A detailed mathematical treatment of the solution of the piezoelectric-wave equations, as applied to layered media, has been described by Farnell.[8] His method of solution involves the evaluation of an nth-order determinantal equation, where $n = 12$ for the general case of one layer over a substrate, although n may be smaller if there are specific crystal symmetries.

Consequently, an extension of this method to multilayered structures is clearly impractical, since each additional layer can cause the order of the determinant to be increased by degree 8.

The present program is comprehensive and can treat a wide range of plane-layer applications. Three examples of its application to multilayered structures are described. In the examples presented, crystal orientation is assumed to be z cut, x propagating. The material constants used were obtained from References 9 and 10, except for the piezoelectric constants of the zinc oxide (ZnO), for which the values used are given in the legend to Fig. 1. These revised values have been obtained from more recent measurements.[11]

Fig. 1 illustrates the phase-velocity dispersion for a film of zinc oxide on a film of yttrium–iron garnet (y.i.g.) on a substrate of gadolinium–gallium garnet (g.g.g.) as a function of $(h_1+h_2)k$. The y.i.g. is acoustically faster than g.g.g., while ZnO is acoustically slower. The interplay of low-velocity ZnO with high-velocity y.i.g. can give rise to a peak in the dispersion characteristic at a value of k dependent on the relative proportions of h_1 and h_2. Over a range of frequencies centred about this peak, the velocity is approximately constant. The dispersion of this structure is clearly intermediate between the dispersion of y.i.g. on g.g.g. ($h_1 = 0$) and the dispersion for ZnO on g.g.g. ($h_2 = 0$). This example illustrates how the use of an extra dispersive layer permits control of the shape of the dispersion characteristic. The magnitude of the surface-wave velocity at a given frequency is thus determined by the relative thicknesses of the two layers.

The second example concerns the determination of the value of $\Delta v/v$ associated with a layer of ZnO on a layer of fused silica on a substrate of silicon, a structure which has been suggested for surface-wave filters.[1] The layer of fused silica is utilised because of its low acoustic loss and because of its compatibility with silicon integrated-circuit technology. Fig. 2 shows the variation of $\Delta v/v$ as a function of $h_1 k$ for the four possible transducer configurations, where, in this example, the two layers have equal thicknesses. These results are of the same basic form as those presented by Solie[7] for ZnO on sapphire. If the transducer is located between the ZnO and the fused silica, the maximum $\Delta v/v$ is 0·0116 at $h_1 k = 2{\cdot}9$. If, however, the transducer is laid on top of the ZnO, the maximum $\Delta v/v$ occurs at a much lower value of $h_1 k$ ($\Delta v/v = 0{\cdot}0044$ at $h_1 k = 0{\cdot}25$). It should be emphasised that the peak value of $\Delta v/v$ on curve a depends on the relative magnitudes of h_1 and h_2. The corresponding peak values for ZnO on fused silica and ZnO on silicon are 0·0117 and 0·0092, respectively.

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NEW ACTIVE *RC* CONFIGURATION FOR REALISING A MEDIUM-SELECTIVITY NOTCH FILTER

Indexing term: Active filters

A new configuration for activating the symmetrical twin-T network to realise a medium-selectivity notch filter is given. The ω_0 and Q factor sensitivities with respect to all circuit components are very low. The frequency-limitation equations for the circuit are derived, based on the 1-pole rolloff model of the operational amplifier.

Several active RC networks for realising a notch filter, with characteristics of the form

$$T(s) = H\frac{s^2+\omega_0{}^2}{s^2+(\omega_0/Q)s+\omega_0{}^2} \quad \ldots \ldots \quad (1)$$

and having a pole Q factor greater than 0·5, have been reported.[1–3] All these methods use the operational amplifier as the active-circuit element, and the analysis is based on the assumption that the operational amplifier is ideal. In examining the Q factor sensitivities of the well known circuit[4–5] shown in Fig. 1, it is found that the relations

$$\left.\begin{aligned} \left| S^{Q}_{R_1, R_2} \right| &\propto Q \\ \left| S^{Q}_{A} \right| &\propto Q \end{aligned}\right\} \quad \ldots \ldots \quad (2)$$

limit the maximum Q factor available with this network[5] to $Q \leqslant 10$ (low-selectivity filters). In this letter, a new configuration for realising a medium-selectivity notch filter ($5 \leqslant Q \leqslant 50$) is given.

