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CLASSIFICATION AND GENERATION OF ACTIVE COMPENSATED NON-INVERTING VCVS BUILDING BLOCKS

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SUMMARY

It is shown that the active compensated non-inverting voltage controlled voltage source (VCVS) structures employing two operational amplifiers (op-amps) may be classified into two major classes. Class 1, in which the phase compensation condition does not depend on the unity-gain bandwidth of the op-amps used, and class 2, in which the phase compensation condition depends on the unity-gain bandwidth of both op-amps. Detailed comparison tables which include the condition for phase compensation, the phase error, the magnitude error and the realizable bandwidth are given.

INTRODUCTION

The non-inverting voltage controlled voltage source (VCVS) shown in Figure 1, which includes the voltage follower as a special case is a basic building block in active *RC* networks. The assumption of an ideal VCVS cannot be sustained except at very low frequencies.[1-3] It is well known that the finite and complex open loop

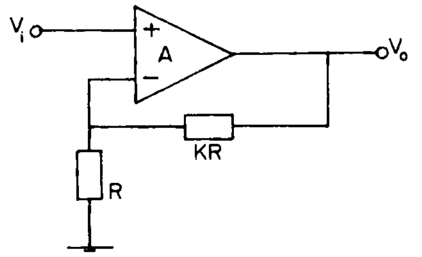

Figure 1. The uncompensated non-inverting VCVS

gain nature of the operational amplifier (op-amp) results in both phase and magnitude errors. The phase error is of a first order magnitude whereas the magnitude error is a second order term.[4] Thus the VCVS structures require only phase compensation. There are two main compensation methods. These are passive compensation and active compensation. Passive compensation for the non-inverting VCVS requires a compensating capacitor to be connected between the inverting op-amp input and ground.[4-5] The compensating capacitor has to be adjusted at specific ambient temperature and power supply voltage to match the op-amp unity-gain bandwidth ω_t according to a certain design constraint. Thus if the temperature or the power supply voltage is changed (which affects ω_t) the compensation will no longer be satisfactory. For this reason active compensation is superior to passive compensation.[6-7] The main idea in passive or active compensation; is, to introduce a negative real zero at a suitable position in order to reduce the phase error to a negligible level. In active compensation the introduction of this negative real zero requires the use of a second op-amp in the circuit which provides compensation for the original op-amp. Recently a limited number of active compensated VCVS circuits have been introduced in the literature.[6-12]

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The purpose of this paper is to present a unified analysis for the generalized active compensated non-interverting VCVS structures employing 2 op-amps. It is shown that all the active compensated non-inverting VCVS networks may be classified into two major classes. This classification leads to the generation of new active compensated circuits. A detailed comparison table which includes the condition for phase compensation, the phase error and the magnitude error is given. A figure of merit for the VCVS is the normalized bandwidth gain product defined by:

$$F = \mathrm{NBWG} = \left[\frac{\text{Bandwidth}}{\omega_t}\right][\text{DC Gain}] \tag{1}$$

For the uncompensated VCVS of Figure 1 it is well known that $F_u = 1$.

ANALYSIS

Figure 2 represents the generalized active compensated amplifier employing 2 op-amps. Let the open loop gain of the op-amp be represented in the form [13–15]

$$A(s) = \frac{A_o \omega_1}{s + \omega_1} = \frac{\omega_t}{s + \omega_1} \tag{2a}$$

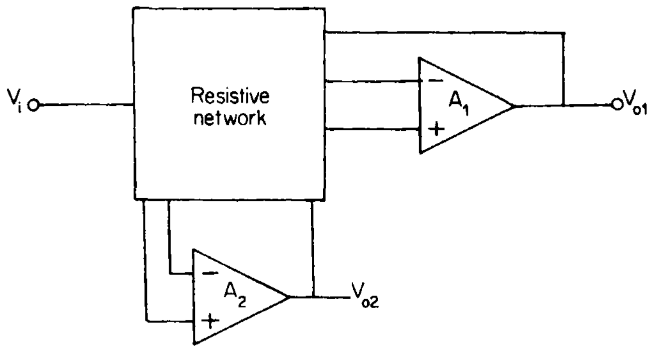


Figure 2. The generalized compensated non-inverting VCVS using 2 op-amps

where

A_o is the open-loop DC gain of the op-amp

ω_1 is the open-loop 3-dB bandwidth

$\omega_t = A_o \omega_1$ is the gain-bandwidth product of the op-amp

In the frequency range of interest $\omega \gg \omega_1$, therefore[2]

