shown in Fig. 2*a*. Analysis now yields the input impedance

$$Z_{in} = R_1 + j2\omega CR_1R_1 \tag{8}$$

Thus with R_1 fixed, adjusting R_2 large value frequency independent inductances with high Q are achieved. Figs. 2*b* and *c* are two more versions that simulate the same RL impedance as in eqn. 8. The structure in Fig. 2*c* uses a grounded capacitance and hence is particularly attractive in view of its microcircuit fabrication.[7] It may be shown that all these active inductances possess very low active sensitivities as in eqns. 5, 6 and 7 above.

Application in bandpass filter realisation: Finally as an application of the simulated grounded inductances, consider the simple bandpass transfer function realisation as shown in Fig. 3. The voltage function is

$$\frac{V_0}{V_i} = \pm \frac{R_0}{\left[Z_{in} + \frac{1}{sC_0}\right]} \tag{9}$$

If $Z_{in} = r + sL$, then a bandpass filter results

$$\frac{V_0}{V_i} = \pm \frac{sC_0R_0}{s^2LC_0 + sC_0r + 1} \tag{10}$$

It may additionally be noted that if ports x and z are interchanged in Fig. 3, a band-reject filter results.

Conclusion: Using the concepts of the CC II, simulation of a moderate-Q grounded inductance is proposed. Circuit techniques next are described for realising frequency independent high-Q inductances with independent L and Q_L controllability. The inductances are shown to be sufficiently stable relative to changes in active parameters. Finally as a simple application, these are used to realise a bandpass transfer function.

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PHASE CORRECTION IN TWO-INTEGRATOR LOOP FILTERS USING A SINGLE COMPENSATING RESISTOR

Indexing terms: Active filters, Compensation, Integrating circuits

A method of phase correction in two-integrator loop filters is described in which a single compensating resistor is added in series with one of the two capacitors necessary for implementing the second-order filter section. This method enables perfect phase compensation. Experimental results are presented.

There exist a number of biquadratic filter sections based on the 2-integrator loop or the analogue-computer method for synthesising a complex-pole pair. A major circuit belonging to this class is the Tow-Thomas biquad[1] of Fig. 1*a* which is a straightforward implementation of the 2-integrator loop principle. Although the circuit exhibits very low sensitivities with respect to the passive components, it suffers from a rather drastic Q-factor enhancement effect due to the op-amp's limited unity-gain bandwidth. The Q-factor enhancement is due to the parasitic phase shift around the loop which is cumulative and lagging. Thomas[1] has shown that the total phase lag ϕ_T at resonance around the loop is the negative reciprocal of the filter Q-factor, i.e.

$$\phi_T \approx -\frac{1}{Q_p} = -\frac{1}{Q_{p_0}} + \frac{4\omega_{p_0}}{\omega_t} \tag{1}$$

where Q_p stands for the actual pole-Q, Q_{p_0} for the ideal pole-Q, ω_{p_0} for the ideal pole frequency and ω_t for the op-amp unity-gain bandwidth. From the above expression it is seen that the term $4\omega_{p_0}/\omega_t$ represents the unwanted phase lag contributed by the finite ω_t.

Thomas[1] has used the conventional concept of adding a compensating capacitor in parallel with one of the loop resistances to produce an amount of phase lead by which phase correction around the loop is done. However, it can be shown that if the second integrator K_2 of Fig. 1*a* is used in one of the compensated forms of Fig. 2, its closed-loop transfer function becomes

$$\frac{V_2}{V_1} = -\frac{1}{sCR}\,\frac{1 + s\tau_c}{1 + \frac{1}{\tau\omega_t} + \frac{s}{\omega_t}\left(1 + \frac{\tau_c}{\tau}\right)} \tag{2}$$

where $\tau = CR$ and $\tau_c = R_cC = C_cR$.

