to Prof. Wagner[4] who pointed out the factors previously dropped.

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X BAND GUNN OSCILLATORS TRIGGERED BY BASEBAND GUNN DIODES

Indexing terms: Gunn oscillators, Amplitude modulation, Trigger circuits

A 155 μm-long Gunn diode produces output pulses of 1·5 V amplitude in resistive loading. These pulses are used to trigger a transistor multivibrator and an X band Gunn oscillator.

In high-bit-rate communication systems, Gunn diodes do not show the frequency limitation of junction devices.[1] It should therefore be possible to develop p.c.m. systems with considerably increased information-flow rates with respect to presently available techniques. The new baseband pulses have to be suitable for the modulation of microwave oscillators, if, for example, some frequency or phase-shift keying is to be employed for wideband communication. This letter reports successful amplitude modulation of X band Gunn oscillators by pulse signals from another Gunn diode in resistive loading.

The baseband diode had an interelectrode distance of $l = 155\ \mu m$, the resistivity was 1·806 Ωcm and its low-field resistance was 500 Ω. The diode could only be operated under bias conditions owing to the heat dissipation problems. The bias pulse of 100 Hz repetition frequency and 220 ns duration was first applied to an integrating circuit, so that the bias frequencies were much lower than the domain-signal frequencies. The bias voltage was set at such an amplitude that a succession of domain pulses occurred at the peak value of applied bias. The Gunn diode was earthed at the anode end. The bias voltage was applied via a 50 Ω coaxial cable, whose impedance formed the series load resistance to the diode.[2] The output pulses were then taken from across the diode and separated from the bias signals via a capacitive filter. The resulting Gunn-effect pulses of 1·5 V amplitude and 750 Mbit/s were first applied to the base terminal of a fast monostable transistor multivibrator,* which was successfully triggered by individual domain pulses. This technique provides a method of studying the occurrance of individual Gunn-diode domains without the use of averaging sampling techniques. The monostable multivibrator had a pulse time constant of 10 ms, which could, of course, easily be displayed on an ordinary oscilloscope.

Subsequently the baseband signals were applied to a low-*Q* factor X band coaxial Gunn-diode oscillator† together with a direct bias voltage of about 5 V. The d.c. bias was adjusted in such a way that, for a particular resonator-tuning-stub position, the diode was operated near the threshold for microwave emission. A small increase in bias voltage then produces a large increase in microwave output power. The output was finally applied to the input of a sampling oscilloscope,‡ which produced a signal trace representing the X band microwave. (Of course, the 10 GHz periods could not be displayed as this was outside the range of the sampling oscilloscope.) This trace showed very low (almost-zero) microwave power, except during production of the baseband pulses. As soon as the 750 Mbit/s pulses were terminated after about 80 ns, the microwave power returned to its original low level. The microwave pulse produced on the oscilloscope was about 13 mV in amplitude. Taking into account several mismatches at transitions employed, we estimate an output power of about 10 μW. This figure is admittedly very low, but could be improved by further work, in particular with a better cavity design and matching. For the coaxial resonator employed with a *Q* factor of about 50, it is not to be expected that the microwave will in fact follow the exact shape of the baseband pulses. Future experiments are to be performed with lower *Q* factors (up to resistive loading) in order to determine ultimate modulation speeds.

It was established by a range of tests that the wave shape observed was, in fact, the baseband-modulated X band output. Reducing the microwave diode bias voltage by only a few percent caused the 80 ns trace to disappear. Equally, an increase in this bias voltage produced high continuous emission of microwaves and no further pulse trace was observed. Detuning of the X band cavity with its tuning screw also caused the pulse trace to vanish. In fact, the quality of the performance was very sensitively dependent on tuning-screw position. A reduction in baseband bias voltage of about 10% eliminated any microwave pulse trace, as no baseband domain signals were then produced, as seen on the sampling scope when fed with an inductive current probe monitoring the baseband diode current. The same effect was also observed when the earthing connection of the baseband diode was removed, so that no current was able to flow through the diode.

It should be possible now with the results obtained to set up, for example, a phase-shift-keying system without the use of additional microwave switches. The system would have to contain two microwave Gunn diodes in very low *Q* factor circuitry, phase locked to each other by a low background output and switched by baseband signals from long Gunn diodes. The phase shifting is obtained by employing a different length of cable for the connection from each of the microwave diodes to a common output terminal.

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* See Ferranti handbook, June 1969, p. 17
† Gunn diodes from Plessey, TEO2B
‡ Tektronix

LOSSLESS MULTIPORTS WITH TERMINATIONS IN SYNTHESIS PROBLEMS

Indexing term: Network synthesis

Use of lossless multiports with terminations in the synthesis of multivariable positive real functions is discussed. An example is included to illustrate the exploitation of the degrees of freedom in multiterminated-lossless-multiport synthesis.