$$A(s) \simeq \frac{\omega_t}{s} \tag{2b}$$

By direct analysis of the circuit in Figure 2, the basic circuit equations are

$$\begin{bmatrix} \dfrac{V_{o1}}{A_1} \\ \dfrac{V_{o2}}{A_2} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} V_i + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} V_{o1} \\ V_{o2} \end{bmatrix} \tag{3}$$

where a_i and b_{ij} are real coefficients having magnitudes $\leqslant 1$ (since they represent transfer functions of a resistive network[8]).

Without any loss of generality let V_{o1} represent the output of the VCVS as indicated in Figure 2. Then the transfer function of the circuit is given by:

$$T(s) \equiv \frac{V_{o1}}{V_i} = \left[\frac{a_2 b_{12} - a_1 b_{22}}{\Delta B}\right] \varepsilon(s) \tag{4}$$

where

$$\Delta B = b_{11} b_{22} - b_{12} b_{21} \tag{5}$$

and

$$\varepsilon(s) = \frac{1 + \left[\frac{a_1}{a_2 b_{12} - a_1 b_{22}}\right] \frac{s}{\omega_{t2}}}{1 + \left[\frac{-b_{22}}{\Delta B}\right] \frac{s}{\omega_{t1}} + \left[\frac{-b_{11}}{\Delta B}\right] \frac{s}{\omega_{t2}} + \left[\frac{1}{\Delta B}\right] \frac{s^2}{\omega_{t1}\omega_{t2}}} \tag{6}$$

$\varepsilon(s)$ is the error function of the compensated circuit. Ideally it must have a unity magnitude and a zero phase. To limit the number of resistors in the circuit and for the coefficient of the s term in the denominator of $\varepsilon(s)$ to depend only on the unity gain bandwidth of one of the two op-amps, it is necessary to have either $b_{11} = 0$ or $b_{22} = 0$. Thus the VCVS structures are classified into two major classes.

CLASS 1 VCVS

In this case $b_{22} = 0$ and equation (4) reduces to:

$$T_1(s) = \left[\frac{-a_2}{b_{21}}\right] \varepsilon_1(s) \tag{7}$$

where

$$\varepsilon_1(s) = \frac{1 + \left[\frac{a_1}{a_2 b_{12}}\right] \frac{s}{\omega_{t2}}}{1 + \left[\frac{b_{11}}{b_{12} b_{21}}\right] \frac{s}{\omega_{t2}} + \left[\frac{-1}{b_{12} b_{21}}\right] \frac{s^2}{\omega_{t1}\omega_{t2}}} \tag{8}$$

In order to reduce the phase error to a negligible level it is necessary that the coefficients of the s-terms in the numerator and denominator of $\varepsilon_1(s)$ be equal.[6] In this class, the phase compensation condition is independent of ω_t of the op-amps and is given by:

$$\frac{a_1}{a_2} = \frac{b_{11}}{b_{21}} \tag{9}$$

If the above condition is satisfied, the phase error is reduced to a negligible level so long as the second order term of the denominator in equation (8) is considered quite small. Examining the coefficients of s and s^2 of the denominator of $\varepsilon_1(s)$, it is seen that both b_{11} and the product $(b_{12} b_{21})$ must be negative. Thus two VCVS types are defined. Type A in which b_{12} is negative and b_{21} is positive, and type B in which b_{12} is postive and b_{21} is negative. For a positive gain the coefficient a_2 must have an opposite sign to b_{21}. From equation (8) a_1 must be positive in order to have a negative real zero. Table I includes the coefficient signs for all VCVS types.

