Current Feedback Operational Amplifier (CFOA) Based Fractional Order Oscillators

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***Abstract*—This paper presents a study of fractional order oscillators based on current feedback operational amplifiers (CFOA). Two general cases have been discussed for the oscillation frequency and condition with the use of two fractional order elements of different orders. Design procedure for the two general cases is illustrated with numerical discussions. Circuit simulations for some special cases are presented to validate the theoretical findings. The simulations have been done using Ad844 commercial CFOA model**

Keywords—Fractional calculus; CFOA; Oscillators.

I. Introduction

Fractional calculus gained considerable attention in the late sixties that's because it gives more accurate description of real objects due to the fact that many structures found in nature can be modeled by fractals [1]. Fractional calculus deals with the generalization of differentiation and integration of non-integer order. The fractional calculus concept invaded wide areas of science and engineering [2-8] such as in control engineering [2], robotics [3], mechanical engineering [4], in the theory of dielectric relaxation [5], fractional-order smith chart [6] and electric circuits and systems [7, 8].

One of the most frequently used definitions for the general fractional derivatives is the Caputo definition which can be expressed as follows [9]

$$D_{t_0}^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_{t_0}^{t} f^{(m)}(u)\,(t-u)^{m-\alpha-1} \qquad (1)$$

where m is an integer such that $(m-1) < \alpha < m$. There are many numerical techniques for the computation of fractional calculus. One of these techniques is the power series expansion (PSE) of a generating function. It is important to note that PSE leads to approximation in the form of polynomials [10]. Another approach in the computation of the fractional order derivatives is the Podlubny's matrix approach [11]. This approach is based on the fact that operation of the fractional calculus can be expressed by matrix.

The fractional derivative gives extra degree of freedom which is the order of the derivative; it increases the flexibility of the design and it adds more fundamentals. It helps to obtain generalized design equations to any system from which integer order can be considered just a special case from the wide fractional order domain. The general design equations are now with fractional order which grabbed the interests of researchers to find different numerical schemes to solve them. Various algorithms are presented to solve the fractional order differential equations. One of these algorithms is the FracPECE subroutine for the numerical solution of differential equations in [12] and that is the method used in this paper.

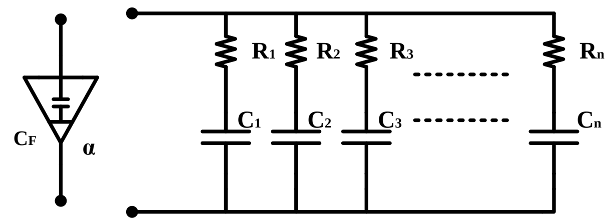

Fig. 1. Approximation the fractional order capacitor with general order.

The method do prediction and correction by evaluating predicted value then use it to evaluate the correct value which allows having more accurate solution.

The aim of this work is to implement a well-known two oscillators using fractional order capacitor which has the impedances $Z(s) = ks^{\alpha}$ and the conventional capacitor could be considered as special case from the general fractional order capacitor and to study its effect on both condition and frequency of oscillation. Many papers presented the implementation of the fractional order capacitor into a feasible device could be used in practical [13,14]. Figure 1 shows the implementation of the fractional capacitor with any order [14]. It is represented by an equivalent circuit composed of RC series branches connected in parallel with each other.

Voltage mode has dominated the analog design for a long period of time till researchers turned to use current mode due to its advantages such as higher signal bandwidth, larger dynamic range, greater linearity and lower power consumption. From this point of view the idea of current feedback operational amplifier (CFOA) was presented [15] as a new block for high frequency applications to replace the op-amp which was a key building block in most of filter circuits and oscillators [16-20] which is the focus of this paper.

The paper is organized as follows section II presents the current feedback operational amplifier (CFOA), Section III illustrates two fractional order oscillators based on CFOA with numerical discussion, Section IV presents the simulation results which verify the reliability of the fractional order oscillator and finally section V drawn the conclusion.

II. Current Feedback Operational Amplifier (CFOA)

The current feedback operational amplifier is a four port network with terminal characteristics described by the following matrix:

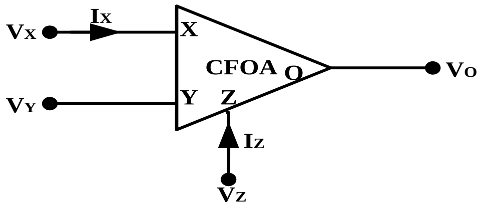

Fig. 2. CFOA symbol diagram.

$$\begin{bmatrix} I_Y \\ V_X \\ I_Z \\ V_O \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_Y \\ I_X \\ V_Z \\ I_O \end{bmatrix} \quad (2)$$

where V_X, V_Y, V_Z, V_O, I_X, I_Y, I_Z and I_O are the voltages and the currents of X-, Y- , Z- and O-terminals respectively. The symbol diagram of CFOA is shown in Fig.2., The non-idealities of the CFOA had been addressed in many papers such as in [20].