Analysis of the circuit shown in Fig. 2 leads to

$$\frac{V_o}{V_i} = \frac{K}{\frac{1}{T}\left(1+\frac{1}{A}\right)\left(1+\frac{K}{A}\right)+(K-1)} \quad \ldots \quad (3)$$

where it is assumed that the operational amplifiers have

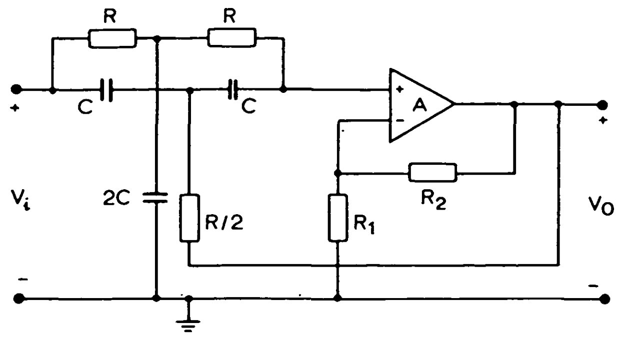

Fig. 1 *Well known configuration for realising a low-selectivity notch filter*

infinite input impedance and gain A,

$$K = 1 + \frac{R_2}{R_1} \quad \quad (4)$$

and

$$T = \frac{V_2}{V_1} \quad \quad (5)$$

For a symmetrical twin-T network,

$$T(s) = \frac{s^2 + (1/CR)^2}{s^2 + (4/CR)\,s + (1/CR)^2} \quad \quad (6)$$

On substituting from eqn. 6 into eqn. 3, and after some approximations based on $Q \gg 1$, one obtains

$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{1}{A}} \; \frac{s^2 + (1/CR)^2}{s^2 + \frac{4}{CR}\left(\frac{\frac{1}{K} + \frac{1}{A}}{1 + \frac{1}{A}}\right) s + \left(\frac{1}{CR}\right)^2} \quad \quad (7)$$

For $A \rightarrow \infty$,

$$\frac{V_o}{V_i} \simeq \frac{s^2 + (1/CR)^2}{s^2 + \left(\frac{4}{KCR}\right) s + \left(\frac{1}{CR}\right)^2} \quad \quad (8)$$

Comparing with eqn. 1,

$$H = 1 \quad \quad (9)$$

$$\omega_0 = 1/CR \quad \quad (10)$$

$$Q = K/4 \quad \quad (11)$$

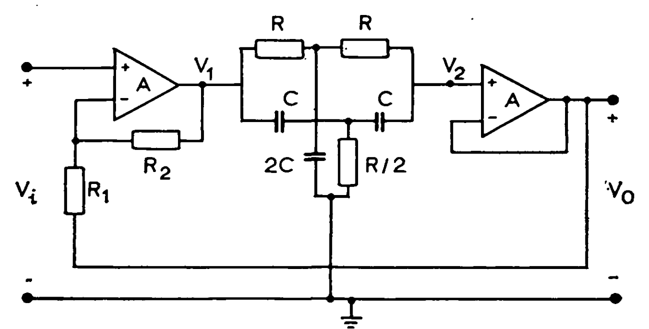

Fig. 2 *New configuration for realising a medium-selectivity notch filter*

The sensitivities with respect to the passive circuit components are

$$\left.\begin{aligned} \left|S_{R,C}^{\omega_0}\right| &= 1 \\ \left|S_{R_1,R_2}^{\omega_0}\right| &= 0 \end{aligned}\right\} \quad \quad (12)$$

$$\left.\begin{aligned} \left|S_{R,C}^{Q}\right| &= 0 \\ \left|S_{R_1,R_2}^{Q}\right| &< 1 \end{aligned}\right\} \quad \quad (13)$$

Next, the sensitivities with respect to the active elements are derived. Consider the case where the gain of the operational amplifier is finite and real, i.e. $A = A_0$. From eqn. 7, one obtains the following actual values for H, ω_0 and Q:

$$H_a = \frac{1}{1 + \frac{1}{A_0}} \quad \quad (14)$$

$$\omega_{0a} = \omega_0 \quad \quad (15)$$

$$Q_a \simeq Q \frac{1 + \frac{1}{A_0}}{1 + \frac{K}{A_0}} \simeq Q\left(1 - \frac{K}{A_0}\right) \text{ for } Q \gg 1 \quad \quad (16)$$

