Available online at www.sciencedirect.com
SCIENCE DIRECT®
Chaos, Solitons and Fractals 21 (2004) 553–561
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

MOS realization of the modified Lorenz chaotic system

A.G. Radwan *, A.M. Soliman, A. El-Sedeek

Faculty of Engineering, Cairo university, Cairo, Egypt

Accepted 24 February 2003

Abstract

A new chaotic oscillator circuit that realizes three attractors, the modified Lorenz system, Lorenz attractor "Butterfly attractor" and unsymmetrical modified Lorenz system [IEEE Trans. Circ. Syst. I 48 (2001) 289] is given in the paper. The general block diagram of this circuit is introduced based on g_m–C integrators. The overall circuit realization using MOS transistors and using low supply voltage is given. The proposed circuit depends on the use of grounded capacitors which provides the freedom to be off chip. A new block diagram called voltage controlled current direction is also introduced and its realization using MOS transistors is given. Numerical and PSpice simulations are also provided to confirm its functionality.
© 2004 Published by Elsevier Ltd.

1. Introduction

Recently, during the last few decades, chaos as a very interesting nonlinear phenomenon has been extensively studied within the scientific, engineering and mathematical communities. Chaos has great potential to be useful in many disciplines which include information processing, encryption, modulation, demodulation and biomedical engineering applications such as research of human brain and heart [2–4]. Realization of chaotic equations has been a powerful area of research due to the previously mentioned applications. The basic element of any chaotic application is the chaotic oscillator. Realization of the chaotic oscillator passed many stages, from the use of diode as in Chua's circuit [5], large block diagrams such as operational amplifier (op-amp) or current feedback op-amp (CFOA) as in [6,7] till reaching transistor level [8].

Chaotic equations are a very great area of research, many mathematicians tried to simplify the chaotic equations as much as they can in order to analyze the chaotic behavior and to study how these equations govern chaos or try to answer the basic question of what is the necessary and sufficient conditions for the differential equations to become chaotic? [9,10]

Lorenz from few decades proved that there is a chaotic behavior in our life from his study of the system of 12 differential equations that model a miniature atmosphere. He was able to transform them into only three differential equations which govern the same characteristics and same attractor which is called "Butterfly attractor" [11]. The main disadvantage of these equations appeared in its realization due to the presence of two multipliers which made it difficult. Modified Lorenz system was introduced in [1] which is represented by three differential equations with no multipliers. This system can capture the essential behavior of Lorenz attractor and can produce the "Butterfly effect", modified Lorenz and unsymmetrical Lorenz systems.

The chaotic oscillator circuit proposed in this paper is designed using MOS transistor level based on Mietec 0.5 micron technology and operates on low supply voltage ±1.5 V and small die area in order to satisfy the required conditions for portable devices. This circuit can realize three chaotic attractors which are the modified Lorenz system, Lorenz attractor "Butterfly effect" and unsymmetrical Lorenz system with very small modifications.

* Corresponding author.

0960-0779/$ - see front matter © 2004 Published by Elsevier Ltd.
doi:10.1016/S0960-0779(03)00077-8

This paper presents a general block diagram for the modified Lorenz system based on g_m–C integrator. The circuit realization of the block diagram is also presented and is based on the use of MOS transistors and three grounded capacitors for realization of the modified Lorenz chaotic system of equations [1] that is the minimum requirement for the implementation of a chaotic oscillator.

2. The modified Lorenz chaotic system

The matrix form which describes the differential equations of the modified Lorenz chaotic system [1] is given by:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} -a & a & 0 \\ 0 & 0 & -K \\ K & 0 & -d \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} 0 \\ bK \\ 0 \end{bmatrix} \tag{1a}$$

where

$$K = \begin{Bmatrix} 1 & X \geqslant 0 \\ -1 & X < 0 \end{Bmatrix} \tag{1b}$$

The values of the chaotic equations parameters a, b and d are the key of the system.

3. Dimension form of the modified Lorenz system

In the pure mathematical domain, equations are dimensionless. But the case is different in the electronic domain, where finding an implementation of an equation requires the use of quantities having dimensions. Hence, the signals X, Y, Z and the dimensionless time τ can be transformed into voltage signals V_X, V_Y, V_Z and time t with dimension [s] through the following relations:

$$X = \frac{V_X}{V_R}, \quad Y = \frac{V_Y}{V_R}, \quad Z = \frac{V_Z}{V_R}, \quad \frac{d}{d\tau} = \frac{c}{g}\frac{d}{dt} \tag{2}$$

where V_R is the reference voltage, g is a transconductance and C is a capacitor.