This means that the compensated integrators of Fig. 1*b* can be looked upon as integrator blocks having a controlled

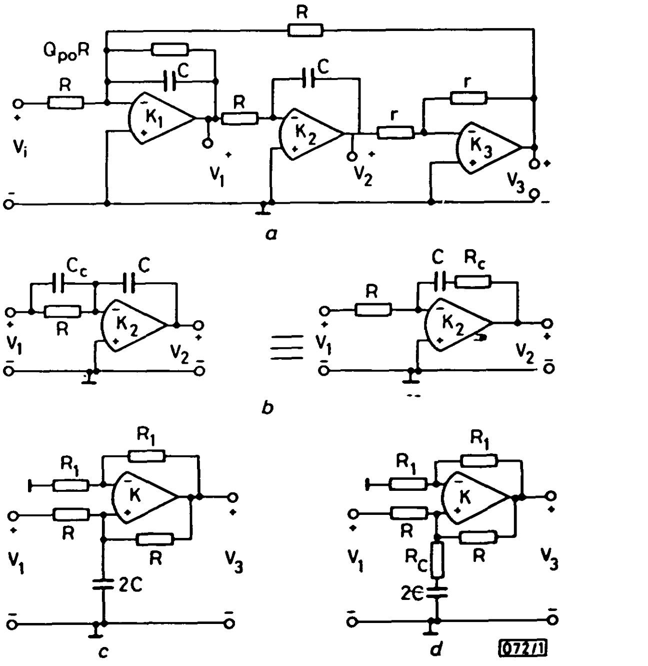

Fig. 1 *Circuit diagrams*

a Tow-Thomas biquad
b Two equivalent compensated inverting integrators
c Deboo noninverting integrator
d Compensated Deboo integrator

amount of phase lead, i.e. the phase shift in these circuits can be made to deviate from the ideal value of $\pi/2$ by a considerable positive amount. Therefore, selecting τ_c as $4/\omega_t$ will result in phase correction around the loop.

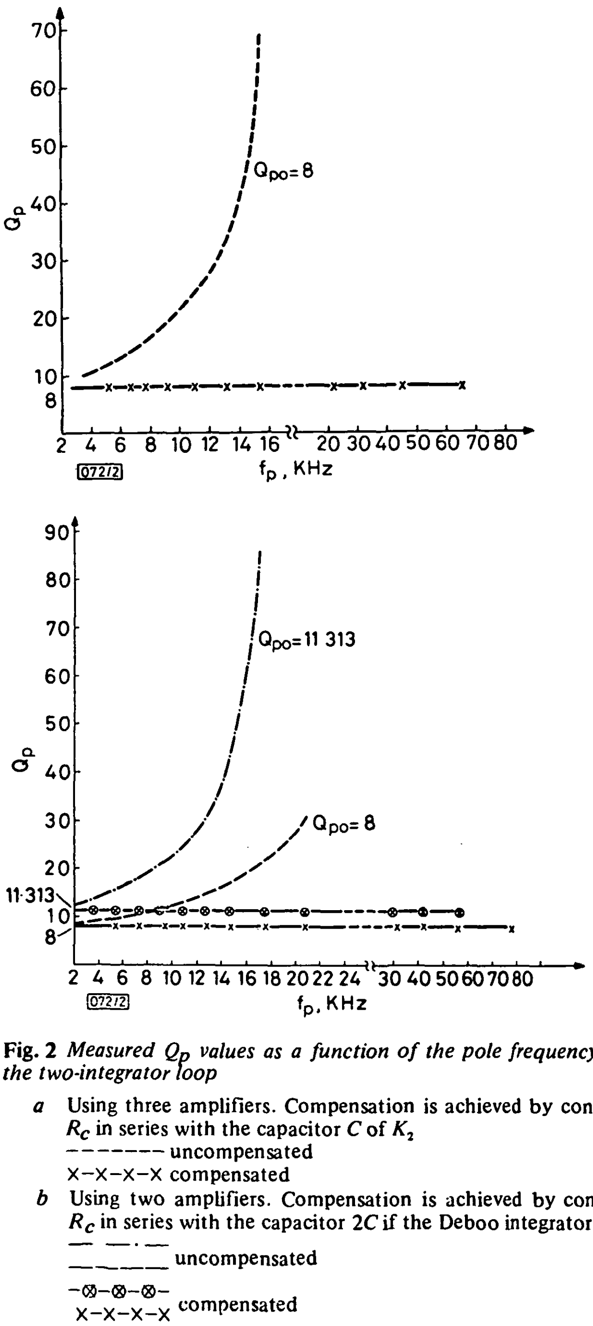

Fig. 2 *Measured Q_p values as a function of the pole frequency f_p for the two-integrator loop*

a Using three amplifiers. Compensation is achieved by connecting R_C in series with the capacitor C of K_2
——— uncompensated
x–x–x–x compensated

b Using two amplifiers. Compensation is achieved by connecting R_C in series with the capacitor $2C$ if the Deboo integrator
— —·— uncompensated
–⊗–⊗–⊗– x–x–x–x compensated