III. CFOA BASED FRACTIONAL ORDER OSCILLATORS

For the oscillator shown in Fig.3(a) which is the generalization of the integer order oscillator presented in [19]. The state matrix describes this oscillator could be written as follows:

$$\begin{bmatrix} D^{\alpha} v_1 \\ D^{\beta} v_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\left(1-\frac{R_3}{R_1}\right)}{C_1 R_3} & \dfrac{-1}{C_1 R_3} \\ \dfrac{1}{C_2 R_2} & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (3)$$

According to the theorem presented in [7]; this system oscillates if there is a value of ω that satisfies the following two equations

$$\omega^{\alpha+\beta}\cos\left(\frac{(\beta+\alpha)\pi}{2}\right) - \frac{\left(1-\frac{R_3}{R_1}\right)}{C_1 R_3}\omega^{\beta}\cos\left(\frac{\beta\pi}{2}\right) + \frac{1}{C_1C_2R_2R_3} = 0 \quad (4a)$$

$$\sin\left(\frac{(\beta+\alpha)\pi}{2}\right) - \frac{\left(1-\frac{R_3}{R_1}\right)}{C_1 R_3}\omega^{-\alpha}\sin\left(\frac{\beta\pi}{2}\right) = 0 \quad (4b)$$

For example, Let R_3 is the design parameter to control the condition of oscillations then,

$$R_3 = \frac{C_2R_1R_2\omega^{\beta}\cos\left(\frac{\beta\pi}{2}\right) - R_1}{C_1C_2R_1R_2\omega^{\alpha+\beta}\cos\left(\frac{(\beta+\alpha)\pi}{2}\right) + C_2R_2\,\omega^{\beta}\cos\left(\frac{\beta\pi}{2}\right)} \quad (5a)$$

$$R_3 = \frac{R_1\,\omega^{-\alpha}\sin\left(\frac{\beta\pi}{2}\right)}{C_1R_1\sin\left(\frac{(\beta+\alpha)\pi}{2}\right) + \omega^{-\alpha}\sin\left(\frac{\beta\pi}{2}\right)} \quad (5b)$$

Eliminating R_3 from both equations; the frequency of oscillation could be written as follows:

$$\omega^{\alpha+\beta}\sin\left(\frac{\alpha\pi}{2}\right) - \frac{\omega^{\alpha}}{C_2R_2}\sin\left(\frac{(\beta+\alpha)\pi}{2}\right) - \frac{\sin\left(\frac{\beta\pi}{2}\right)}{C_1C_2R_1R_2} = 0 \quad (6)$$

Some special cases are investigated in Table I where the integer case is considered as a special case each of them in the fractional order oscillator and many design configurations could be achieved from this structure.

For example, taking equal C design with different R_1 values; the relation between the condition and the frequency of oscillation versus the fractional order parameter are shown in Fig.4.

TABLE I. SPECIAL CASES OF THE 1ST OSCILLATOR

	Oscillation parameters	
	***R*₃** → R_3	***Freq.***
α = β	$\dfrac{R_1}{1+2C_1R_1\ \omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)}$	$\omega^{2\alpha} = \frac{2\omega^{\alpha}}{C_2R_2}\cos\left(\frac{\alpha\pi}{2}\right) + \frac{1}{C_1C_2R_1R_2}$
α = 1	$\dfrac{R_1}{1+C_1R_1\omega\cot\left(\frac{\beta\pi}{2}\right)}$	$\omega^{1+\beta} = \frac{\omega}{C_2R_2}\cos\left(\frac{\beta\pi}{2}\right) + \frac{\sin\left(\frac{\beta\pi}{2}\right)}{C_1C_2R_1R_2}$
β = 1	$\dfrac{R_1}{1+C_1R_1\ \omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)}$	$\omega^{\alpha+1}\sin\left(\frac{\alpha\pi}{2}\right) = \frac{\omega^{\alpha}}{C_2R_2}\cos\left(\frac{\alpha\pi}{2}\right) + \frac{1}{C_1C_2R_1R_2}$