Applying the above transformations, the resulting voltage mode differential equations in the matrix form become:

$$\begin{bmatrix} \dot{V}_X \\ \dot{V}_Y \\ \dot{V}_Z \end{bmatrix} = \left(\frac{g}{C}\right) \left\{ \begin{bmatrix} -a & a & 0 \\ 0 & 0 & -K \\ K & 0 & -d \end{bmatrix} \begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} + \begin{bmatrix} 0 \\ bV_R K \\ 0 \end{bmatrix} \right\} \tag{3a}$$

where

$$K = \begin{cases} 1 & V_X \geqslant 0 \\ -1 & V_X < 0 \end{cases} \tag{3b}$$

The system described in Eqs. (3a) and (3b) represents the dimension form of the modified Lorenz system which will be transformed into electronic circuit elements for realization.

4. Current integrator

Due to the simplicity of current mode techniques, and the progress in the design of transconductors (g_m), the general block diagram of the chaotic oscillator will be presented based on transconductors and this imposed the need for the use of current integrators in the design of the circuit. Therefore the realizations of such circuits are based on the use of g_m–C integrators. Due to the presence of three differential equations, there is a need for three current integrators as shown in Fig. 1, where

$$\dot{V}_X = \frac{1}{C_1} I_1, \quad I_X = g_{m1} V_X \tag{4a}$$

$$\dot{V}_Y = \frac{1}{C_2} I_2, \quad I_Y = g_{m2} V_Y \tag{4b}$$

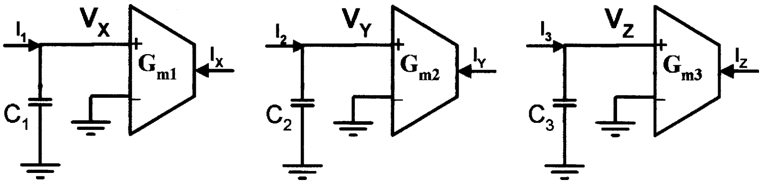

Fig. 1. Three current integrators.

$$\dot{V}_Z = \frac{1}{C_3} I_3, \quad I_Z = g_{m3} V_Z \tag{4c}$$

Taking $g_{m1} = g_{m2} = g_{m3} = g_m$, $C_1 = C/a$ and $C_2 = C_3 = C$. Comparing Eqs. (3) and (4), the following relations are obtained:

$$I_1 = (-I_X + I_Y) \tag{5a}$$

$$I_2 = -KI_Z + KI_R \tag{5b}$$

$$I_3 = KI_X - dI_Z \tag{5c}$$

where $I_R = bg_m V_R$ and K is described by Eq. (3b). The above equations indicate the relationship between the input current for each current integrator with respect to the output currents from the three current integrators. The modified Lorenz chaotic system will be obtained based on the practical realization of the above equations.

Despite that the previous relations are very simple, the existence of the sign term K results in some problems for the realization. A new block diagram taking the effect of the parameter K into consideration and defined as the voltage controlled current direction will be introduced in Section 5.

5. MOS realization of voltage controlled current direction (VCCD)

The VCCD block diagram can be considered as a transconductor cell followed by a direction stage depending on the sign of V_X. The block diagram and the graphical relations are shown in Fig. 2(a). The functionality of the VCCD can be described by the following relation:

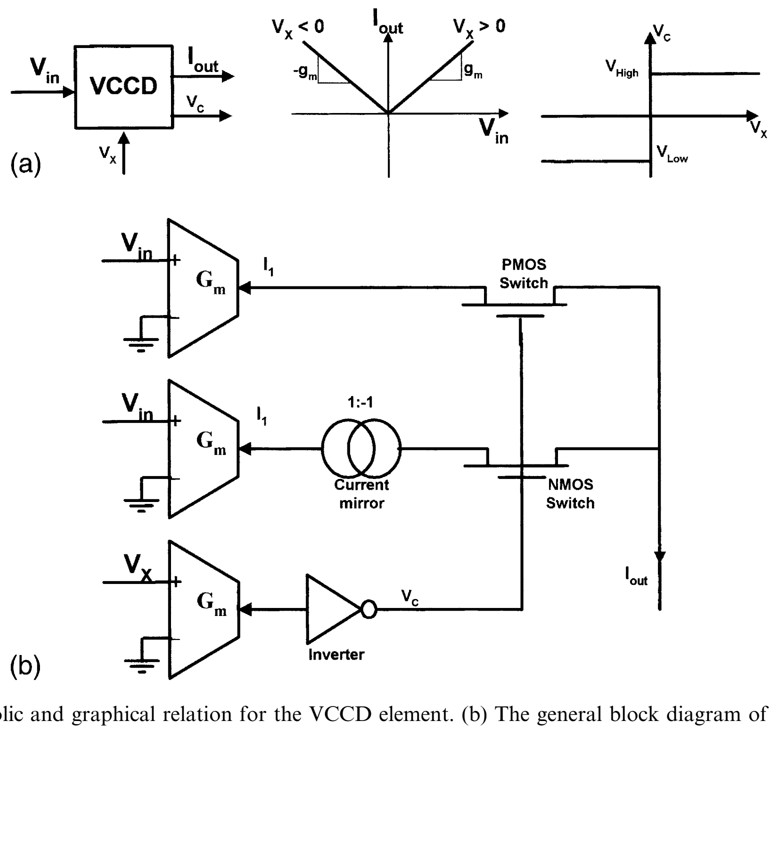

Fig. 2. (a) Symbolic and graphical relation for the VCCD element. (b) The general block diagram of the VCCD circuit.

$$I_{\text{out}} = \begin{Bmatrix} g_m V_{\text{in}} & V_X \geqslant 0 \\ -g_m V_{\text{in}} & V_X < 0 \end{Bmatrix} \tag{6}$$

Special case: when $V_{\text{in}} = V_X$, then the VCCD will be transformed into an absolute transconductor and Eq. (6) will be changed into:

$$I_{\text{out}} = g_m|V_X| \tag{7}$$

The general block diagram of the VCCD based on the transconductor is shown in Fig. 2(b) showing three g_m, current mirror, inverter and two switches. The inverter output V_C is the controlling signal that controls both of NMOS and PMOS switches, the relation between V_C and V_X is given by:

$$V_C = \begin{Bmatrix} \text{High, NMOS ON} & V_X > 0 \\ \text{Low, PMOS ON} & V_X < 0 \end{Bmatrix} \tag{8}$$

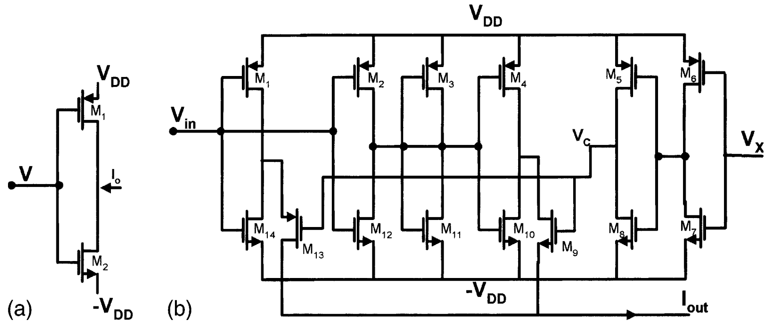

Fig. 3. (a) The typical transconductor circuit. (b) The MOS realization of the VCCD circuit.

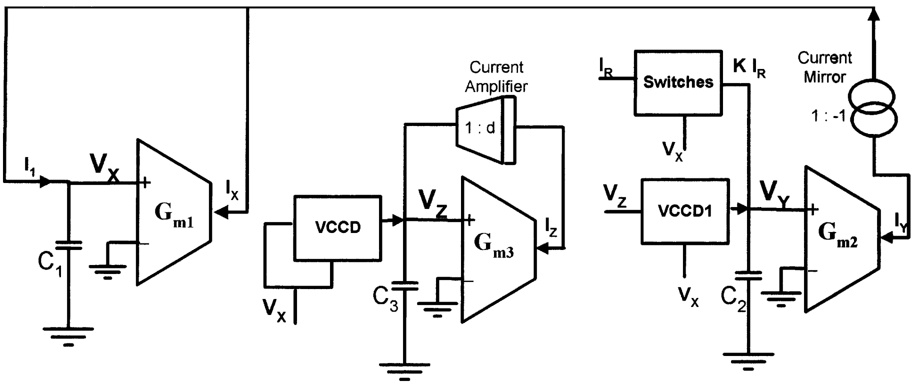

Fig. 4. The general block diagram of the modified Lorenz chaotic circuit.