Lossless networks with suitable terminations have been used in various network problems. Darlington showed[1] that, if a real rational function of a single variable can be realised by terminating a lossless 2-port by a resistor, the function is positive real. Similarly, a special case of Koga's recent general results is that any multivariable reactance function, bilinear in the variable p_n, can be realised by terminating a particular lossless 2-port by p_n. The extension of this result to that of any multivariable positive real function $Z(p_1, p_2, \ldots, p_n)$ follows after generating first the multivariable reactance function $Z_1(p_1, p_2, \ldots, p_{n+1})$ in $(n+1)$ variables along the lines of Koga[2] and Rao and Newcomb.[3] Hence, if

$$Z(p_1, p_2, \ldots, p_n) = \frac{m_1(p_1, p_2, \ldots, p_n)+n_1(p_1, p_2, \ldots, p_n)}{m_2(p_1, p_2, \ldots, p_n)+n_2(p_1, p_2, \ldots, p_n)}$$

then

$$Z_1(p_1, p_2, \ldots, p_{n+1}) = \frac{m_1+p_{n+1}n_1}{p_{n+1}m_2+n_2}$$

where m_i $(i = 1, 2)$ are even polynomials and n_j $(j = 1, 2)$ are odd polynomials in the variables $p_1, p_2, \ldots, p_n$. Putting $p_{n+1} = 1$, makes $Z_1 = Z$, and, as the reactance function Z_1 can be realised by terminating a lossless 2-port by p_{n+1}, Z can also be realised by terminating with a unit resistor the same lossless 2-port. This result is the multi-variable counterpart of Darlington's original result in the single-variable case.

Often, the realisation of the matrix in the n variables $p_1, p_2, \ldots, p_n$ characterising the lossless 2-port requires considerable manipulative effort. For 2-variable reactance functions, Ansell,[4] Koga,[5] and, very recently, Rao[6] used a multiterminated lossless network. Ansell's synthesis technique involves the development of methods of finding the relevant polynomial decomposition, which is a difficult problem. Furthermore, Ansell's procedure is limited in scope, as it cannot be used to realise all 2-variable reactance functions. Koga's realisation often includes gyrators in the lossless multiport. The elimination of the gyrators demands nonminimal realisation, which might be undesirable. Rao's final realisation is similar to Koga's but the procedure is different. Here, terminations similar to Ansell's are used, which sometimes leads to the absence of gyrators in the minimal synthesis, and the approach suggested avoids the polynomial-decomposition problem encountered by Ansell. The procedure is best illustrated by an example (that chosen by Ansell and Koga, for convenience in comparison).

Example: Realise

$$Z(p_1, p_2) = \frac{1}{Y(p_1, p_2)} = \frac{4p_2^2+5p_2p_1+p_1^2+4}{5p_2^2p_1+p_1^2p_2+p_2+p_1} \quad (1)$$

as an impedance function.

If $Y = \{y_{ij}(p_1)\}$ $i = 1, 2, 3$ $j = 1, 2, 3$ is the admittance matrix of the lossless 3-port terminated by p_2 and $1/p_2$ at the output ports, as in Ansell's configuration, the driving-point admittance is

$$Y(p_1, p_2) = \frac{p_2^2\left(y_{11}-\frac{y_{12}y_{21}}{y_{22}}\right)+p_2\left(y_{11}y_{33}+\frac{y_{11}}{y_{22}}-\frac{y_{11}y_{23}y_{32}}{y_{22}}-\frac{y_{12}y_{21}y_{33}}{y_{22}}+\frac{y_{12}y_{31}y_{23}}{y_{22}}+\frac{y_{21}y_{13}y_{32}}{y_{22}}-y_{13}y_{31}\right)+\left(\frac{y_{11}y_{33}}{y_{22}}-\frac{y_{13}y_{31}}{y_{22}}\right)}{p_2^2+p_2\left(y_{33}+\frac{1}{y_{22}}-\frac{y_{23}y_{32}}{y_{22}}\right)+\frac{y_{33}}{y_{22}}} \quad (2)$$

The losslessness of the 3-port implies that

$$\left.\begin{aligned} y_{12}(p_1) &= -y_{21}(-p_1)\\ y_{13}(p_1) &= -y_{31}(-p_1)\\ y_{23}(p_1) &= -y_{32}(-p_1)\end{aligned}\right\} \quad \ldots\ldots \quad (3)$$