Figure 3(a) represents a class 1 type A VCVS network.[9] The approximate phase and magnitude errors are summarized in Table II. The figure of merit for this amplifier is given in Table III.

Figure 3(b) represents a new VCVS network which belongs to class 1 type B. The condition for phase compensation is given by:

$$K_1 + 1 = \frac{K_2(K_3 + 1)}{(K_2 + 1)} \tag{10}$$

Table I. The coefficients a_i, b_{ij} signs for all VCVS structures

Non-inverting VCVS									
Class No.	Sub-class No.	Type	a_1	a_2	b_{11}	b_{12}	b_{21}	b_{22}	Necessary condition
1	—	A	+	−	−	−	+	0	—
	—	B	+	+	−	+	−	0	—
2	I	A	+	0	0	−	+	−	—
		B	+	0	0	+	−	−	—
	II	A	+	−	0	−	+	−	—
		B	+	−	0	+	−	−	$a_2b_{12}-a_1b_{22}>0$
	III	A	+	+	0	−	+	−	$a_2b_{12}-a_1b_{22}>0$
		B	+	+	0	+	−	−	—

Two possible designs are given in Table II. The figure of merit given in Table III corresponds to the second design which has smaller phase and magnitude errors than the first design.

CLASS 2 VCVS

Here $b_{11}=0$ and equation (4) reduces to:

$$T_2(s)=\left[\frac{a_2b_{12}-a_1b_{22}}{-b_{12}b_{21}}\right]\varepsilon_2(s) \tag{11}$$

where

$$\varepsilon_2(s)=\frac{1+\left[\dfrac{a_1}{a_2b_{12}-a_1b_{22}}\right]\dfrac{s}{\omega_{t2}}}{1+\left[\dfrac{b_{22}}{b_{12}b_{21}}\right]\dfrac{s}{\omega_{t1}}+\left[\dfrac{-1}{b_{12}b_{21}}\right]\dfrac{s^2}{\omega_{t1}\omega_{t2}}} \tag{12}$$

As seen from the above equation, the condition for phase compensation depends on the unity gain bandwidth of both op-amps. Thus it is recommended to use matched op-amps with this class; in this case the condition for phase compensation is given by:

$$\frac{a_1}{a_2b_{12}-a_1b_{22}}=\frac{b_{22}}{b_{12}b_{21}} \tag{13}$$

Examining the coefficient of s^2 in the denominator of the error function $\varepsilon_2(s)$, it is seen that the product $b_{12}b_{21}$ must be negative. Therefore the coefficient b_{22} also must be negative in order to have a positive s-term in the denominator of $\varepsilon_2(s)$. From equation (11) it is clear that the term $(a_2b_{12}-a_1b_{22})$ must be positive for a positive gain. This implies that a_1 must be positive in order to have a positive s-term in the numerator of $\varepsilon_2(s)$. From the above coefficient analysis it is seen that three possible subclasses may be defined as follows:

Subclass I: Here $a_2=0$.
Subclass II: In this case a_2 is negative.
Subclass III: In this case a_2 is positive.

Table II. Approximate phase and magnitude errors and the phase compensation condition for all types of the active compensated VCVS structures. (It is assumed that $\omega_{t1} = \omega_{t2} = \omega_t$)