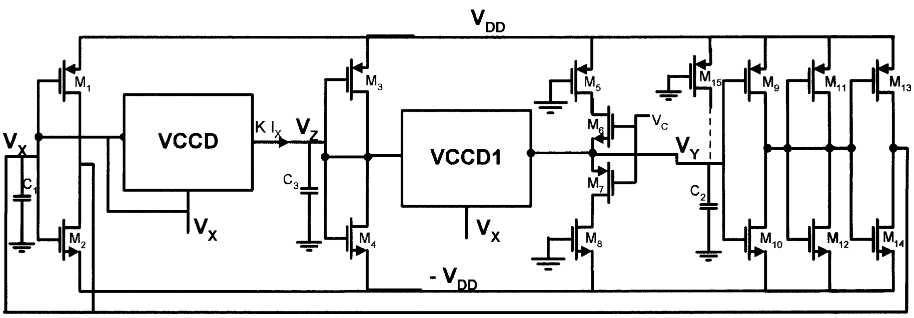

Fig. 5. The MOS realization of the modified Lorenz chaotic circuit.

If we interchange the positions of NMOS and PMOS, naming the final block as VCCD1 whose final relation is given by:

$$I_{\text{out}} = \left\{ \begin{array}{ll} -g_m V_{in} & V_X \geqslant 0 \\ g_m V_{in} & V_X < 0 \end{array} \right\} \tag{9}$$

The simple transconductor using two transistors [12] is chosen as the basis for any transconductor in the paper. The typical transconductor is simply an inverter as shown in Fig. 3(a), the relation between I_0 and V is given by:

$$I_0 = g_m V, \quad g_m = 2K_1(V_{DD} - V_T) \tag{10}$$

where $K_1 = k(W/L)$, k is the process conductance parameter, (W/L) is the transistor aspect ratio and V_T is the threshold voltage ($V_T = V_{Tn} \approx |V_{T_p}|$). The VCCD realization is shown in Fig. 3(b).

6. MOS realization of the modified Lorenz system

The relations in Eq. (5) are now simple to construct using the VCCD block diagram. The general block diagram of the modified Lorenz system is given in Fig. 4 showing three transconductors, two current mirrors, VCCD, VCCD1, current amplifier, some switches and three grounded capacitors. It is clear that the use of VCCD simplifies the general block diagram.

Table 1
Transistor aspect ratios of the VCCD circuit

Transistor	Aspect ratio (W/L) (μm/μm)
M_2, M_3, M_4, M_5, M_6, M_7	30/15
M_7, M_8, M_{10}, M_{12}, M_{14}	30/65.5
M_9, M_{13}	20/2

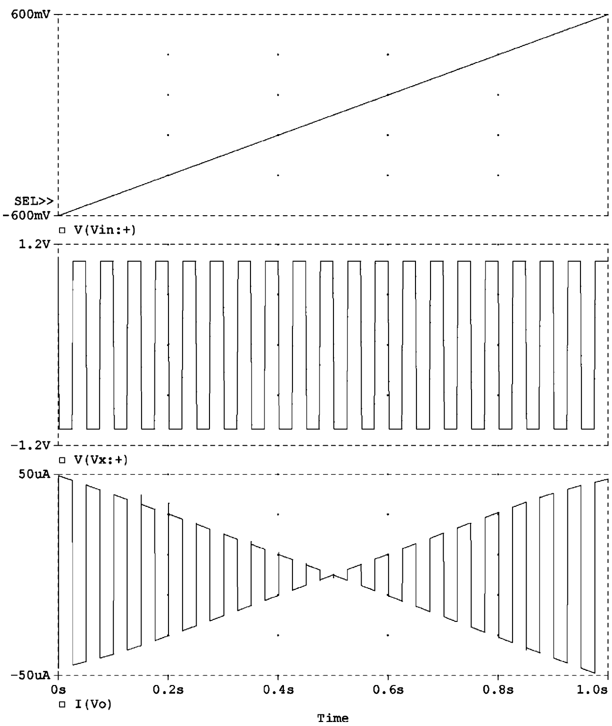

Fig. 6. PSpice results for the VCCD when V_{in} (ramp) and V_X (square wave).

The complete circuit realization is shown in Fig. 5 which consists of MOS transistors and three capacitors. The constant current I_R is obtained from M_5 or M_8. The transistors M_6 and M_7 represent the switching elements on which depends the decision of V_c.