Moreover, comparison of coefficients results in the following five equations, implying a total of eight equations in the nine unknown elements of $\{y_{ij}(p_1)\}$:

$$y_{33}+\frac{1}{y_{22}}-\frac{y_{23}y_{32}}{y_{22}} = \frac{5}{4}p_1 \quad \ldots\ldots \quad (4)$$

$$\frac{y_{33}}{y_{22}} = \frac{p_1^2+4}{4} \quad \ldots\ldots \quad (5)$$

$$y_{11}-\frac{y_{12}y_{21}}{y_{22}} = p_1 \quad \ldots\ldots \quad (6)$$

$$y_{11}y_{33}+\frac{y_{11}}{y_{22}}-\frac{y_{11}y_{23}y_{32}}{y_{22}}-\frac{y_{12}y_{21}y_{33}}{y_{22}}+\frac{y_{12}y_{31}y_{23}}{y_{22}}+\frac{y_{21}y_{13}y_{32}}{y_{22}}-y_{13}y_{31} = \frac{p_1^2+1}{4} \quad (7)$$

$$\frac{y_{11}y_{33}}{y_{22}}-\frac{y_{13}y_{31}}{y_{22}} = \frac{p_1}{4} \quad \ldots\ldots \quad (8)$$

Eight equations in nine unknowns implies a degree of freedom. A systematic way of utilising the degree of freedom is by considering eqns. 4 and 5 resulting from the denominator

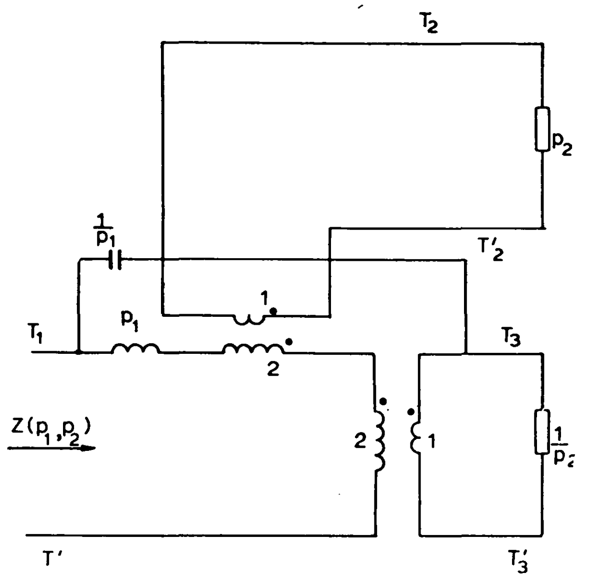

Fig. 1

coefficients of $Y(p_1, p_2)$ in eqn. 2. From eqns. 4 and 5, one obtains

$$y_{22}\left(\frac{p_1^2+4}{4}\right)+\frac{1}{y_{22}}-\frac{y_{23}y_{32}}{y_{22}} = \frac{5}{4}p_1 \quad \ldots \quad (9)$$

For this type of configuration, the degree of freedom can be chosen to be

$$y_{23}y_{32} = xy_{22}^2 \quad \ldots\ldots \quad (10)$$

where x is evaluated such that y_{22} is an odd rational function in p_1.

Using eqns. 9 and 10, and solving for an odd rational y_{22}, one obtains $x = +1$, implying that

$$y_{22} = y_{32} = y_{23} = \frac{4}{p_1} \quad \ldots\ldots \quad (11)$$

Using eqn. 5,

$$y_{33} = \frac{p_1^2+4}{p_1}$$

Inspecting the degree of p_1 in $Y(p_1, p_2)$, one writes

$$y_{11}(p_1) = ap_1 + \frac{b}{p_1}$$

$$y_{12} = c + dp_1 + \frac{e}{p_1}$$

$$y_{13}(p_1) = f + gp_1 + \frac{h}{p_1}$$

(If gyrators are absent in the lossless 3-port, $c = f = 0$.) Substituting in eqns. 6, 7 and 8 and using eqns. 3 and 11, one obtains, as a possible solution,

$$Y(p_1, p_2) = \begin{bmatrix} \frac{p_1^2+1}{p_1} & \frac{2}{p_1} & \frac{p_1^2+2}{p_1} \\ \frac{2}{p_1} & \frac{4}{p_1} & \frac{4}{p_1} \\ \frac{p_1^2+2}{p_1} & \frac{4}{p_1} & \frac{p_1^2+4}{p_1} \end{bmatrix}$$

