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LETTERS TO THE EDITOR

A NEW PHASE AND MAGNITUDE COMPENSATED WEIGHTED SUMMER USING THREE OPERATIONAL AMPLIFIERS

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INTRODUCTION

It is well known that the finite and complex open loop gain nature of the operational amplifier (opamp) degrades significantly the phase and the magnitude of weighted summer structures. With the introduction of low-cost dual and quad opamps having closely matched characteristics which track with changes in temperature and voltage, the active compensation technique proved to be very attractive. Recently several active phase compensated weighted summers using two opamps have been introduced in the literature.[1-4] It has been shown that for low frequencies the two opamps compensated summer has phase and magnitude errors proportional to $(\omega/\omega_t)^3$ and $(\omega/\omega_t)^2$, where ω_t is the unity gain bandwidth of the opamp. That is, the phase error of the two opamps summer is reduced to a negligible level, whereas the magnitude error remains a second order term as that of the uncompensated summer.

Most recently active compensated amplifiers using three opamps have been considered.[5-6] These circuits however are not suitable by their nature for realizing generalized weighted summers for both positive and negative gains.

The purpose of this letter is to introduce a novel active phase and magnitude compensated generalized weighted summer. The proposed summer employs two opamps and six resistors more than the uncompensated weighted summer. At low frequencies the phase and the magnitude errors are proportional to $(\omega/\omega_t)^3$ and $(\omega/\omega_t)^4$, respectively. That is both the phase and the magnitude errors are reduced to negligible levels. The design equations are derived assuming mismatched opamps are used. The special case of matched opamps is also considered. The tuning procedure is discussed. A comparison of the proposed summer with the two opamps compensated summers and the single opamp uncompensated summer illustrates the advantages of the new summer.

THE PROPOSED SUMMER

The novel three opamps active compensated generalized weighted summer is shown in Figure 1. The voltages $V_{11}, V_{12}, \ldots, V_{1m}$ represent the m inverting inputs and the voltage $V_{21}, V_{22}, \ldots, V_{2n}$ are the n non-inverting inputs. Let the open loop gain of each of the three opamps be represented by the single pole model given by:

$$A_i(s) \simeq \frac{\omega_{ti}}{s}, \qquad (i = 1, 2, 3) \tag{1}$$

where ω_t is the unity gain bandwidth of the opamp and is ideally infinity. By direct analysis of the circuit, the generalized expression of the output voltage V_0 is given by:

$$V_0 = \left[\frac{(K+1)}{G^+} \sum_{i=1}^{n} (V_{2i}G_{2i}) - \frac{K}{G} \sum_{i=1}^{m} (V_{1i}G_{1i})\right]\left[\frac{K_1+1}{K_2+1}\right]\varepsilon(s) \tag{2}$$

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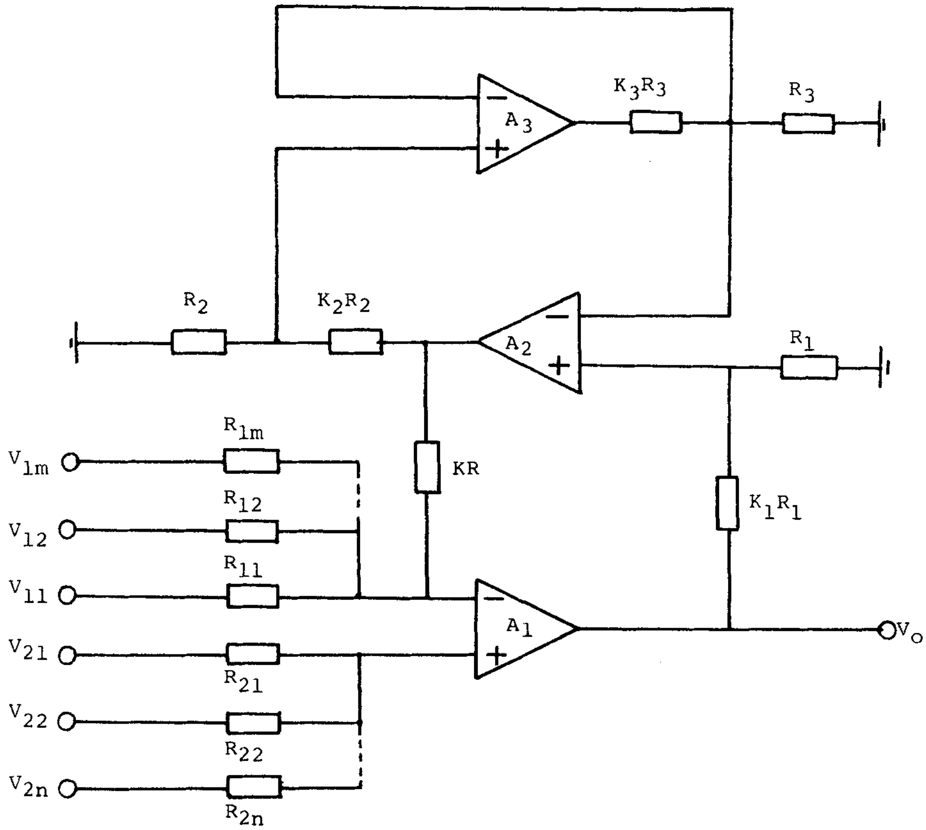