Non-inverting VCVS Circuit									
Class no.	Subclass no.	Type	Figure no.	Reference	Phase compensation conditions	DC gain	Approximate phase error $\phi = \arg.[\in(j\omega)]$	Approximate magnitude error $\gamma = \lvert\in(j\omega)\rvert - 1$	Frequency for which ϕ and γ are valid $\omega \ll$
uncompensated			1	1	—	$K+1$	$-(K+1)\frac{\omega}{\omega_t}$	$-\frac{1}{2}\left[(K+1)\frac{\omega}{\omega_t}\right]^2$	$\frac{\omega_t}{(K+1)}$
1	—	A	3(a)	9	$K_1 = K_2 = K$	$K+1$	$-\frac{1}{K^2}\left[(K+1)\frac{\omega}{\omega_t}\right]^3$	$\frac{1}{K}\left[(K+1)\frac{\omega}{\omega_t}\right]^2$	$\frac{K\omega_t}{(K+1)}$ for $K<1$ $\frac{\omega_t}{(K+1)}$ for $K>1$
	—	B	3(b)	—	$K_1 = K$ $K_2 = K_3 = K+1$ or $K_1 = K$, $K_2 = 1$ $K_3 = 2K+1$	$K+1$	$-(K+1)^2(K+2)\left(\frac{\omega}{\omega_t}\right)^3$ $-2(K+1)\left(\frac{\omega}{\omega_t}\right)^3$	$(K+1)(K+2)\left(\frac{\omega}{\omega_t}\right)^2$ $2(K+1)\left(\frac{\omega}{\omega_t}\right)^2$	$\frac{\omega_t}{(K+2)}$ $\frac{\omega_t}{2(K+1)}$
2	I	A	4(a) 4(b) 4(c) 4(d) 4(e)	6 10 — — 10	$K_1 = K_2 = K$ $K_2 = K$ $K_1 = K(K+2)$ $a = K+1$ $a = K(K+1)$	$K+1$	$-\left[(K+1)\frac{\omega}{\omega_t}\right]^3$	$\left[(K+1)\frac{\omega}{\omega_t}\right]^2$	$\frac{\omega_t}{(K+1)}$
		B	4(f)	—	$K_1 + 1 = \frac{K_2+1}{K_2^2}$	K_1K_2	$-\left[K_2(K_1+1)\frac{\omega}{\omega_t}\right]^3$	$\left[K_2(K_1+1)\frac{\omega}{\omega_t}\right]^2$	$\frac{\omega_t}{K_2(K_1+1)}$
	II	A	4(g)	11	$K_1 = K_2 = K$	$K+1$	$-(K+1)\left(\frac{\omega}{\omega_t}\right)^3$	$(K+1)\left(\frac{\omega}{\omega_t}\right)^2$	$\frac{\omega_t}{(K+1)}$
		B	4(h)	—	$K_2 = \frac{2K_1+1}{K_1(K_1-1)}$	$K_1 - \frac{1}{K_2}$ $(K_1K_2>1)$	$-\left[\frac{K_1+2}{2K_1+1}\right]\left[(K_1+1)\frac{\omega}{\omega_t}\right]^3$	$\left[\frac{K_1+2}{2K_1+1}\right]\left[(K_1+1)\frac{\omega}{\omega_t}\right]^2$	$\frac{\omega_t}{(K_1+1)}$
	III	A	4(i)	—	$K_1 = K(K+1)$ and $K_2 = K_3 = K$	$K+1$	$-(K+1)\left(K+1+\frac{1}{K}\right)^2\left(\frac{\omega}{\omega_t}\right)^3$	$(K+1)\left(K+1+\frac{1}{K}\right)\left(\frac{\omega}{\omega_t}\right)^2$	$\frac{\omega_t}{(K+1+1/K)}$
		B	4(j)	—	$(K_1+1)K_2$ $= \frac{K_1(K_2+1)}{K_1K_2+(K_2+1/K_3+1)}$	$K_1K_2 + \frac{K_2+1}{K_3+1}$	$-K_2(K_2+1)(K_1+1)^2\left(\frac{\omega}{\omega_t}\right)^3$	$(K_1+1)(K_2+1)\left(\frac{\omega}{\omega_t}\right)^2$	$\frac{\omega_t}{K_2(K_1+1)}$

