Fractional Order Two Port Network oscillator with Equal Order

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***Abstract*—Most of electric circuits can be viewed as a two port network with two terminals defined as input and output ports. In this paper, two different concepts are combined together which are the two port network concept and the fractional calculus to design a general fractional order two port network with equal order. An oscillator case study with three impedances structure has been presented. The three impedances are two equal order fractional capacitors and a resistor. Two different two port network are studied which are Op-amp based circuit and non-ideal gyrator circuit. The general oscillation frequency and condition for each case have been derived and discussed numerically using Matlab. Spice simulations are presented for some cases to validate the proposed idea where the fractional order oscillator has more degrees of freedom than the integer order.**

Keywords*— *Two port network; oscillators; fractional calculus.

I. INTRODUCTION

The fractional calculus has been known since the integer calculus thirty decades ago [1]. Nowadays, researchers pay much attention to it because its use in the field of science and engineering [2-7]. The most commonly used definition for the general fractional order derivative is the Riemann-Liouville (RL) shown in (1),

$$D_{t_0}^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_{t_0}^{t} f(u)\,(t-u)^{m-\alpha-1}\,du \qquad (1)$$

where m is the integer such that $(m-1) < \alpha < m$. Another old valuable technique in circuit theory is the two-port network concept [8-9] because most of electric circuits and electronic modules have at least two ports, namely input and output ports as shown in Fig.1.The parameters of a two-port network completely describes its behavior in terms of the voltage and current at each port. The knowledge of the two-port network parameters enables users to treat it as a black-box placed within a larger network. It will help dissolving large circuits into smaller ones and dealing with each one separately. There exist six different ways to describe the relationship between the input, output voltages and currents depending on which two of the four variables are given. The parameters used in order to describe a two-port network are the following: impedance matrix Z, admittance matrix Y, hybrid matrix H, inverse hybrid matrix G and transmission matrix A. The transmission parameters, which are well-suited for networks with feedback, are used through this paper and can be described by (2),

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [A] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad (2)$$

The concept could be utilized in lots of basic electronic circuits such as amplifiers, filters and oscillators [10-13]. The aim of this paper is to utilize the two techniques together in one of basic building blocks of electronic circuits which is oscillators to get the advantages of both techniques the extra degree of freedom proposed by the fractional order parameter and the simplicity of design with the two port network concept without getting through internal currents and voltages of the used device (black box).

This paper is organized as follows; section II presents the general configuration of the fractional order common-B two-port network oscillator. Section III discusses practical example with numerical discussions. section IV illustrates the simulation results for different cases and finally the conclusion.