Table 2
Transistor aspect ratios of the modified Lorenz chaotic circuit

Transistor	Aspect ratio (W/L) (μm/μm)
M_1, M_9, M_{11}, M_{11}, M_{13}	30/15
M_2, M_{10}, M_{12}, M_{14}	30/65.5
M_6, M_7	20/2
M_3	4.5/15
M_4	4.5/65.5
M_5	24/15
M_8	24/65.5
M_{15}	3/15

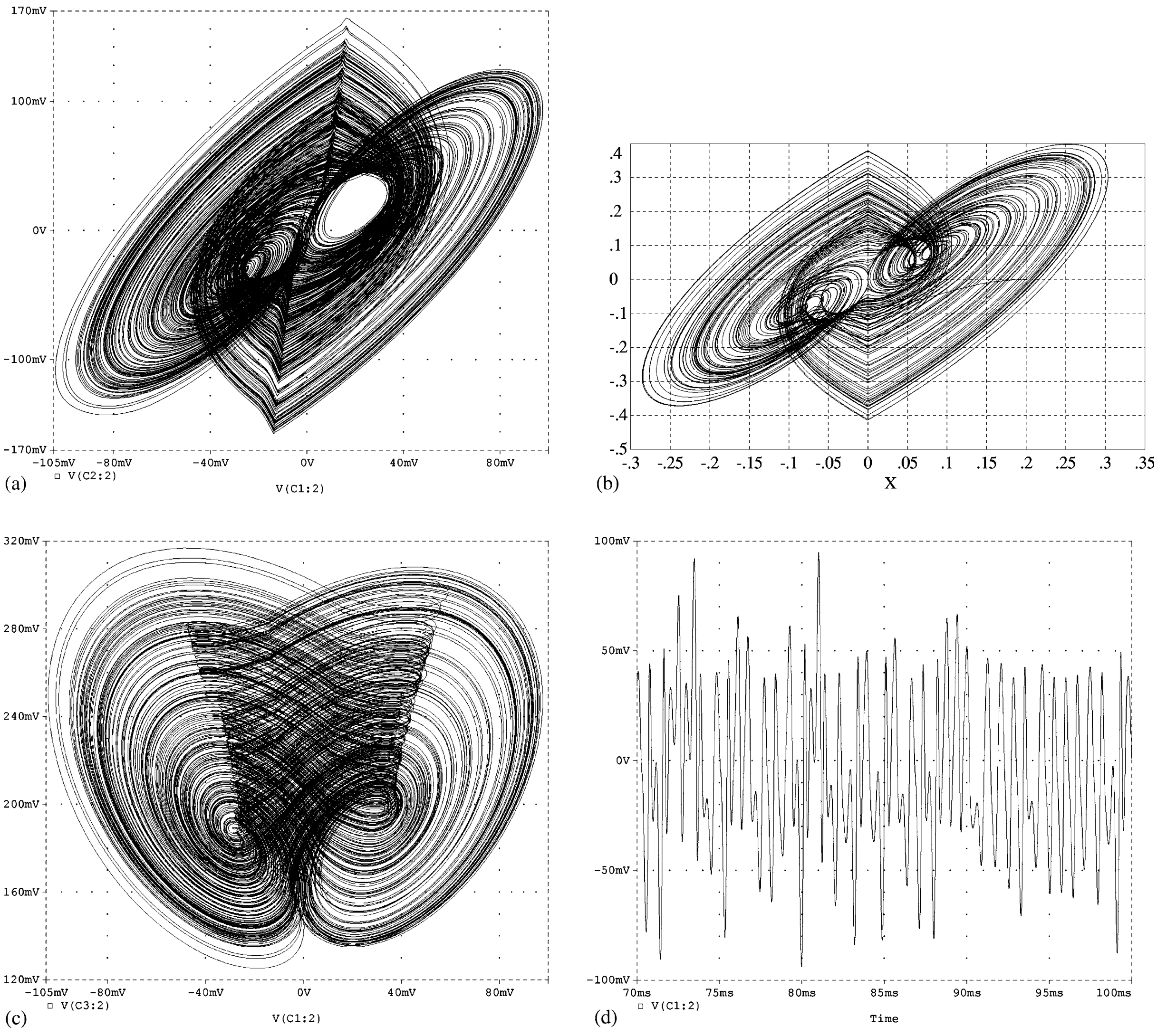

Fig. 7. (a) V_X–V_Y projection of PSpice simulation for the modified Lorenz. (b) X–Y numerical projection for the modified Lorenz. (c) V_X–V_Z projection of PSpice simulation for the modified Lorenz. (d) V_X time waveform of PSpice simulation for the modified Lorenz.