Thus

$$\left.\begin{aligned} \left|S_{A_0}^{\omega_0}\right| &= 0 \\ \left|S_{A_0}^{Q_a}\right| &\simeq \frac{4Q}{A_0} \text{ for } Q \gg 1 \end{aligned}\right\} \quad \quad (17)$$

The above sensitivities are suitable for realising medium-Q factor notch filters. The frequency limitation of the above notch filter is discussed next. Using the 1-pole rolloff model for the operational amplifier,

$$A = \frac{A_0}{1 + \frac{s}{\omega_1}} \simeq \frac{GB}{s} \quad \quad (18)$$

where A_0 is the open-loop d.c. gain, ω_1 is the open-loop 3 dB bandwidth in radians per second and $GB = A_0\omega_1$ is the gain–bandwidth product. On substituting eqn. 18 into eqn. 7 and normalising with respect to ω_0, one obtains for the denominator of the transfer function, after some approximations based on $Q \gg 1$,

$$D(s_n) \simeq \left\{s_n^2 + \left(\frac{1}{Q}\right) s_n + 1\right\} + \frac{1}{GB_n} s_n(s_n^2 + 4s_n + 1) \quad \quad (19)$$

where

$$\left.\begin{aligned} s_n &= \frac{s}{\omega_0} \\ GB_n &= \frac{GB}{\omega_0} \end{aligned}\right\} \quad \quad (20)$$

Using Budak–Petrela technique,[6] one identifies

$$\left.\begin{aligned} P_1(s_n) &= s_n^2 + \left(\frac{1}{Q}\right) s_n + 1 \\ P_2(s_n) &= s_n(s_n^2 + 4s_n + 1) \end{aligned}\right\} \quad \quad (21)$$

$$\frac{dP_n}{d\left(\frac{1}{GB_n}\right)} = -\left.\frac{P_2(s_n)}{\frac{dP_1(s_n)}{ds_n}}\right|_{s_n = P_n}$$

$$\equiv -\frac{d\alpha_n}{d\left(\frac{1}{GB_n}\right)} + j\frac{d\beta_n}{d\left(\frac{1}{GB_n}\right)} \quad \quad (22)$$

where

$$P_n \equiv -\alpha_n + j\beta_n = -\frac{1}{2Q} + j\surd\{1 - (1/4Q^2)\} \quad \quad (23)$$

Using the above relations, and after some manipulation, one obtains

$$\frac{d\alpha_n}{d\left(\frac{1}{GB_n}\right)} = -\frac{1}{2Q}\left(4-\frac{1}{Q}\right) \quad . \quad . \quad . \quad . \quad . \quad (24)$$

$$\frac{d\beta_n}{d(1/GB_n)} = -\frac{1}{2\sqrt{\{1-(1/4Q^2)\}}}\left(4-\frac{1}{Q}\right)\left(1-\frac{1}{2Q^2}\right) \quad (25)$$

Finally, on substituting in the relations[6]

$$\frac{d\omega_0}{d(1/GB_n)} = \omega_0\left\{\alpha_n\frac{d\alpha_n}{d(1/GB_n)} + \beta_n\frac{d\beta_n}{d(1/GB_n)}\right\} \quad (26)$$

$$\frac{dQ}{d(1/GB_n)} = Q\left\{\left(\alpha_n - \frac{1}{\alpha_n}\right)\frac{d\alpha_n}{d(1/GB_n)} + \beta_n\frac{d\beta_n}{d(1/GB_n)}\right\} \quad (27)$$

it follows that

$$\frac{\Delta\omega_0}{\omega_0} = -2\left(1-\frac{1}{4Q}\right)\frac{\omega_0}{GB} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$\frac{\Delta Q}{Q} = 2\left(1-\frac{1}{4Q}\right)\frac{\omega_0}{GB} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Hence the actual values of ω_0 and Q are

$$\omega_{0a} = \omega_0\left\{1-2\left(1-\frac{1}{4Q}\right)\frac{\omega_0}{GB}\right\} \quad . \quad . \quad . \quad . \quad (30)$$

$$Q_a = Q\left\{1+2\left(1-\frac{1}{4Q}\right)\frac{\omega_0}{GB}\right\} \quad . \quad . \quad . \quad . \quad . \quad (31)$$

On examining the well known configuration shown in Fig. 1 by the same analysis, it is found that $\Delta\omega_0/\omega_0$ and $\Delta Q/Q$ are twice those given in eqns. 28 and 29, respectively. Therefore, for prescribed allowable changes in ω_0 and Q, for a specified Q factor and using the same OA, the new circuit given here can realise a notch frequency twice that which is realisable using the notch filter shown in Fig. 1.