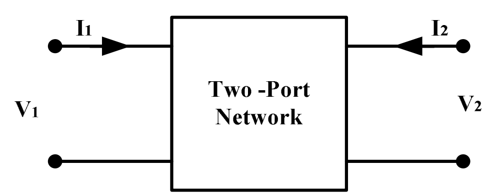

Fig. 1. Two port network

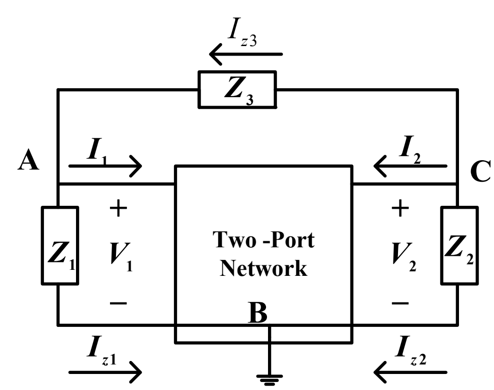

Fig. 2. Common- B topology

TABLE I. GENERAL OSCILLATION PARAMETERS

#	impedances			General characteristics for each case
	$\mathbf{Z_1}$	$\mathbf{Z_2}$	$\mathbf{Z_3}$	
1	$\frac{1}{s^\alpha C_1}$	R	$\frac{1}{s^\alpha C_2}$	$s^{2\alpha} + \left(\frac{a_{12} + a_{11} R}{C_2 R a_{12}} + \frac{a_{12} + (a_{11} + a_{22} - (1+A))R}{C_1 R a_{12}}\right) s^\alpha + \frac{a_{22} + a_{21} R}{C_1 C_2 R a_{12}} = 0$
				$a_{12} = -\frac{(C_2 R(a_{11}-1)(1-a_{22}) + C_1 R\, a_{11})\omega^\alpha \cos(0.5\alpha\pi) + a_{22} + R\, a_{21}}{C_1 C_2 R \omega^{2\alpha} \cos(\alpha\pi) + (C_2(R\, a_{21} + 1) + C_1)\omega^\alpha \cos(0.5\alpha\pi)}$ $a_{12} = -\frac{(C_2 R(a_{11}-1)(1-a_{22}) + C_1 R\, a_{11})}{2 C_1 C_2 R \omega^\alpha \cos(0.5\alpha\pi) + C_2(R\, a_{21}+1) + C_1}$
				$C_1 C_2 R(C_2 R(a_{11}-1)(1-a_{22}) + a_{11} C_1 R)\omega^{2\alpha} + 2 C_1 C_2 R(a_{22} + R\, a_{21})\omega^\alpha \cos(0.5\alpha\pi) + (a_{22} + R\, a_{21})(C_2(R\, a_{21}+1) + C_1) = 0$
2	$\frac{1}{s^\alpha C_1}$	$\frac{1}{s^\alpha C_2}$	R	$s^{2\alpha} + \left(\frac{a_{12} + a_{11} R}{C_2 R a_{12}} + \frac{a_{12} + a_{22} R}{C_1 R a_{12}}\right) s^\alpha + \frac{a_{11} + a_{22} + a_{21} R - (1+A)}{C_1 C_2 R a_{12}} = 0$
				$a_{12} = -\frac{(C_2 R a_{22} + C_1 R a_{11})\omega^\alpha \cos(0.5\alpha\pi) + (a_{11}-1)(1-a_{22}) + R\, a_{21}}{C_1 C_2 R \omega^{2\alpha} \cos(\alpha\pi) + (C_1 + C_2)\omega^\alpha \cos(0.5\alpha\pi) + a_{21}}$ $a_{12} = -\frac{(C_2 R a_{22} + C_1 R a_{11})}{2 C_1 C_2 R \omega^\alpha \cos(0.5\alpha\pi) + C_1 + C_2}$
				$C_1 C_2 R(C_2 R a_{22} + a_{11} C_1 R)\omega^{2\alpha} + 2 C_1 C_2 R\big((a_{11}-1)(1-a_{22}) + R\, a_{21}\big)\omega^\alpha \cos(0.5\alpha\pi) + C_2\big((1-a_{22})(a_{11} + R\, a_{21} - 1)\big)$ $+ C_1\big((a_{11}-1)(1-a_{22} - R\, a_{21})\big) = 0$
3	R	$\frac{1}{s^\alpha C_1}$	$\frac{1}{s^\alpha C_2}$	$s^{2\alpha} + \left(\frac{a_{12} + a_{22} R}{C_2 R a_{12}} + \frac{a_{12} + (a_{11} + a_{22} - (1+A))R}{C_1 R a_{12}}\right) s^\alpha + \frac{a_{11} + a_{21} R}{C_1 C_2 R a_{12}} = 0$
				$a_{12} = -\frac{(C_2 R(a_{11}-1)(1-a_{22}) + C_1 R\, a_{22})\omega^\alpha \cos(0.5\alpha\pi) + a_{11} + R\, a_{21}}{C_1 C_2 R \omega^{2\alpha} \cos(\alpha\pi) + (C_2(R\, a_{21}+1) + C_1)\omega^\alpha \cos(0.5\alpha\pi)}$ $a_{12} = -\frac{C_2 R(a_{11}-1)(1-a_{22}) + C_1 R\, a_{22}}{2 C_1 C_2 R \omega^\alpha \cos(0.5\alpha\pi) + C_2(R\, a_{21}+1) + C_1}$
				$C_1 C_2 R(C_2 R(a_{11}-1)(1-a_{22}) + a_{22} C_1 R)\omega^{2\alpha} + (a_{11} + R\, a_{21})\big(2 C_1 C_2 R \omega^\alpha \cos(0.5\alpha\pi) + (C_2(R\, a_{21}+1) + C_1)\big) = 0$

II. FRACTIONAL ORDER OSCILLATOR

The oscillator shown in Fig.2 is a two port network based with three impedance structure. The employed impedances through this paper are two equal order fractional capacitors and a resistor.