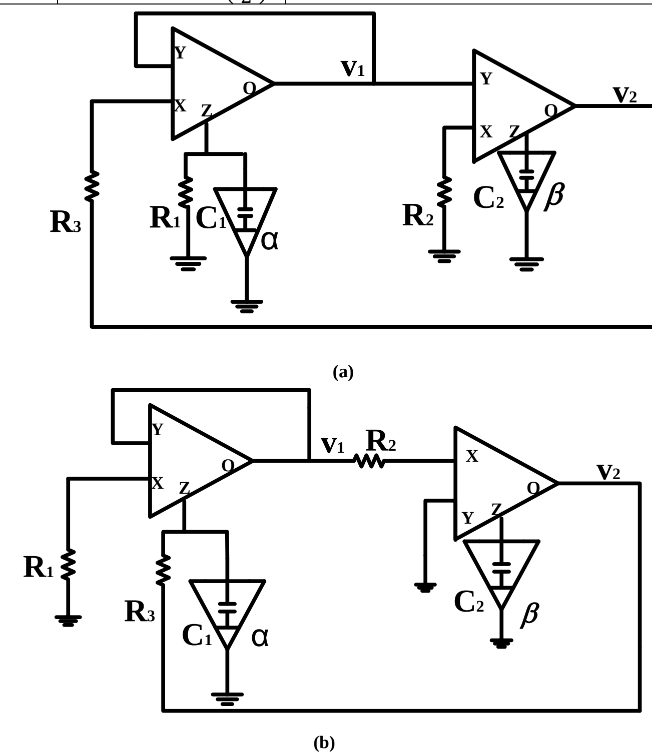

Fig. 3. (a) &(b)Fractional order CFOA oscillators.

It is clear that the oscillation frequency decays as α increases under constant values of R_1. Moreover, the frequency range increases in the equal order design than the other two designs. Note that the integer-order case is special case from all these figures when the fractional-order equals to 1. The effect of changing R_1 on the oscillation frequency is also shown in these subplots where for fixed values of R_1 ; the frequency changes according to the fractional order parameters α & β but increasing R_1 will decrease the frequency range provided by α & β .

For the same results in Fig.4, the oscillation condition is increasing with the fractional order parameter and wider range exists in the last special case. Moreover, this oscillation condition increases as the parameter R_1 increase as shown in Fig.4. Therefore, the extra degree of freedom in these three special cases shows more flexibility and adds more fundamentals that are not exist in the conventional case.

For the oscillator shown in Fig.3(b) is the generalization of the integer order oscillator presented in [17]. The state matrix describes this oscillator could be written as follows:

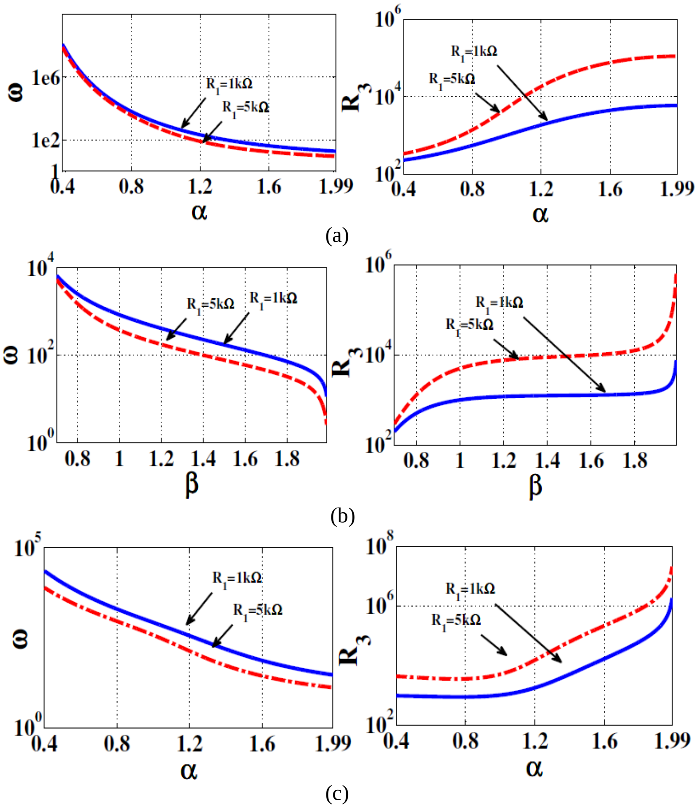

Fig. 4. Numerical analysis for the 1st oscillator (a) $\alpha = \beta$ (b) $\alpha = 1$(c) $\beta = 1$