Figure 1. The active compensated weighted summer

where

$$G^{+}=\sum_{i=1}^{n} G_{2i}, \qquad G_{2i}=\frac{1}{R_{2i}}, \qquad (i=1,2,\ldots,n) \tag{3}$$

$$G=\frac{1}{R}=\sum_{i=1}^{m} G_{1i}, \qquad G_{1i}=\frac{1}{R_{1i}}, \qquad (i=1,2,\ldots,m) \tag{4}$$

$\varepsilon(s)$ is the remaining error function of the compensated circuit and is given by:

$$\varepsilon(s)=\frac{1+s\tau_2+s^2\tau_2\tau_3}{1+s\tau_3+\left(\dfrac{K+1}{K_2+1}\right)[s\tau_1+s^2\tau_1\tau_2+s^3\tau_1\tau_2\tau_3]} \tag{5}$$

where

$$\tau_i=\frac{K_i+1}{\omega_{ti}}, \qquad (i=1,2,3) \tag{6}$$

Choosing $K_2=K$, equations (2) and (5) become:

$$V_0=\left[\frac{(K_1+1)}{G^{+}}\sum_{i=1}^{n}(V_{2i}G_{2i})-\frac{K(K_1+1)}{(K+1)G}\sum_{i=1}^{m}(V_{1i}G_{1i})\right]\varepsilon(s) \tag{7}$$

$$\varepsilon(s)=\frac{1+s\tau_2+s^2\tau_2\tau_3}{1+s(\tau_1+\tau_3)+s^2\tau_1\tau_2+s^3\tau_1\tau_2\tau_3} \tag{8}$$

Examining the above equation for the remaining phase and magnitude errors, it is seen that by taking

$$\tau_1 = \frac{\tau_2}{2} = \tau_3 \tag{9}$$

will yield relatively negligible phase and magnitude errors over a prescribed frequency range. The compensated error function reduces to:

$$\varepsilon_c(s) = \frac{1 + 2\tau_1 s + 2\tau_1^2 s^2}{1 + 2\tau_1 s + 2\tau_1^2 s^2 + 2\tau_1^3 s^3} \tag{10}$$

From the above equation, it is seen that the phase and the magnitude errors of the compensated circuit are given, respectively, by:

$$\left.\begin{aligned} \phi &\equiv \arg[\varepsilon_c(j\omega)] \simeq 2(\tau_1\omega)^3 = 2\left[(K_1+1)\left(\frac{\omega}{\omega_{t1}}\right)\right]^3 \\ \gamma &\equiv |\varepsilon_c(j\omega)| - 1 \simeq 4(\tau_1\omega)^4 = 4\left[(K_1+1)\left(\frac{\omega}{\omega_{t1}}\right)\right]^4 \end{aligned}\right\} \quad \omega\tau_i \ll 1 \qquad (i = 1, 2, 3) \tag{11}$$

Thus, with the conditions of equation (9) being satisfied and at frequencies such that $\omega\tau_i \ll 1 (i = 1, 2, 3)$, the phase error is reduced to a third order term and the magnitude error is reduced to a fourth order term.