The conventional capacitor can be considered a special case from the fractional order capacitor whose impedance $Z(s) = 1/(Cs^\alpha)$ when $\alpha = 1$.The implementation of fractional order capacitor proposed in [14] is used through this work.

The general characteristic equation of this oscillator could be written as

$$a_{11} Z_2 (Z_3 + Z_1) + a_{12}(Z_1 + Z_2 + Z_3) + a_{21} Z_1 Z_2 Z_3 + a_{22} Z_1 (Z_2 + Z_3) - (1+A) Z_1 Z_2 = 0 \tag{3}$$

where $A = a_{11} a_{22} - a_{12} a_{21}$ is the determinant of the transmission matrix of the two port network. Three possible topologies could be extracted from these combinations.

Table I shows the impedance combination with the characteristic equation for each case from which; investigation for each case is presented. The characteristic equation for each oscillator is used to generate oscillation frequency and condition according to theorem presented in [3]. Taking an example the characteristic equation of case 1 and according to [3] since the system is oscillatory then $s = \omega e^{\pm j\pi/2}$ must be two roots of the characteristic equation. The necessary condition for oscillation can be written in (4a) and (4b)

$$\omega^{2\alpha} \cos(\alpha\pi) + \left(\frac{R\, a_{21} + 1}{C_1 R} + \frac{(a_{11}-1)(1-a_{22})}{C_1 a_{12}} + \frac{1}{C_2 R} + \frac{a_{11}}{C_2 a_{12}}\right)\omega^\alpha \cos\left(\frac{\alpha\pi}{2}\right) + \frac{a_{22} + R\, a_{21}}{C_1 C_2 R a_{12}} = 0 \tag{4a}$$

$$2\omega^\alpha \cos\left(\frac{\alpha\pi}{2}\right) + \left(\frac{R\, a_{21} + 1}{C_1 R} + \frac{(a_{11}-1)(1-a_{22})}{C_1 a_{12}} + \frac{1}{C_2 R} + \frac{a_{11}}{C_2 a_{12}}\right) = 0 \tag{4b}$$

The condition of oscillation could be controlled by design parameters which could be any of the three impedances or the two port network elements. For the three cases a_{12} is chosen to be the design parameter. By eliminating a_{12} from both of the above equations; the general equation for the frequency of oscillation could be deduced. Similar procedure could be done for the other two cases and Table I summarizes the general oscillation frequency and condition for the three cases with the use of two equal order fractional capacitors.

III. CASE STUDIES

In this section, the transmission matrices for some practical two port networks are studied to achieve oscillations.

A. Op-amp based two port network

The transmission matrices of op-amp based two port network structure shown in Fig.3 can be written as

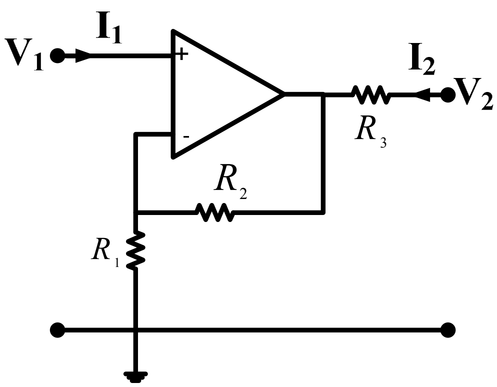

Fig. 3. Op-amp based two port

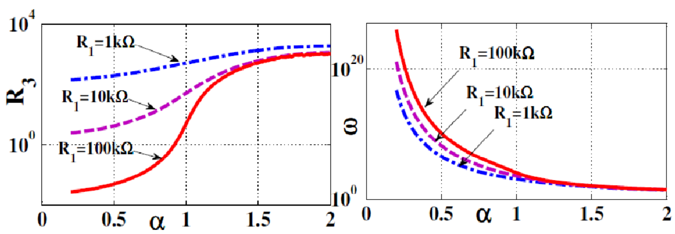

Fig. 4. Numerical study for op-amp with different values of R_1

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \frac{R_1}{R_1+R_2} & \frac{R_1R_3}{R_1+R_2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} k & kR_3 \\ 0 & 0 \end{bmatrix} \tag{5}$$

