Example 5: Given

$$D_5(s) = (s^2+2s+1)(s^2+3s-2)$$

$$\textit{test:}\ \beta_2 = -2 \not> 0.$$

The test fails and $D_5(s)$ is not $RC:RL$ decomposable, since (s^2+3s-2) does not have complex conjugate zeros.

Three necessary conditions for the $RC:RL$ decomposition of a 4th-order polynomial $D(s)$ have been derived. When taken simultaneously, the three conditions become necessary and sufficient. Since the conditions are given in terms of the quadratic factor Qs and coefficients α_i and β_i, there is some computational advantage, as shown in examples 2 and 4.

In some cases the application of the conditions may be accomplished by a simple inspection, as shown in examples 3 and 5. It is believed the conditions will be helpful in synthesis since they provide limits on the Qs and coefficients of the quadratic factors.

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NOVEL APPROACH TO SYNTHESIS OF MULTIVARIABLE POSITIVE REAL FUNCTIONS

The possibility of synthesis of multivariable positive real functions, bilinear in one variable, using a simple configuration is demonstrated. The result is illustrated by two examples.

The realisation of multivariable positive real functions, bilinear in one variable, has been a subject of considerable interest.[1, 2] The methods known are usually quite cumbersome. Here, a method will be presented which is easy to implement, whenever it is possible. Consider a 2-port, characterised by the chain matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

where each element of the matrix is a function of the complex variables $p_1, p_2, \ldots, p_{n-1}$. If this 2-port is terminated by an impedance p_n, the driving-point impedance at the remaining pair of terminals is known to be

$$Z(p_1, p_2, \ldots, p_n) = \frac{Ap_n+B}{Cp_n+D} \qquad (1)$$

Any multivariable positive real function, bilinear in the variable p_n, can be written in the form

$$Z(p_1, p_2, \ldots, p_n) = \frac{ap_n+b}{cp_n+d} \qquad (2)$$

where a, b, c and d are polynomials of the $n-1$ variables $p_1, p_2, \ldots, p_{n-1}$. Dividing the numerator and denominator by a polynomial $P(p_1, p_2, \ldots, p_{n-1})$, yet to be determined, the above expression can be written as

$$Z(p_1, p_2, \ldots, p_n) = \frac{(a/P)p_n+(b/P)}{(c/P)p_n+(d/P)} \qquad (3)$$

Comparing eqns. 1 and 3, the chain matrix of the 2-port terminated by p_n can be identified as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} a/P & b/P \\ c/P & d/P \end{bmatrix} \qquad (4)$$

If the above chain matrix is positive real,[3] synthesis is definitely possible.[4] In many cases, the 2-port can be realised by inspection. For reciprocal networks, $AD-BC = 1$,[5] which implies that $P^2 = ad-bc$; this enables one to determine P uniquely. For nonreciprocal 2-ports, P is not unique, and has to be determined by inspection so as to yield a realisable 2-port.

Next, it will be shown that $ad-bc$ is a perfect square if, and only if, the 2-port under consideration is reciprocal. Suppose the open-circuit impedance matrix characterising the 2-port, shown in Fig. 1 is

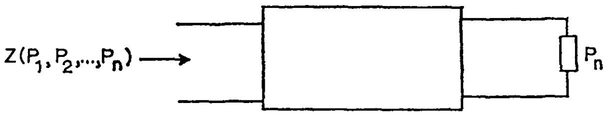

Fig. 1

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

where $z_{11}, z_{12}, z_{21}, z_{22}$ are functions of $p_1, p_2, \ldots, p_{n-1}$.

Then,

$$Z(p_1, p_2, \ldots, p_n) = z_{11} - \frac{z_{12}z_{21}}{z_{22}+p_n}$$

Then

$$\frac{\partial Z}{\partial P_n} = \frac{z_{12}z_{21}}{(z_{22}+p_n)^2} \qquad (5)$$

Also, using eqn. 2,

$$\frac{\partial Z}{\partial P_n} = \frac{ad-bc}{(Cp_n+d)^2} \qquad (6)$$

Equating eqns. 5 and 6, it follows that $ad-bc$ is a perfect square if, and only if, $z_{12} = z_{21}$. The result arrived at is illustrated by two examples.