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SIMPLE MULTIFREQUENCY-TONE DETECTOR*

Indexing terms: Digital filters, Signal detection, Signalling

An outline is given of a simple method of detecting digitally encoded multifrequency tones.

Signalling in telephone networks is often carried out by transmission in pulse-code-modulation (p.c.m.) form of a combination of tones from some given set. The touch-tone receiver is an example of a device which must detect such signals. Each dialling digit is represented by a pair of frequency components selected from the sets {697, 770, 852, 941} and {1209, 1336, 1477, 1633}, respectively. Frequency tolerance is ±2%. The detection method should be insensitive to the phase of the received signal, and must allow variations of up to 2·5 in the relative amplitudes of the components. The p.c.m. sampling rate is 8 kHz.

Present methods of reception are analogue filters, which require conversion of the p.c.m. signal to analogue form, and standard digital filters. The latter may be synthetised by the usual methods, with any desired frequency response, and have been used in commercial touch-tone receivers.[1] However, this method of detection involves needless complexity, because it requires, first, bandpass filtering of the signal eight times, and, secondly, rectification of the resulting components.

The approach to the problem used here is motivated by Fourier-analysis techniques. It is shown that the multi-frequency tones may be recognised in 12 ms.

The proposed receiver is shown in Fig. 1. ROM represents a stored sequence of approximately 100 sign bits. For example, the contents of the first (697 Hz) ROM may be represented by the sequence A (665666566656665666), denoting six '+', six '−', five '+', ...

G_A represents a sign gate and pulse counter. It changes the sign of samples passing through it, according to the sequence stored in ROM. G_B is similar, except that its first sign inversion occurs after counting a number of samples equal to the integral value of half the first ROM count. It deals normally with the remaining counts. In the example given above, G_B would operate according to the sequence B (365666566656665666).

The two sequences (A and B) are designed to ensure that the corresponding square waves are approximately one-quarter of a period out of phase, so that the phase of the observed signal is unimportant. The device of halving the first count saves the storage of a second complete sequence, but introduces an error into the translation, such that, for the highest frequency involved (1633 Hz), the two 'testing' square waves are only one-sixth of a period out of phase.

MAX selects the maximum of the absolute values of accumulators A and B. After 12 ms, the values of the outputs 1–4 are examined. The maximum of these corresponds to the low-frequency component present. A similar test on outputs 5–8 establishes the high-frequency component present.

The operation of the detector may be explained as follows: Consider a component of amplitude h at radian frequency α. Samples will be $x(m\tau) = h\cos(m\alpha\tau + \phi)$, where ϕ is the initial phase, and $\tau = 0{\cdot}000125$, the sampling interval. The 'testing' waveform is essentially a square wave of frequency ω, represented by the Fourier sine expansion

$$z(t) = \frac{4}{\pi}\left(\sin\omega t + \frac{\sin 3\omega t}{3} + \ldots\right)$$

Assume, first, that the testing samples are equivalent to $y(m\tau) = 4/\pi \sin m\omega\tau$. Then, after counting n samples, an

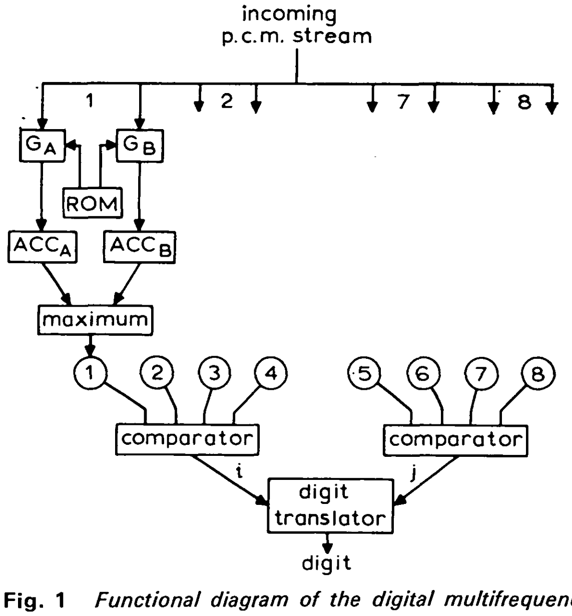

Fig. 1 *Functional diagram of the digital multifrequency-tone detector*

* Australian patent application PB 24, 1972