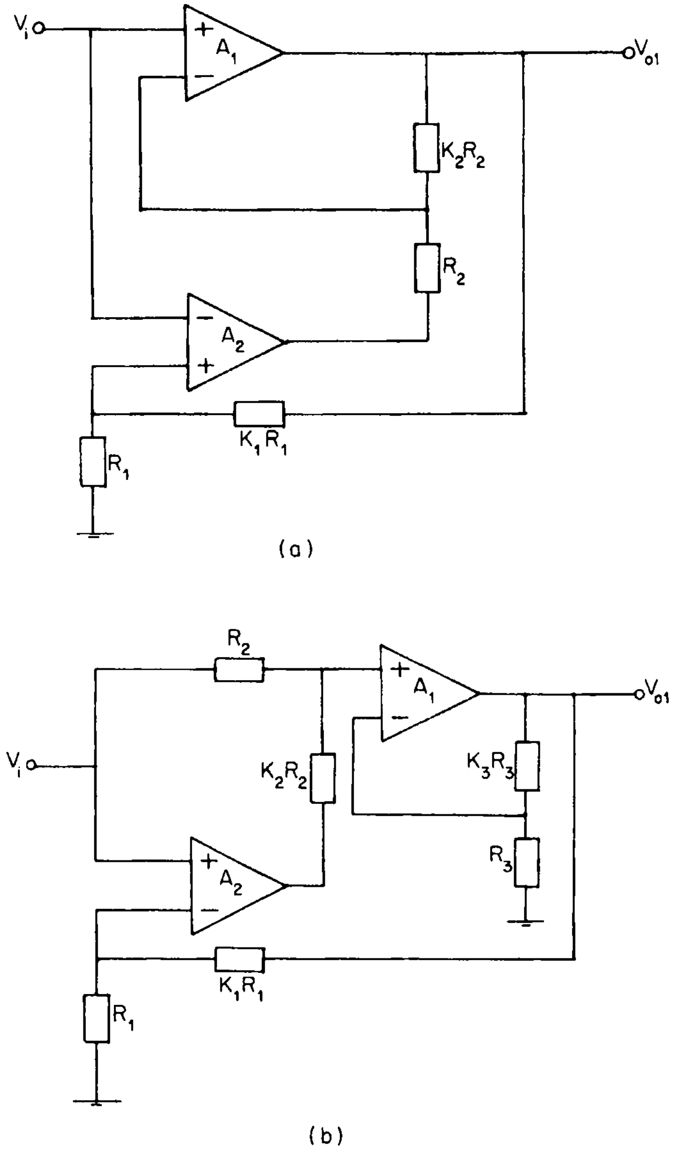


Figure 3. (a) Class 1, Type A amplifier[9]. (b) Class 1, Type B amplifier

For each of these subclasses two types of amplifiers may be defined. Type A VCVS in which b_{12} is negative and b_{21} is positive, and type B in which b_{12} is positive and b_{21} is negative. Note that the condition $(a_2b_{12}-a_1b_{22})>0$ must be satisfied in both Subclass II type B and Subclass III type A.

Several circuits may be introduced which belong to subclass I type A VCVS. Five of these circuits are shown in Figures 4(a) to 4(e). For a DC gain of 2, the realizable bandwidth, the phase and the magnitude errors obtained using this VCVS type are identical to those obtained using the VCVS of Figure 3(a). For a DC gain >2 (<2) the circuit of Figure 3(a) has smaller errors (larger errors) than the circuits of Figures 4(a) to 4(e).

As seen from Table III, the class 2 subclass I type A VCVS structure has the advantage of having a constant figure of merit (independent of K). The VCVS circuit of Figure 4(a) may be employed as a differential VCVS, whereas those of Figures 4(b) to 4(e) are not suitable by their nature for the three port