7. Simulation results

The proposed circuits were simulated using PSpice simulation and using the model of Mietec 0.5 micron technology. Also the numerical analysis has been carried out using the Runge–Kutta fourth order with step size of 0.005 [1].

(I) *VCCD circuit*: The transistor aspect ratios of the VCCD circuit shown in Fig. 3(b) are listed in Table 1. The output current I_{out} when the input voltage increases linearly from −0.6 to 0.6 V and V_X is a square waveform are shown in Fig. (6).

(II) *Modified Lorenz system*: PSpice simulation results are used for circuit simulation with $C_1 = 10$ nF and $C_2 = C_3 = 6$ nF. The value of parameters a, b and d are changed as shown in the following three cases.

Table 3
Transistor aspect ratio changes for Butterfly attractor

Transistor	Aspect ratio (W/L) (μm/μm)
M_3	13.5/15
M_5	19/15
M_4	13.5/65.5
M_6	19/65.5

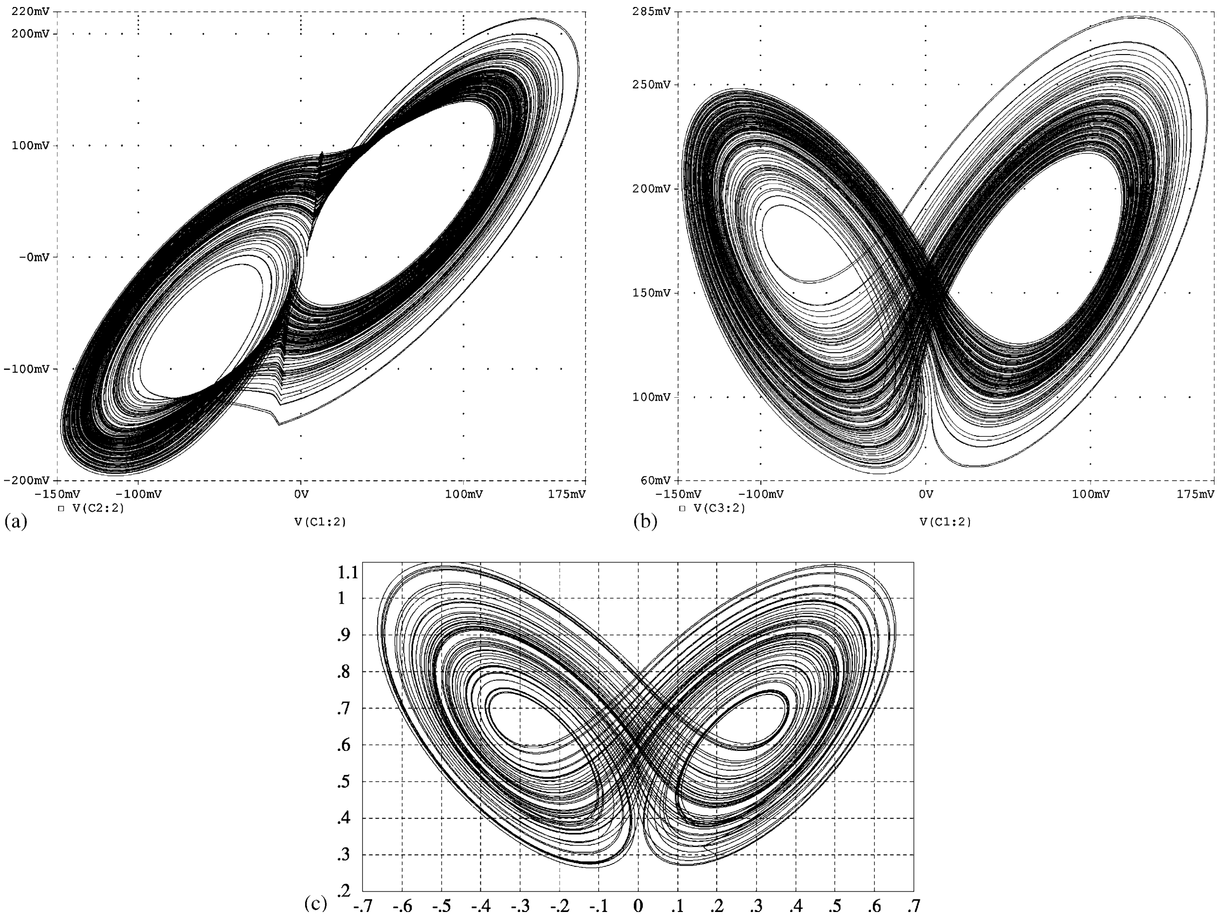

Fig. 8. (a) V_X–V_Y projection of PSpice simulation for the Butterfly attractor. (b) V_X–V_Z projection of PSpice simulation for the Butterfly attractor. (c) X–Z numerical projection for the Butterfly attractor.