The gain requirements are controlled by the parameter K_1. The compensation conditions can be satisfied by tuning the resistors KR, K_2R_2 and K_3R_3. The design equation for K, K_2 and K_3 are obtained from equations (6) and (9) and are given by:

$$K = K_2 = 2(K_1+1)\left(\frac{\omega_{t2}}{\omega_{t1}}\right) - 1 \tag{12}$$

$$K_3 = (K_1+1)\left(\frac{\omega_{t3}}{\omega_{t1}}\right) - 1 \tag{13}$$

Thus, it is seen that it is not necessary to use matched opamps with this generalized summer. If matched opamps are used however, the design equations simplify to:

$$K = K_2 = 2K_1 + 1 \tag{14}$$

$$K_3 = K_1 \tag{15}$$

It should be noted that the above design is based on the choice $K = K_2$; other choices for the parameter K are possible.

CONCLUSIONS

A new active compensated weighted summer is given. The proposed summer is superior to the single opamp summer since it reduces the phase error from a first order to a third order term and reduces the magnitude error from a second order to a fourth order. The compensation is achieved by using two extra opamps and six resistors. Comparing the proposed summer with the two opamps summers,[1–4] it is seen that the magnitude error is reduced from a second order term to a fourth order term.

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A GENERAL FORM FOR THE INPUT–OUTPUT DIFFERENCE EQUATION OF SWITCHED-CAPACITOR NETWORKS

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INTRODUCTION

Although in the analysis and synthesis of switched capacitor networks (SCNs) one often deals with an input–output difference equation, the general form for such a relation has not been presented in the literature. Rather, matrix analysis has been emphasized through which input–output relations are derived directly in the frequency domain. Deriving from these the form of, say, a single-input, single-output difference equation would be unnecessarily convoluted, just as it would be unnecessarily convoluted to derive the general form of the input–output differential equation of classical linear networks by going through the frequency domain. In this work the general form of the input–output difference equation for linear voltage-driven SCNs consisting of capacitors, switches, and VCVSs, will be derived directly in the time domain through the use of 'shift' operators.

THE DIFFERENCE EQUATION

Consider an SCN as above, driven by a voltage input which can be continuous or piecewise-constant. We define as a switching instant t_k each instant at which at least one switch *assumes* a new state, or the input *assumes* a new value after a discontinuity, if piecewise-constant. In this way, in the interval $[t_k, t_{k+1})$ all switch states are fixed and the input is continuous. The fact that the above interval is *closed* on the left is instrumental in the unified formulation to follow. The SCN is assumed to be periodically switched with time period T, and $K+1$ subintervals in each major interval;[1] each subinterval starts at a switching instant of the form $t_k = nT + \tau_l$, where n is an integer and $nT + \tau_{l+K+1} = nT + T + \tau_l$, with $\tau_0 \equiv 0$. Let I_l be the collection of all subintervals starting at $nT + \tau_l$, all n, $l = 0, 1, \ldots, K$. We will assume the SCN is driven by a single input for simplicity, and we will denote that input by $x(t)$. Let $\mathbf{v}(.)$ represent the vector of the circuit variables (assumed m in number), whether they be node voltages or other sets of appropriately selected voltages.[1-3] It is known that the circuit variables can be expressed for a given subinterval in the form

$$\mathbf{v}(t) = \mathbf{d}_l x(t) + \mathbf{E}_l \mathbf{v}(nT + \tau_l^-), \qquad t \in (nT + \tau_l, nT + \tau_{l+1}) \tag{1}$$

with $\mathbf{d}_l$, $\mathbf{E}_l$ an appropriate vector and matrix, respectively.[1-3] Writing this equation for $t = nT + \tau_l^+$ gives:

$$\mathbf{v}(nT + \tau_l^+) = \mathbf{d}_l x(nT + \tau_l^+) + \mathbf{E}_l \mathbf{v}(nT + \tau_l^-) \tag{2}$$

Let us identify one of the circuit variables, say $v_j(\cdot)$, as the output and denote it by $y(\cdot)$. Subtracting (2) from (1) and taking the jth row, we obtain:

$$y(t) = y(nT + \tau_l^+) + d_{lj}[x(t) - x(nT + \tau_l^+)], \qquad t \in (nT + \tau_l, nT + \tau_{l+1}) \tag{3}$$

where d_{lj} is the jth element of $\mathbf{d}_l$. Thus, in order to know $y(t)$ for $t \in I_l$, it suffices to know the scalar d_{lj},

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