where $k = R_1/(R_1 + R_2)$. By substituting in Table I with (5); Table II shows the oscillation frequency and condition for each case with the use of op-amp network from which it could be concluded that only case 3 can achieve oscillations. For case 1 the condition of oscillation could be achieved but there will be no frequency and case 2 need negative resistance to be implemented. Applying equal C design with $R_2 = R$ and different values of R_1 to control the oscillation frequency and condition; Fig.4 shows that increasing R_1 could increase the obtained frequency range. It shows also the inverse relation between the condition of oscillation and the increase of R_1. The frequency is decreasing with the increase of the fractional order parameter α while the condition of oscillation increases with α.

B. Non-ideal gyrator circuit

A gyrator is a two-port nonreciprocal device with the property that the input impedance is proportional to the reciprocal of the load impedance [15]. The Non ideal gyrator circuit shown in Fig.5 was introduced by Sedra [16] but with equal resistors. To achieve non-ideal gyrator circuit; different resistors are used. The transmission matrix of a non-ideal gyrator circuit can be written as follows

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & R_1 \\ \frac{1}{R_2} & 0 \end{bmatrix} \tag{6}$$

TABLE II. OSCILLATION PARAMETERS USING OP-AMP

#	R_3	Freq.
1	$\frac{-(C_2R(k-1)+C_1R\,k)/k}{2C_1C_2R\omega^{\alpha}\cos(0.5\alpha\pi)+C_2+C_1}$	$\omega^{2\alpha}=0$
2	$\frac{-\,C_1R}{2C_1C_2R\omega^{\alpha}\cos(0.5\alpha\pi)+C_1+C_2}$	$kC_2(C_1R)^2\omega^{2\alpha} + 2C_1C_2R(k-1)\omega^{\alpha}\cos(0.5\alpha\pi) + (k-1)(C_2+C_1)=0$
3	$\frac{C_2R(1-k)/k}{2C_1C_2R\omega^{\alpha}\cos(0.5\alpha\pi)+C_2+C_1}$	$C_1(C_2R)^2(k-1)\omega^{2\alpha} + k\left(2C_1C_2R\omega^{\alpha}\cos(0.5\alpha\pi) + (C_2+C_1)\right)=0$