TABLE II. GENERAL CHARACTARISTICS OF THE 2ND OSCILLATOR

General characteristics Oscillation parameters	
R_3	$= \frac{C_2R_1R_2\omega^{\beta}\cos\left(\frac{\beta\pi}{2}\right)+R_1}{C_2R_2\,\omega^{\beta}\cos\left(\frac{\beta\pi}{2}\right)-C_1C_2R_1R_2\omega^{\alpha+\beta}\cos\left(\frac{(\beta+\alpha)\pi}{2}\right)}$ $= \frac{R_1\,\omega^{-\alpha}\sin\left(\frac{\beta\pi}{2}\right)}{\omega^{-\alpha}\sin\left(\frac{\beta\pi}{2}\right)-C_1R_1\sin\left(\frac{(\beta+\alpha)\pi}{2}\right)}$
Freq.	$\omega^{\alpha+\beta}\sin\left(\frac{\alpha\pi}{2}\right)+\frac{\omega^{\alpha}}{C_2R_2}\sin\left(\frac{(\beta+\alpha)\pi}{2}\right)-\frac{\sin\left(\frac{\beta\pi}{2}\right)}{C_1C_2R_1R_2}=0$

TABLE III. SPECIAL CASES OF THE 2ND OSCILLATOR

#	Oscillation parameters	
	R_3	Freq.
$\alpha = \beta$	$\frac{R_1}{1-2C_1R_1\,\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)}$	$\omega^{2\alpha}+\frac{2\omega^{\alpha}}{C_2R_2}\cos\left(\frac{\alpha\pi}{2}\right)-\frac{1}{C_1C_2R_1R_2}=0$
$\alpha = 1$	$\frac{R_1}{1-C_1R_1\omega\cot\left(\frac{\beta\pi}{2}\right)}$	$\omega^{1+\beta}+\frac{\omega}{C_2R_2}\cos\left(\frac{\beta\pi}{2}\right)-\frac{\sin\left(\frac{\beta\pi}{2}\right)}{C_1C_2R_1R_2}=0$
$\beta = 1$	$\frac{R_1}{1-C_1R_1\,\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)}$	$\omega^{\alpha+1}\sin\left(\frac{\alpha\pi}{2}\right)+\frac{\omega^{\alpha}}{C_2R_2}\cos\left(\frac{\alpha\pi}{2}\right)-\frac{1}{C_1C_2R_1R_2}=0$

$$\begin{bmatrix} D^{\alpha}V_1 \\ D^{\beta}V_2 \end{bmatrix} = \begin{bmatrix} \frac{\left(1-\frac{R_1}{R_3}\right)}{C_1\,R_1} & \frac{1}{C_1\,R_3} \\ \frac{-1}{C_2\,R_2} & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (7)$$

Similar analysis could be performed for the general condition and frequency of oscillation for this oscillator. By choosing also R_3 to be the design parameter and applying theorem in [7]; Table II summarizes the general characteristics of this oscillator.

Investigation of different special cases is illustrated in Table III. Taking equal C design with different R_1 values; Fig.5 shows the relation between the condition and the frequency of oscillation versus the fractional order parameter. The oscillation condition R_3 is inversely proportional to the fractional order parameter in the first two cases and has a peak in the last case as shown in Fig.5(c). Its range increases with increasing the value of R_1. It is clear from Fig.5 that the range of frequency increases with lowering the value of R_1.The oscillation frequency is also inversely proportional to the fractional order parameter in the 1st and 3rd cases and a peak arises in the 2nd case as illustrated in Fig.5(b).

IV. SIMULATION RESULTS

Some cases are chosen to be simulated to show the reliability of the proposed design.

For the first oscillator; the simulation parameters are chosen to be $C_1 = C_2 = 1.2 * 10^{\wedge} - 6$ and$R_1 = R_2 = 1k\Omega$; R_3 and the oscillation frequency are calculated for each simulated case. For example; on using integer order with fractional order with $\alpha = 0.7$, $\beta = 1$ (case 3) ; $R_3 = 866\Omega$ and the frequency of oscillation equals to 506 Hz. Fig.6 shows the ideal simulations versus the practical simulations with percentage of error 6%. Using equal order with $\alpha = \beta = 0.8$ (case 1); $R_3 = 544\,\Omega$ and the frequency of oscillation equals to 1.04 kHz. Fig.7 shows the ideal simulations versus the practical simulations with percentage of error 7%.