The complete realisation is shown in Fig. 1, which is the same

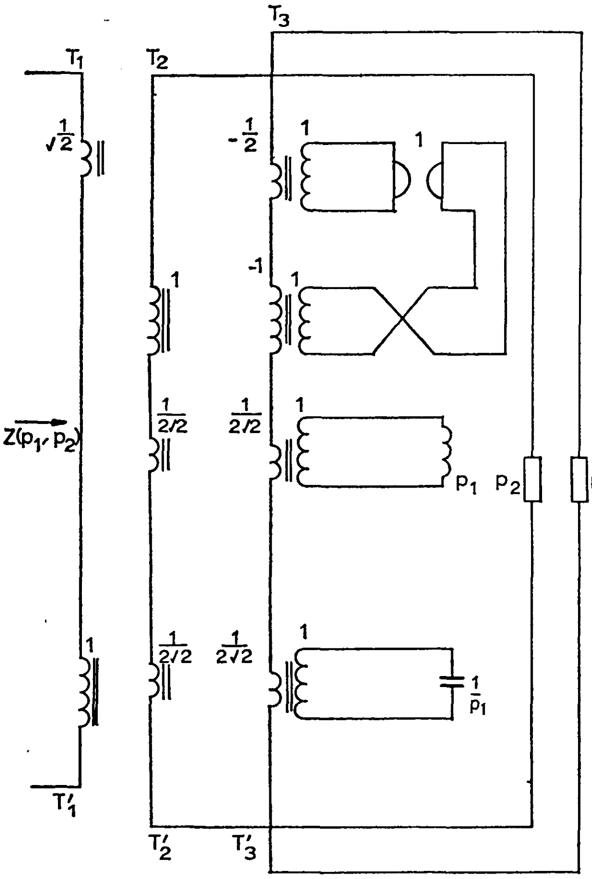

Fig. 2

as Ansell's, arrived at here using a very different and simpler approach. Koga and Rao's realisations of the same function is shown in Fig. 2, for comparison. In Fig. 2, it will be noticed that gyrators are present in the lossless 3-port.

Higher-degree functions can also be solved by this approach. In general, it can be shown that the number of degrees of freedom for the complete realisation of a function $Z(p_1, p_2)$ is $m(m-1)/2$, where $m = \deg_{p_2} Z$, when p_2 is the variable used in terminating the lossless multiport $\{y_{ij}(p_1)\}_{(m+1)\times(m+1)}$. Note that, if $Z(p_1, p_2)$ is bilinear in any one of the variables, the degree of freedom is zero.

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FREQUENCY MULTIPLICATION USING THE PROPERTIES OF AN ANGLE-MODULATED WAVE

Indexing terms: Frequency multipliers, Phase modulation

Frequency multiplication by an integral number is inherent in the spectrum of an angle-modulated wave. A system using this property is described, which enables very high orders of frequency multiplication to be achieved, with minimum circuit complexity and maximum tolerance to component variation.

It is well known that the spectrum of a wave modulated in phase or frequency by a repetitive signal contains lines separated by the repetition frequency. These extend, in theory, over all frequencies, but, in practice, the spectrum can usually be regarded as bandlimited to a range not much greater than the deviation from the carrier frequency.

One method of using this property to achieve frequency multiplication by means of frequency modulation* is illustrated in Fig. 1. Assuming that multiplication by a factor $2n$ of an input signal at f hertz is required, filters A and B select, respectively, the nth-ordered sideband above and below the carrier frequency f_c. The difference, $2nf$ hertz, is selected by a mixer, ideally a product detector, if unwanted high-frequency components are to be minimised.

This technique uses sinusoidal modulation, which does not give the optimal spectral distribution. Some difficulties in implementation can also arise, but nevertheless it is possible to achieve multiplications of the order of 20 in a single stage.

The method proposed by the author is illustrated in Fig. 2. Some of the implementation difficulties are removed, filter B has been eliminated and filter C now selects the wanted component $2nf$ hertz from the adjacent sidebands. Since filter C operates at a lower frequency than filters A and B, a lower Q factor is permissible. Further, filter A can be allowed to pass some sidebands adjacent to $f_c + nf$, again allowing a reduction in Q factor. This has the additional advantage that the tight limits otherwise required on oscillator carrier frequency may be relaxed.

These advantages are enhanced if the modulating waveform is chosen to achieve optimal spectral distribution. This normally implies a rectangular modulation, the repetition frequency being f as before. The deviation may then be set so that the power density is greatest at $f_c \pm nf$. If filter C admits components adjacent to $2nf$, these will appear almost entirely as amplitude modulation on a carrier of frequency nf, and the modulation can be removed by a simple limiter.

* From a suggestion by M. G. T. Hewlett, Plessey Radar, Cowes, Isle of Wight