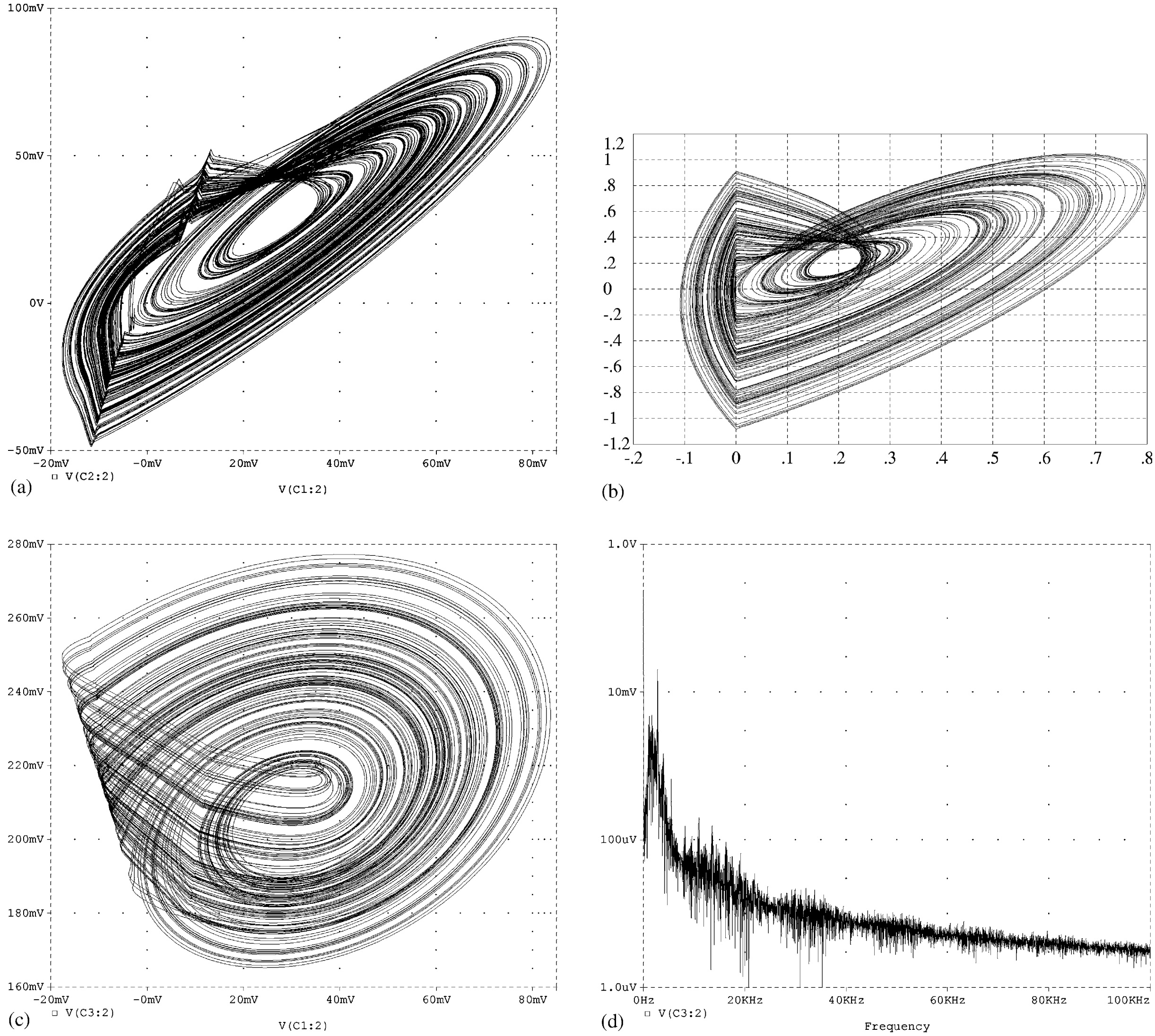

Fig. 9. (a) V_X–V_Y projection of PSpice simulation for unsymmetric Lorenz. (b) X–Y numerical projection for unsymmetric Lorenz. (c) V_X–V_Z projection of PSpice simulation for unsymmetric Lorenz. (d) PSpice frequency response of V_z for unsymmetric Lorenz.