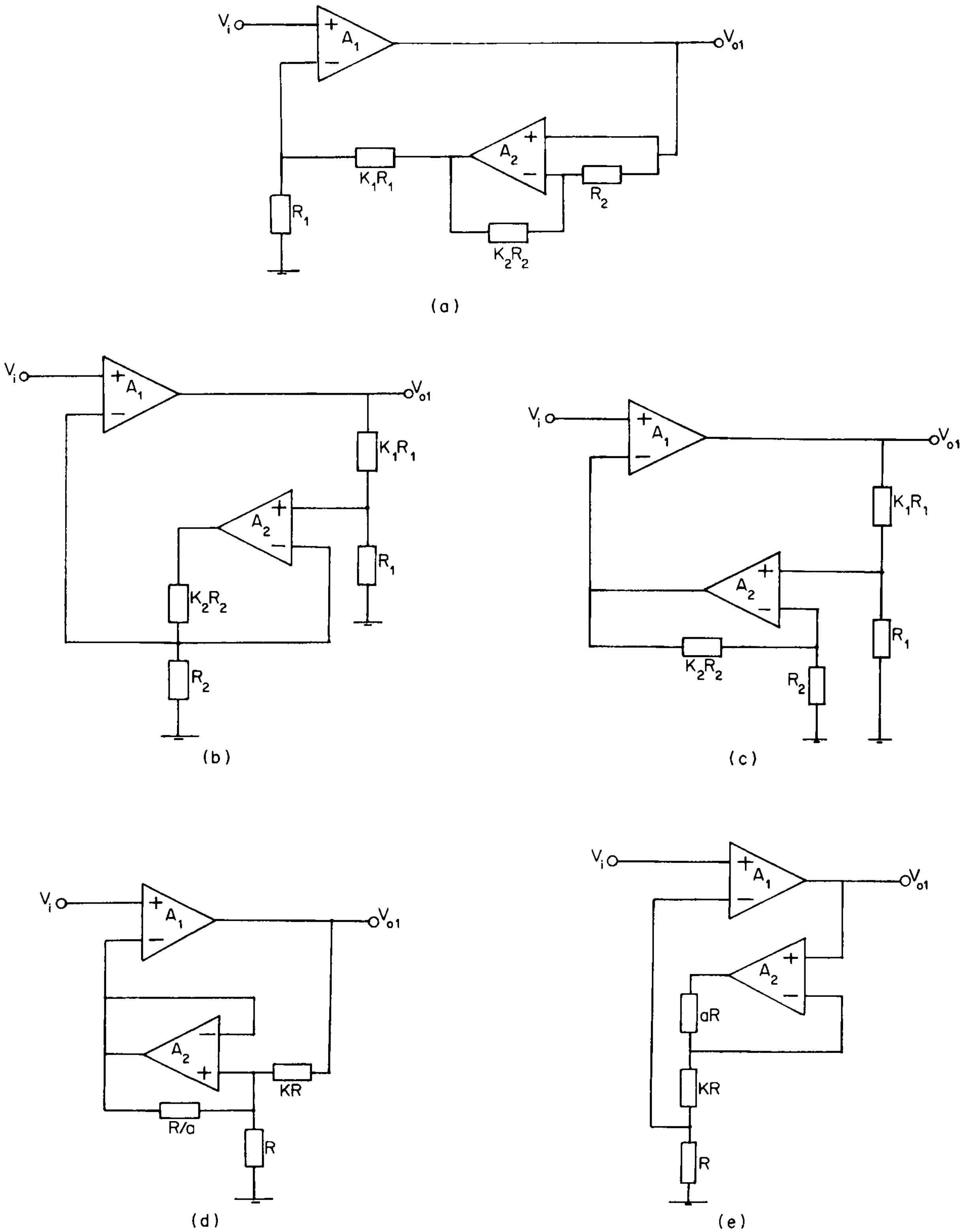
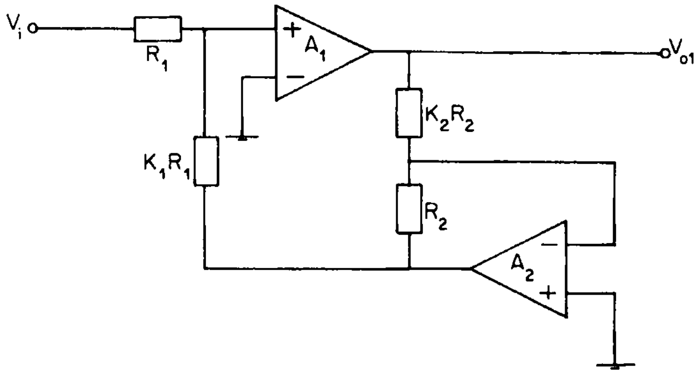
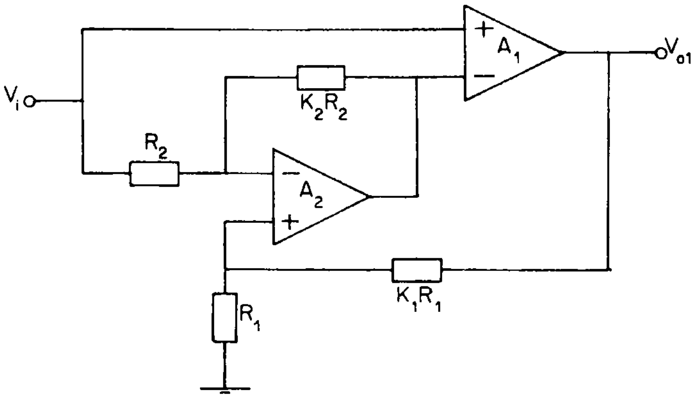