The well-known Deboo integrator of Fig. 1*c* can be used to realise the noninverting integrator realised in Fig. 1*a* by cascading an inverter K_3 to the second integrator K_2. This will save one op-amp at the expense of increased sensitivity to passive components.[1] It can be shown that the total parasitic phase lag around the loop in this case is equal to $3\omega_{p_0}/\omega_t$. If the Deboo integrator is used in the modified form of Fig. 1*d*, in which a compensating resistor is connected in series with the integrator capacitor and selected as $3/(2C\omega_t)$, phase correction around the loop can be done.

On the other hand, it is found that the addition of a compensating resistor R_c in series with the feedback capacitor C of the first integrator K_1 of Fig. 1*a* will result also in phase correction around the loop. The compensating resistor is selected in this case as $K/(C\omega_t)$, where K is either 4 (if K_2 is cascaded by an inverter to realise a noninverting integrator) or 3 (if the Deboo integrator is used). For all cases discussed above, the compensating component design value usually does not result in perfect phase correction. This may be due to the fact that the amplifier ω_t is not accurately known. However, resistors have generally better trimming properties than capacitors. So, trimming a compensating resistor, about its design value, will result in a perfect phase correction.

Figs. 2*a* and 2*b* show the measured pole-Q of circuits (compensated and uncompensated) realised with op-amps type μA 741 (National Semiconductors), having f_t = 500 kHz.

In conclusion, it can be said that the addition of a single compensating resistor in series with one of the two biquad capacitors according to a certain design constraint will result in perfect compensation for the unwanted phase shift produced by the real op-amps used.

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INTEGRATION-FREE ALGORITHMS FOR FIXED END-POINT REGULATOR PROBLEMS

Indexing terms: Control system synthesis, Numerical methods

A new formulation is suggested for the fixed end-point regulator problem, which, in conjunction with the recently developed integration-free algorithms, provides an efficient means of obtaining numerical solutions to such problems.

Introduction: In a recent paper, Bierman and Sidhu[1] detail an integration-free doubling algorithm for obtaining solutions to the Riccati equations that arise in estimation and control problems. In this paper, we use a dual version of this algorithm together with a new formulation of the fixed end-point problem[2,3] to derive efficient computational algorithms. An added advantage of the formulation is the manner in which it allows us to handle different sets of terminal constraints with minimal increase in computational complexity.

The fixed end-point problem: Consider the dynamical system and the performance index defined by

$$\dot{x}(t) = A(t)x(t) + B(t)u(t); x(t_0) = x_0 \quad (1)$$

$$J = \tfrac{1}{2}x'(T)Fx(T) + \tfrac{1}{2}\int_{t_0}^{T} [x'(t)Q(t)x(t) + u'(t)R(t)u(t)]\,dt \quad (2)$$

with the entries of $A(t), B(t)$ continuous for all t in the interval $[t_0, T]$. $Q(t)$ and $R(t)$ are continuous, symmetric, and nonnegative-definite and positive-definite matrices, respectively. F is a nonnegative definite, symmetric matrix. The fixed end-point problem seeks to determine the control law $u(t)$ which minimises expr. 2, subject to the constraint that a linear function of the terminal states should satisfy

$$Zx(T) = \beta; \;\; \beta \text{ is a } (q \times 1) \text{ vector} \quad (3)$$

We solve this problem by first reformulating it as an extended regulator problem by considering the Lagrange multiplier vector w defined below in eqn. 4 as an unknown, constant parameter of the extended state. This approach leads to a different, and more straightforward method of solution than that presented in Bryson and Ho,[2] and appears to be the dual version of that suggested by Zachrisson[6] for fixed-point