Example 1: Consider the positive real function

$$Z(p_1, p_2) = \frac{p_1p_2+4p_1+p_2}{9p_1+p_2+1}$$

$$= \frac{(p_1+1)p_2+4p_1}{p_2+(9p_1+1)}$$

It follows that

$$a = p_1+1 \qquad b = 4p_1 \qquad c = 1 \qquad d = (9p_1+1)$$

so that

$$p = 3p_1+1$$

Therefore,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} (p_1+1)/(3p_1+1) & 4p_1/(3p_1+1) \\ 1/(3p_1+1) & (9p_1+1)/(3p_1+1) \end{bmatrix}$$

The corresponding open-circuit impedance matrix

$$\begin{bmatrix} p_1+1 & 3p_1+1 \\ 3p_1+1 & 9p_1+1 \end{bmatrix}$$

of a reciprocal lossy 2-port can be easily realised. Fig. 2*a* gives the complete realisation of $Z(p_1, p_2)$.

Example 2: Consider the positive real function

$$Z(p_1, p_2) = \frac{4p_1^3p_2+4p_1^2+8p_1p_2+4}{4p_1^3+5p_1^2p_2+4p_1+p_2}$$

In this case, $Z(p_1, p_2)$ can be written as

$$Z(p_1, p_2) = \frac{P_2'(4p_1^2+4)+(4p_1^3+8p_1)}{P_2'(4p_1^3+4p_1)+(5p_1^2+1)}$$

where $P_2' = 1/P_2$.

In this case, the termination has been chosen as $1/p_2$ instead of p_2, so as to make the 2-port physically realisable without generalised gyrators.[6]

Then

$$a(p_1) = 4(p_1^2+1) \qquad b(p_1) = 4p_1(p_1^2+2)$$

$$c(p_1) = 4p(p_1^2+1) \qquad d(p_1) = (5p_1^2+1)$$

$ad-bc = -16p_1^6-28p_1^4-8p_1^2+4$ is not a perfect square. By inspection, the polynomial $P(p)$ is chosen to be

$$P(p_1) = 4p_1^3+2p_1^2+4p_1+2$$

From this, the chain matrix of the 2-port is obtained. The

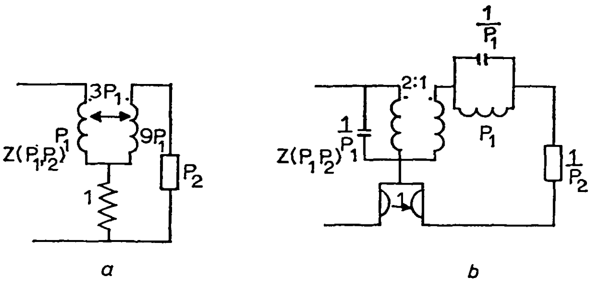

Fig. 2

open-circuit impedance matrix of the foregoing 2-port is

$$\begin{bmatrix} 1/p_1 & -1+(1/2p_1) \\ 1+(1/2p_1) & (1/4p_1)+p_1/(p_1^2+1) \end{bmatrix}$$

The complete realisation is shown in Fig. 2*b*.

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DISCREPANCIES IN WAVEGUIDE-ATTENUATION EXPRESSIONS

Waveguide-attenuation expressions given in the literature disagree by as much as 0·2%, which exceeds the precision customarily associated with waveguide computations. In addition, the UK DEF standard waveguide-attenuation expression is shown to be misleading. A rectangular waveguide is specifically treated and standard waveguide-attenuation expressions are suggested.

Waveguide attenuation is customarily stated as a constant or constants times an expression involving the internal waveguide dimensions and ratios of operating/cutoff frequencies or wavelengths. Various numerical constants given in the literature disagree and at least one standards authority gives a misleading expression for attenuation. This letter points out these discrepancies and suggests that an attenuation expression be adopted that is at least accurate to within the number of significant figures customarily associated with standard waveguide computations.