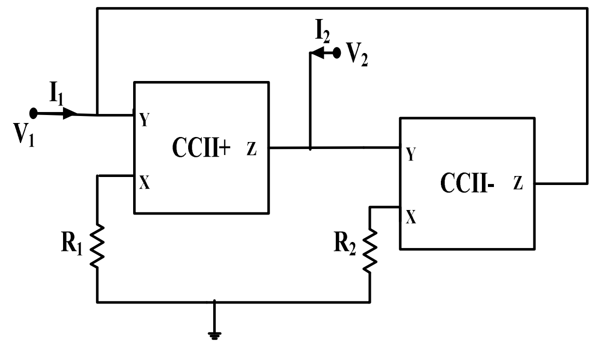

Fig. 5. Non-ideal gyrator

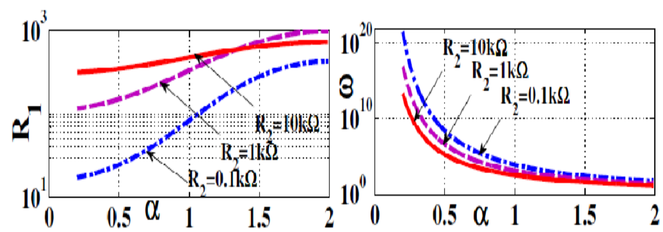

Fig. 6. Numerical study for gyrator cases 1 and 3

TABLE III. OSCILLATION PARAMETERS USING GYRATOR

#	R_1	Freq.
1	$\frac{C_2R}{2C_1C_2R\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)+C_2\left(\frac{R}{R_2}+1\right)+C_1}$	$C_1(C_2R)^2\omega^{2\alpha} = \left(\frac{R}{R_2}\right)\left(2C_1C_2R\omega^{\alpha}\cos(0.5\alpha\pi) + \left(C_2\left(\frac{R}{R_2}+1\right)+C_1\right)\right)$
2	0	$\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right) = (C_2+C_1)/2C_1C_2R$
3	$\frac{C_2R}{2C_1C_2R\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)+C_2\left(\frac{R}{R_2}+1\right)+C_1}$	$C_1(C_2R)^2\omega^{2\alpha} = \left(\frac{R}{R_2}\right)\left(2C_1C_2R\omega^{\alpha}\cos(0.5\alpha\pi) + \left(C_2\left(\frac{R}{R_2}+1\right)+C_1\right)\right)$

By substituting in Table I with (6); Table III shows that with the use of gyrator cases 1 and 3 are exactly the same and both can achieve oscillations while case 2 cannot produce any oscillations. Applying equal C design with different values of R_2 to control the oscillation condition; Fig.6 shows that the condition of oscillation increases with the fractional order parameter α unlike the frequency which decays with α . The oscillation frequency range increases with lower values of R_2.

IV. SIMULATION RESULTS

In this section, simulations of some cases are introduced to verify the reliability of the proposed oscillator. The simulation parameters chosen for the 1st two port network used (op-amp circuit) are $C_1 = C_2 = 1.2 * 10^{-6}$ and $R = R_1 = R_2 = 1k\Omega$. Order 0.8 is chosen to be simulated from which; R_3 is calculated to be 324Ω and the frequency of oscillation of this case is 1.44 kHz. Fig.7 shows the spice simulations results for this case.

For the non-ideal gyrator circuit, the simulation parameters are $C_1 = C_2 = 1.2 * 10^{-6}$ and $R = R_2 = 1k\Omega$. Order 0.8 is also chosen to be simulated from which; R_1 is equal to 233Ω and the frequency of oscillation is 1.7 kHz. Fig.8 shows the spice simulations results for this case. The spice macro model of AD844 is used to simulate the op-amp and the CCII.

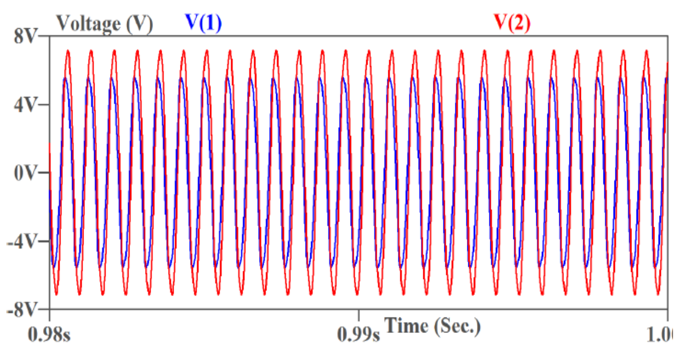

Fig. 7. Spice simulations for Op-amp two port case 3

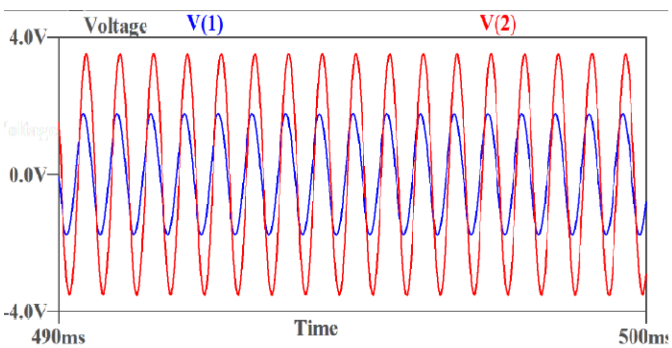

Fig. 8. Spice simulations for gyrator two port cases 1 and 3

V. CONCLUSION

A study of a fractional equal order two port network was presented in this paper. The study involved oscillator with three impedances structure (two fractional equal order capacitors and a resistor). Three possible cases were obtained from different combinations of these three impedances. The general characteristic equation, oscillation frequency and condition for each case were derived and summarized in a Table. Two different two port network devices were investigated to verify the theoretical findings which were op-amp based circuit and a non-ideal gyrator circuit. Simulation for some cases was done by spice to verify the concept's reliability.

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