(a) *Modified Lorenz system*: This type can be achieved by putting $a = b = 0.6$ and $d = 0.15$ [1]. The transistor aspect ratios are stated in Table 2. The V_X–V_Y projection for the circuit realization and X–Y numerical results are shown in Fig. 7(a) and (b) respectively. The V_X–V_Z projection for the circuit realization is shown in Fig. 7(c). Time waveform of the V_X signal is shown in Fig. 7(d).

(b) *Lorenz attractor (Butterfly attractor)*: This Butterfly attractor can be obtained by putting $a = b = 0.6$ and $c = 0.45$ [1]. The modified transistor aspect ratios are listed in Table 3. The V_X–V_Y projection for the circuit realization is shown in Fig. 8(a). The V_X–V_Z projection for the circuit realization and X–Z numerical results are shown in Fig. 8(b) and (c) respectively.

(c) *Unsymmetric modified Lorenz system*: Further modification can be done by adding a constant m to the equation $\dot{Y}$. This would give rise to one half only of the Lorenz attractor [1] (one wing of the Butterfly attractor). When introducing $m = 1$ in the numerical simulation which is equivalent to connecting the transistor M_{15} to the Y terminal (which is dotted in Fig. 5) on the circuit implementation level, the following simulation results were obtained using $a = b = 0.6$ and $c = 0.15$ [1]. The transistor aspect ratios are stated in Table 2 taking into consideration the value of M_{15}. V_X–V_Y projection for the circuit realization and X–Y numerical results are shown in Fig. 9(a) and (b) respectively. V_X–V_Z projection for the circuit realization is shown in Fig. 9(c). The frequency response of V_X is shown in Fig. 9(d). The frequency response can be easily scaled by changing the values of g_m or C or both of them.

8. Conclusion

This paper presents the general block diagram of the modified Lorenz chaotic equations using the typical transconductor that gives the same results in the numerical simulation. The presented circuit can capture three kinds of attractors through small changes in the circuit, the modified Lorenz system, Butterfly attractor and unsymmetrical Lorenz system. The circuit realization is given using MOS transistor level and operates on low supply voltage of ± 1.5 V with the use of three grounded capacitors which is the best requirement for any chaotic system suitable for portable devices. This paper also introduces a new building block called the voltage controlled current direction showing its functionality, general block diagram and overall circuit realization that simplify the synthesis of the chaotic circuit.

References

[1] Elwakil AS, Kennedy MP. Construction of classes of circuit-independent chaotic oscillators using passive-only nonlinear devices. IEEE Trans Circ Syst I 2001;48:289.
[2] Carroll TL. Noise-robus synchronized chaotic communication. IEEE Trans Circ Syst I 2001;48:1519.
[3] Williams C. Chaotic communication over radio channels. IEEE Trans Circ Syst I 2001;48:1394.
[4] Dachselt F, Schwartz W. Chaos and cryptography. IEEE Trans Circ Syst I 2001;48:1498.
[5] Matsumoto T, Chua LO, Komuro GM. The double scroll. IEEE Trans Circ Syst I 1985;32:798.
[6] Elwakil AS, Soliman AM. A family of Wien-type oscillators modified for chaos. Int J Circ Theory Appl 1997;25:561.
[7] Elwakil AS, Kennedy MP. Chaotic oscillator configuration using a frequency dependent negative resistor. J Circ Syst Comp 1999;9:229.
[8] Gonzales OA, Han G, De Gyvez J, Sanchez-Sinencio E. Lorenz-based chaotic cryptosystem: a monolithic implementation. IEEE Trans Circ Syst I 2000;47:1243.
[9] Sprott JC. Simple chaotic systems and circuits. Am J Phys 2000;68:758.
[10] Sprott JC. Algebraically simple chaotic flows. Int J Chaos Theory Appl 2000;5:2000.
[11] Lorenz EN. Deterministic non-periodic flow. J Atmos Sci 1963;20:130.
[12] Sedra AS, Smith KC. Microelectronic circuits. Oxford University Press, Inc; 1998.