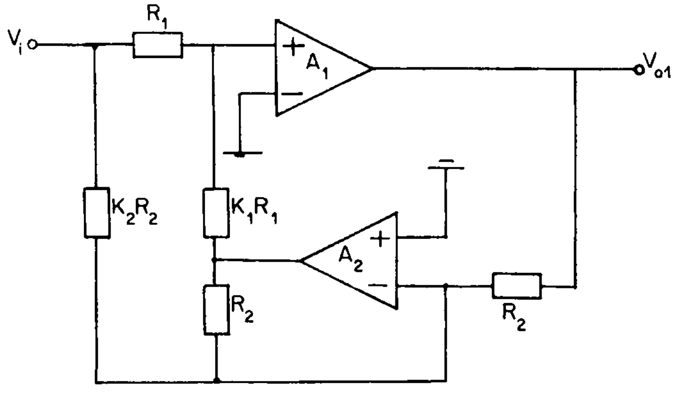
Figure 4(a) Class 2, Subclass I, Type A amplifier[6]. (b) Class 2, Subclass I, Type A amplifier[10]. (c) Class 2, Subclass I, Type A amplifier. (d) Class 2, Subclass I, Type A amplifier. (e) Class 2, Subclass I, Type A amplifier[10]



(f)

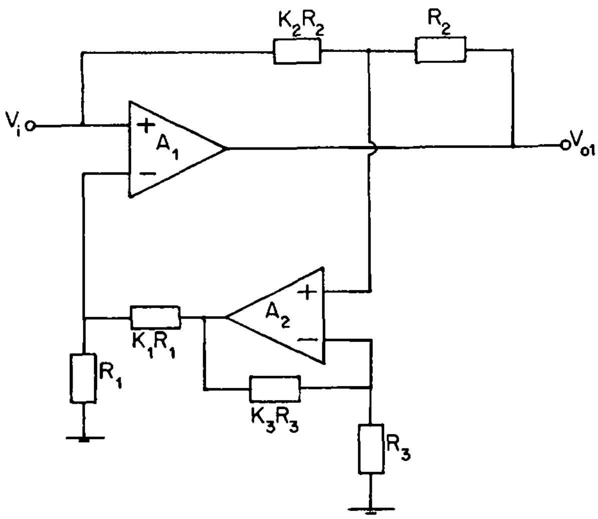


(g)

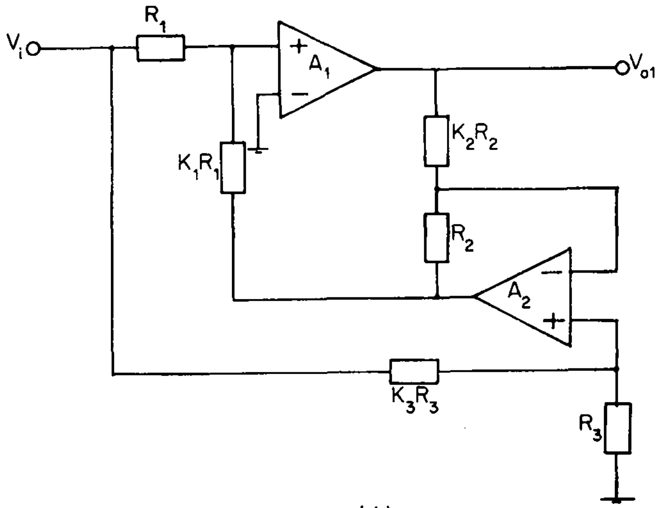


(h)

Figure 4 (continued). (f) Class 2, Subclass I, Type B amplifier. (g) Class 2, Subclass II, Type A amplifier[11]. (h) Class 2, Subclass II, Type B amplifier. (i) Class 2, Subclass III, Type A amplifier. (j) Class 2, Subclass III, Type B amplifier



(i)



(j)

mode of operation. More circuits[12] may be generated which belong to this type and are not included here to limit the length of the paper.

