB}{s} \tag{14}$$

where A_0 is the open loop d.c. gain.

ω_1 is the open loop 3 dB bandwidth in radians per second, and $GB = A_0\omega_1$.

Thus the denominator of the transfer function is given by :

$$D(s) \simeq \left(\frac{s^2}{\omega_0{}^2} + \frac{s}{\omega_0 Q} + 1\right) + \frac{(6Q+1)}{GB} \cdot s\left(\frac{s^2}{\omega_0{}^2} + \frac{s}{\omega_0 Q} + \frac{1}{9Q^2}\right) \tag{15}$$

Type of filter	OAs	Z_{in}	Z_{out}	$\left\lvert S^{\omega_0}_{R,C} \right\rvert$	$\left\lvert S^{Q}_{R,C} \right\rvert$	$A_0 \, S^{\omega_0}_{A_0}$	$A_0 \, S^{Q}_{A_0}$	Roll-off of OA gain	
								$GB \frac{\Delta\omega_0}{\omega_0}$	$GB \frac{\Delta Q}{Q}$
Sallen–Key negative feedback	1	Low	Very low	$\leqslant \frac{1}{2}$	$\leqslant \frac{1}{2}$	$\frac{25}{2} Q^2$	$\frac{25}{2} Q^2$	$\frac{5}{2} \omega_0 Q$	$25 \omega_0 Q^3$
New circuit	2	Very high	Very low	$\leqslant \frac{1}{2}$	$\leqslant \frac{1}{2}$	$3 Q$	$3 Q$	0	$6 \omega_0 Q^2$

Following the Budak–Petrela analysis (1972) thus

$$\frac{\Delta\omega_0}{\omega_0}=0 \tag{16}$$

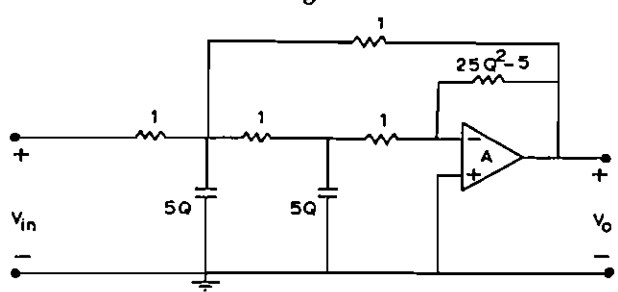
$$\frac{\Delta Q}{Q}=\left(1-\frac{1}{9Q^2}\right)(1+6Q)\frac{\omega_0 Q}{GB}\simeq\frac{6Q^2\omega_0}{GB} \quad \text{for} \quad Q>\tfrac{1}{2} \tag{17}$$

It is seen that ω_0 is absolutely insensitive to the roll-off of the OA gain.

7. Comparison with the Sallen-Key filter

Figure 2 represents the well-known Sallen–Key negative feedback low-pass filter (1955) which uses also two earthed capacitors. A detailed comparison between the two filters for $Q>\frac{1}{2}$ is given in the table.

Fig. 2



Sallen–Key negative feedback low-pass filter. Circuit components are given for $\omega_0=1$.

8. Conclusions

A canonic active *RC* low-pass filter having both capacitors earthed and having ω_0 absolutely insensitive to the gain-bandwidth of the operational amplifier is given. It is noted that a high-pass filter having ω_0 insensitive to the roll-off of the OA gain is obtainable directly from the given filter by interchanging R_i and C_i $(i=1, 2)$.

References

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