For simplicity, the discussion will be limited to mechanically perfect hollow metal rectangular waveguides.* To the author's knowledge, the only complete expression for the propagation constant γ of a rectangular waveguide propagating power in the TE_{mn} mode is that published by Käch (Reference 1, eqn. 43):

$$\gamma = \sqrt{\left\{\gamma_0^2+(1-j)\frac{\mu_m}{\mu}\delta\left(\frac{m^2\,2\pi^2}{a^3}+\frac{k^2}{b}\right)\right\}} \quad . \quad . \quad (1)$$

where $\delta = \sqrt{(\eta f \mu_m \sigma_m)}$ is the skin depth, $k = \omega\sqrt{(\mu\varepsilon)}$ is the wave number, the subscript m refers to the metal involved, and $\gamma_0 = k^2-(m\pi/a)^2$, following the notation first introduced by Kerns and Hedberg.[2] The attenuation constant α is found from eqn. 1 by separating it into its real and imaginary parts and approximating for frequencies lower than, equal to, and greater than, the cutoff frequency f_c (Reference 1, eqns. 48 and 50). Above f_c for the TE_{0m} mode,

$$\alpha = \sqrt{\left(\frac{\pi\mu_m}{2\mu^2\,c\sigma_m}\right)}\left[\frac{\sqrt{m}}{b\sqrt{a}}\,\frac{(f/f_c)^{\frac{1}{2}}+(2b/a)(f/f_c)^{-\frac{1}{2}}}{\sqrt{\{(f/f_c)^2-1\}}}\right] \quad (2)$$

The quantity $(\pi\mu_m/2\mu^2\,c\sigma_m)^{\frac{1}{2}} = (R_s/\eta)(c/2f)^{\frac{1}{2}}$, where η is the TEM wave impedance and $R_s = 1/\sigma_m\delta$ is the surface resistivity. This quantity is customarily normalised to free space and pure copper conductors:

$$\sqrt{(\pi\mu_m/2\mu^2\,c\sigma_m)} = (R_s/\eta)(c/2f)^{\frac{1}{2}} = K\sqrt{\varepsilon_r}\sqrt{(\rho/\rho_0)} \quad (3)$$

where ε_r is the relative permittivity, ρ is the resistivity of the wall material (now restricted to being nonmagnetic), ρ_0 is the resistivity of pure copper, and K is a constant as indicated by the equation. Eqn. 2 then becomes (for the TE_{01} mode)

$$\alpha = K\sqrt{\varepsilon_r}\sqrt{\frac{\rho}{\rho_0}}\left[\frac{1}{b\sqrt{a}}\,\frac{(f/f_c)^2+(2b/a)}{\sqrt{(f/f_c)}\sqrt{\{(f/f_c)^2-1\}}}\right] \quad . \quad (4)$$

where a and b are, respectively, the larger and smaller internal waveguide dimensions. The units of α depend on the units chosen within the equation and on the constants used in determining K. If a and b are in metres so that eqn. 4 is dimensionally selfconsistent, K is approximately $7{\cdot}36\times10^{-5}$ and α is in decibels per metre. The values of K stated by (or derived from the work of) six different authors are given in Table 1. As can be seen from Table 1, the Ks disagree in the second and third decimal places. Careful workers in standards avoid numerical values of K by computing constants by individual cases.[9]

Table 1 VALUES OF K

Reference	K
IEC[3]	7·35962
Moreno[4]	7·35112
Ramo, Whinnery, and Van Duzer[5]	7·36222
Matthaei, Young, and Jones[6]	7·35614
Lewin[7]	7·36222
Cox and Rupp[8]	7·33355

$\rho_0 = 1{\cdot}7241\times10^{-8}$ Ωm for Cu, 1 Neper = 8·68588 dB, $\eta = 120\pi$, $c = 2{\cdot}997925\times10^8$ m/s

The DEF standard[10] expression for α is

$$\alpha = 0{\cdot}421\left[\frac{(f/f_c)^2+1}{\sqrt{(f/f_c)}\sqrt{\{(f/f_c)^2-1\}}}\right]\alpha_0 \quad . \quad . \quad . \quad (5)$$

Table 2 STANDARD WAVEGUIDES FOR WHICH $a \neq 2b$

IEC 153 IEC-R	RCSC WG	a/b	$2b/a$
32	10	2·1194	0·9437
35		2·2498	0·8897
	11	2·1141	0·9460
41		2·2503	0·8888
48	12	2·1468	0·9316
70	14	2·2058	0·9067
84	15	2·4547	0·8148
100	16	2·2500	0·8888
220	20	2·4706	0·8095

* A parallel situation exists for circular waveguides; the conclusions reached and suggestions made here are implicitly valid for the circular-waveguide case