The VCVS circuits representing subclasses II and III are shown in Figure 4 and the results are summarized in Tables II and III. Among all these circuits it is seen that the VCVS of Figure[11] 4(g) has the smallest phase error and the largest figure of merit.

STABILITY ANALYSIS

The commercially available op-amps, that are compensated for a uniform 6 dB/octave gain rolloff, still have a second pole at $\omega_2(\omega_2 > \omega_t)$. Taking this second pole into account, the op-amp gain can be expressed as[16]

$$A(s) \simeq \frac{\omega_t}{s(1+s/\omega_2)} \tag{14}$$

Table III. Figure of merit and the stability condition for some of the VCVS structures. (It is assumed that $\omega_{t1} = \omega_{t2} = \omega_t$)

Non-inverting VCVS circuit						
Class no.	Subclass no.	Type	Figure no.	Reference	Figure of merit = NBWG = $\left[\frac{BW}{\omega_t}\right] \cdot [\text{DC gain}]$	Stability condition
1	—	A	3(a)	9	$\sqrt{\{K+\frac{1}{2}) + \sqrt{[2K^2 + K + \frac{1}{4}]}\}}$	$\frac{\omega_2}{\omega_t} > 2 - \frac{2 \cdot 5}{(K+1)}$
	—	*B*	3(*b*)	—	$\sqrt{\left\{\left(\frac{K}{2}+\frac{5}{8}\right) + \sqrt{\left[\frac{1}{2}K^2 + \frac{9}{8}K + \frac{41}{64}\right]}\right\}}$	$\frac{\omega_2}{\omega_t} > 2 - \frac{0 \cdot 25}{(K+1)}$
2	I	A	4(a) 4(b), (e) 4(c) 4(d)	6 10 — —	$\sqrt{\left(\frac{3+\sqrt{13}}{2}\right)} = 1 \cdot 817$	$\frac{\omega_2}{\omega_t} > \frac{1 \cdot 5}{(K+1)}$
	II	A	4(g)	11	$\sqrt{\{(K+\frac{3}{2}) + \sqrt{[2K^2 + 5K + \frac{13}{4}]}\}}$	$\frac{\omega_2}{\omega_t} > 2 - \frac{0 \cdot 5}{(K+1)}$

Substituting for $A(s)$ from equation (14) into equation (3) (assuming matched op-amps are used) and after routine stability analysis, it follows that a necessary condition for the stability of the VCVS is:

$$\frac{\omega_2}{\omega_t} > \frac{b_{ii}}{2} + \frac{2b_{12}b_{21}}{b_{ii}} \tag{15}$$

where $i = 1, 2$ for classes 1 and 2 respectively. The stability condition for some of the VCVS networks is included in Table III.

CONCLUSIONS

It has been shown that the active compensated non-inverting VCVS building blocks employing two op-amps may be classified into two major classes. Class 1: in which the coefficients of the s-terms in both the numerator and the denominator of the error function depend only on one of the two op-amps. Class 2: in which the coefficients of the s-terms depend on both op-amps, and this includes three subclasses. Each subclass includes two types, classified as types A and B. Several novel circuits are generated in this paper, and the results are summarized in Tables. Some of the circuits that are given here may also be considered as special cases from the generalized active *R* filters.[17–18] As far as this paper is concerned, however, the intention is in the active compensation of the non-inverting VCVS structures which employ two op-amps. Similar analysis for classifying the inverting VCVS structures is possible which will also lead to the generation of novel inverting VCVS structures.

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