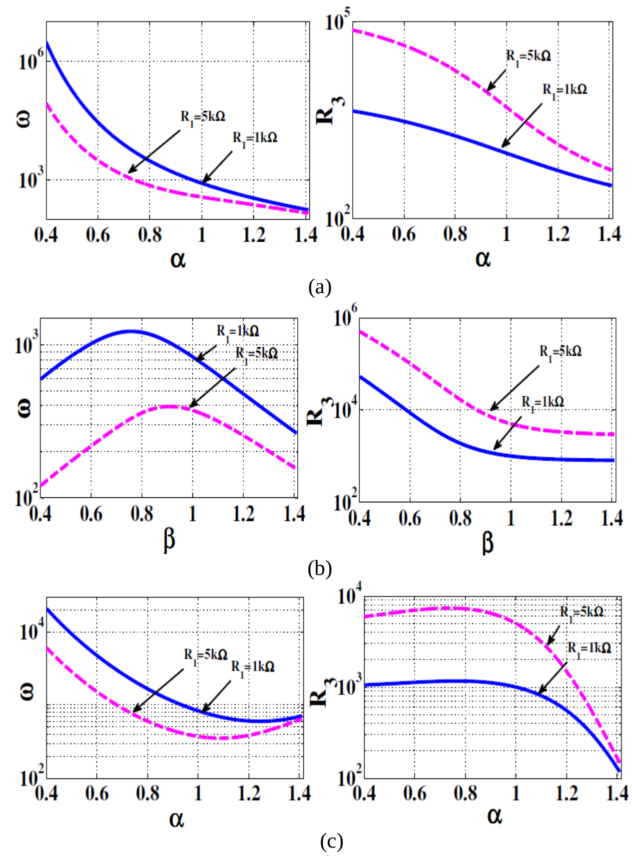

Fig. 5. Numerical analysis for the 2nd oscillator (a) $\alpha = \beta$ (b) $\alpha = 1$(c) $\beta = 1$

For the 2nd oscillator; the simulation parameters are chosen to be $C_1 = C_2 = 1.2 * 10^{-6}$ and $R_1 = R_2 = 1k\Omega$. From the numerical discussion for case 2 a peak was found in the frequency curve it happens when $\beta = 0.8$; Fig.8 shows the ideal versus practical simulations with percentage of error 4%. R_3 is equal to $1.88k\Omega$ with frequency equals to 191Hz. Note that all the practical simulations were done using spice with the use of commercial model AD844 to simulate the CFOA block.

From Figs. (6-8); the phase difference of the oscillator outputs is dependent on the fractional orders $\alpha\&\beta$ and can be controlled through changing the order which is an advantage provided by using the fractional order element . Phase control is very important in some applications in signal processing and wireless communication systems [21, 22].

V. Conclusion

In this paper, a generalization for the condition and the frequency of oscillation for the fractional order oscillators based on CFOA are derived. Also, some special cases were discussed and summarized in tables. Finally, simulation results are presented with comparison with the ideal performance to show the reliability of the design.

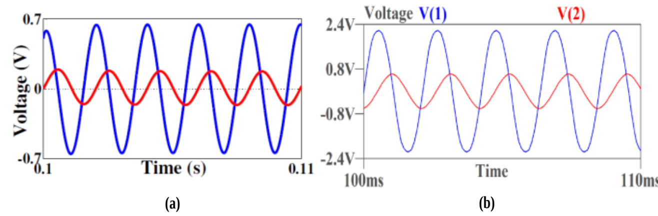

Fig. 6. (a) Matlab simulations(b)Spice simulations for special case 3

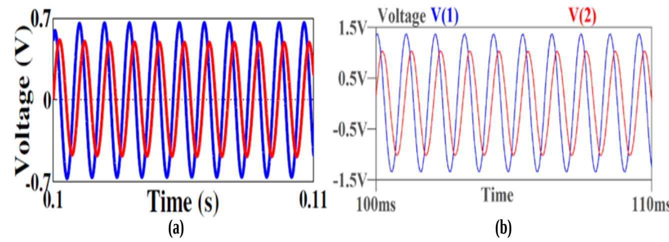

Fig. 7. (a) Matlab simulations(b)Spice simulations for special case 1

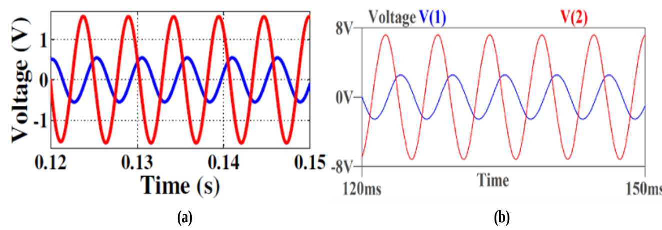

Fig. 8. (a) Matlab simulations(b)Spice simulations for special case 2

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